

Modified Linear Theory for Spinning or Non-Spinning Projectiles

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Abstract: Static and dynamic stability are the most important phenomena for stable flight atmospheric motion of spin and fin stabilized projectiles. If the aerodynamic forces and moments and the initial conditions are accurately known, an essentially exact simulation of the projectile's synthesized pitching and yawing motion can be readily obtained by numerical methods. A modified trajectory linear theory of the same problem implies an approximate solution.

INTRODUCTION

More than 80 years ago, English ballisticians [1] constructed the first rigid six-degree-of-freedom projectile exterior ballistic model. Their model contained a reasonably complete aerodynamic force and moment expansion for a spinning shell and included aerodynamic damping along with Magnus force and moment. Guided by an extensive set of yaw card firings, these researchers also created the first approximate analytic solution of the six-degree-of-freedom projectile equations of motion by introducing a set of simplifications based on clever linearization by artificially separating the dynamic equations into uncoupled groups. The resulting theory is commonly called projectile linear theory. Kent [2], Neilson and Syngé [3], Kelley and McShane [4], and Kelley *et al.* [5] made refinements and improvements to projectile linear theory.

Projectile linear theory has proved an invaluable tool in understanding basic dynamic characteristics of projectiles in atmospheric flight, for establishing stability criteria for fin- and spin-stabilized projectiles, and for extracting projectile aerodynamic loads from spark range data.

In the present work, the full six degrees of freedom (6-DOF) projectile flight dynamics atmospheric model is considered for the accurate prediction of short and long range trajectories of high spin and fin-stabilized projectiles. It takes into consideration the influence of the most significant forces and moments, in addition to gravity.

Projectiles, which are inherently aerodynamically unstable, can be stabilized with spin. For this condition, the spin rate must be high enough to develop a gyroscopic moment, which overcomes the aerodynamic instability, and the projectile is said to be gyroscopically stable. This is the case for the most gun launched projectiles (handguns, rifles, cannons,

etc.) where the rifling of the barrel provides the required axial spin to projectile. In describing this condition, a gyroscopic stability factor can be applied, which is obtained from the roots of the modified linear theory in the equations of projectile motion.

Also, dynamic stability is defined as the condition where a system is perturbed and the ensuing oscillatory has a tendency to either decrease or increase. Note that this definition assumes that the static stability is present, otherwise the oscillatory motion would not occur.

PROJECTILE MODEL

The present analysis considers two different types of representative projectiles: a spin-stabilized of 105mm and a mortar fin-stabilized of 120 mm.

Basic physical and geometrical characteristics data of the above-mentioned 105 mm HE M1 and the non-rolling, finned 120 mm HE mortar projectiles are illustrated briefly in Table 1.

Table 1. Physical and Geometrical Data of 105 mm and 120mm Projectiles Types

Characteristics	105 mm HE M1 projectile	120 mm HE mortar projectile
Reference diameter, mm	114.1	119.56
Total Length, mm	494.7	704.98
Total mass, kg	15.00	13.585
Axial moment of inertia, kg-m ²	2326x10 ⁻²	2335x10 ⁻²
Transverse moment of inertia, kg-m ²	23118x10 ⁻²	23187x10 ⁻²
Centre of gravity from the base, mm	113.4	422.9

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TRAJECTORY FLIGHT SIMULATION MODEL

Flight mechanics of most projectile configurations can be captured using a rigid body six degrees of freedom dynamic model. The six degrees of freedom flight analysis comprise the three translation components (x, y, z) describing the position of the projectile's center of mass and three Euler angles (ϕ, θ, ψ) describing the orientation of the projectile body with respect to (Fig. 1).

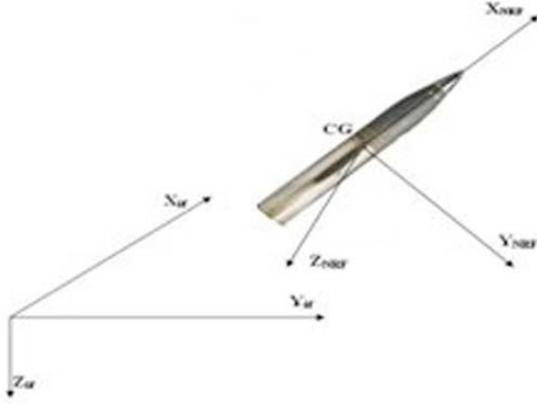


Fig. (1). No-roll (moving) and fixed (inertial) coordinate systems for the projectile trajectory analysis.

Two main coordinate systems are used for the computational approach of the atmospheric flight motion. The one is a plane fixed (inertial frame) at the firing site. The other is a no-roll rotating coordinate system moving with the projectile body (no-roll-frame, NRF, $\phi=0$) with the X_{NRF} axis along the projectile axis of symmetry and Y_{NRF}, Z_{NRF} axes oriented so as to complete a right hand orthogonal system.

Newton's laws of the motion state that the rate of change of linear momentum must equal the sum of all the externally applied forces (1) and the rate of change of angular momentum must equal the sum of the externally applied moments (2), respectively.

$$m \frac{d\vec{V}}{dt} = \sum \vec{F} + m\vec{g} \quad (1)$$

$$\frac{d\vec{H}}{dt} = \sum \vec{M} \quad (2)$$

The total force acting on the projectile comprises the weight, the aerodynamic force and the Magnus force. The total moment acting on the projectile comprises the moment due to the standard aerodynamic force, the Magnus aerodynamic moment and the unsteady aerodynamic moment. The dynamic equations of motion [6-9] are derived in the non-rolling frame and provided in equations (3) up to (6):

$$\begin{Bmatrix} \ddot{x}_{if} \\ \ddot{y}_{if} \\ \ddot{z}_{if} \end{Bmatrix} = \begin{bmatrix} \cos\theta \cos\psi & -\sin\psi & \sin\theta \cos\psi \\ \cos\theta \sin\psi & \cos\psi & \sin\theta \sin\psi \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{Bmatrix} \ddot{u}_{NRF} \\ \ddot{v}_{NRF} \\ \ddot{w}_{NRF} \end{Bmatrix} \quad (3)$$

for the position of projectile's center of mass and

$$\begin{Bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & t_\theta \\ 0 & 1 & 0 \\ 0 & 0 & 1/\cos\theta \end{bmatrix} \begin{Bmatrix} \ddot{p}_{NRF} \\ \ddot{q}_{NRF} \\ \ddot{r}_{NRF} \end{Bmatrix} \quad (4)$$

and for the orientation of the flight body with the classical Euler angles ϕ, θ, ψ . From the two laws of Newton's motion the following equations (5) and (6) are derived, respectively:

$$\begin{Bmatrix} \ddot{u}_{NRF} \\ \ddot{v}_{NRF} \\ \ddot{w}_{NRF} \end{Bmatrix} = \begin{Bmatrix} \tilde{F}x_{TOTAL} / m \\ \tilde{F}y_{TOTAL} / m \\ \tilde{F}z_{TOTAL} / m \end{Bmatrix} + \begin{bmatrix} 0 & \tilde{r}_{NRF} & -\tilde{q}_{NRF} \\ -\tilde{r}_{NRF} & 0 & -\tilde{r}_{NRF} t_\theta \\ \tilde{q}_{NRF} & \tilde{r}_{NRF} t_\theta & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_{NRF} \\ \ddot{v}_{NRF} \\ \ddot{w}_{NRF} \end{Bmatrix} \quad (5)$$

$$\begin{Bmatrix} \ddot{p}_{NRF} \\ \ddot{q}_{NRF} \\ \ddot{r}_{NRF} \end{Bmatrix} = [I^{-1}] \begin{Bmatrix} \tilde{L}_{TOTAL} \\ \tilde{M}_{TOTAL} \\ \tilde{N}_{TOTAL} \end{Bmatrix} - \begin{bmatrix} 0 & -\tilde{r}_{NRF} & \tilde{q}_{NRF} \\ \tilde{r}_{NRF} & 0 & \tilde{r}_{NRF} t_\theta \\ -\tilde{q}_{NRF} & -\tilde{r}_{NRF} t_\theta & 0 \end{bmatrix} \begin{Bmatrix} \ddot{p}_{NRF} \\ \ddot{q}_{NRF} \\ \ddot{r}_{NRF} \end{Bmatrix} \quad (6)$$

The total force acting on the projectile in equation (5) comprises the weight W_f , the aerodynamic force A_f and Magnus force M_f :

$$\begin{Bmatrix} \tilde{F}x_{TOTAL} \\ \tilde{F}y_{TOTAL} \\ \tilde{F}z_{TOTAL} \end{Bmatrix} = \begin{Bmatrix} \tilde{X}_{Wf} \\ \tilde{Y}_{Wf} \\ \tilde{Z}_{Wf} \end{Bmatrix} + \begin{Bmatrix} \tilde{X}_{Af} \\ \tilde{Y}_{Af} \\ \tilde{Z}_{Af} \end{Bmatrix} + \begin{Bmatrix} \tilde{X}_{Mf} \\ \tilde{Y}_{Mf} \\ \tilde{Z}_{Mf} \end{Bmatrix} \quad (7)$$

The total moment acting on the projectile in equation (6) comprises the moment due to the standard aerodynamic force A_m , due to Magnus aerodynamic force M_m and the unsteady aerodynamic moment UA_m :

$$\begin{Bmatrix} \tilde{L}_{TOTAL} \\ \tilde{M}_{TOTAL} \\ \tilde{N}_{TOTAL} \end{Bmatrix} = \begin{Bmatrix} \tilde{L}_{Am} \\ \tilde{M}_{Am} \\ \tilde{N}_{Am} \end{Bmatrix} + \begin{Bmatrix} \tilde{L}_{Mm} \\ \tilde{M}_{Mm} \\ \tilde{N}_{Mm} \end{Bmatrix} + \begin{Bmatrix} \tilde{L}_{UAm} \\ \tilde{M}_{UAm} \\ \tilde{N}_{UAm} \end{Bmatrix} \quad (8)$$

All aerodynamic coefficients are based on Mach number and the aerodynamic angles of attack and sideslip:

$$\alpha = \tan^{-1} \left(\frac{\tilde{w}_{NRF}}{\tilde{u}_{NRF}} \right) \quad (9)$$

$$\beta = \tan^{-1} \left(\frac{\tilde{v}_{NRF}}{\tilde{u}_{NRF}} \right) \quad (10)$$

The total aerodynamic velocity given in equation:

$$V_T = \sqrt{\tilde{u}_{NRF}^2 + \tilde{v}_{NRF}^2 + \tilde{w}_{NRF}^2} \quad (11)$$

The weight force in no-roll system is:

$$\begin{Bmatrix} \tilde{X}_{Wf} \\ \tilde{Y}_{Wf} \\ \tilde{Z}_{Wf} \end{Bmatrix} = mg \begin{Bmatrix} -\sin\theta \\ 0 \\ \cos\theta \end{Bmatrix} \quad (12)$$

The aerodynamic force, which acts on the projectile at aerodynamic center of pressure, is:

$$\begin{Bmatrix} \tilde{X}_{Af} \\ \tilde{Y}_{Af} \\ \tilde{Z}_{Af} \end{Bmatrix} = \frac{1}{2} \rho V_T^2 S_{ref} \begin{Bmatrix} -C_{X0} - C_{X2} \frac{\tilde{w}_{NRF}^2}{V_T^2} - C_{X2} \frac{\tilde{v}_{NRF}^2}{V_T^2} \\ -C_{NA} \frac{\tilde{v}_{NRF}}{V_T} \\ -C_{NA} \frac{\tilde{w}_{NRF}}{V_T} \end{Bmatrix} \quad (13)$$

The Magnus, which acts on projectile at the Magnus force center of pressure, is:

$$\begin{Bmatrix} \tilde{X}_{Mf} \\ \tilde{Y}_{Mf} \\ \tilde{Z}_{Mf} \end{Bmatrix} = \frac{1}{2} \rho V_T^2 S_{ref} \begin{Bmatrix} 0 \\ \frac{\tilde{p}_{NRF} D C_{NPA} \tilde{w}_{NRF}}{2V_T^2} \\ -\frac{\tilde{p}_{NRF} D C_{NPA} \tilde{v}_{NRF}}{2V_T^2} \end{Bmatrix} \quad (14)$$

The moment due to aerodynamic force is:

$$\begin{Bmatrix} \tilde{L}_{Am} \\ \tilde{M}_{Am} \\ \tilde{N}_{Am} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -R_{\oplus MAC} \\ 0 & R_{\oplus MAC} & 0 \end{bmatrix} \begin{Bmatrix} \tilde{X}_{Af} \\ \tilde{Y}_{Af} \\ \tilde{Z}_{Af} \end{Bmatrix} \quad (15)$$

The moment due to Magnus force is:

$$\begin{Bmatrix} \tilde{L}_{Mm} \\ \tilde{M}_{Mm} \\ \tilde{N}_{Mm} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -R_{\oplus MAX} \\ 0 & R_{\oplus MAX} & 0 \end{bmatrix} \begin{Bmatrix} \tilde{X}_{Mf} \\ \tilde{Y}_{Mf} \\ \tilde{Z}_{Mf} \end{Bmatrix} \quad (16)$$

In addition, for the unsteady moment U_{Am} is:

$$\begin{Bmatrix} \tilde{L}_{UAm} \\ \tilde{M}_{UAm} \\ \tilde{N}_{UAm} \end{Bmatrix} = \frac{1}{2} \rho V_T^2 D S_{ref} \begin{Bmatrix} \frac{\tilde{p}_{NRF} D C_{LP}}{2V} \\ \frac{\tilde{q}_{NRF} D C_{MQ} + C_{MA}}{2V} - \frac{C_{MA}}{V} \\ \frac{\tilde{r}_{NRF} D C_{MQ} - C_{MA}}{2V} \end{Bmatrix} \quad (17)$$

The dynamic equations of motion (3-6) are highly non-linear. Thus, numerical integration is commonly used to obtain solutions to this initial value problem.

MODIFIED PROJECTILE LINEAR THEORY

To develop the modified projectile linear theory [7], the following sets of simplifications are employed: the axial velocity \tilde{u}_{NRF} can be replaced by the total velocity V_T because the side velocities \tilde{v}_{NRF} and \tilde{w}_{NRF} are small.

The aerodynamic angles of attack α and sideslip β are small for the main part of the atmospheric trajectory

$$\alpha \approx \tilde{w}_{NRF} / V_T, \beta \approx \tilde{v}_{NRF} / V_T$$

and the projectiles are geometrically symmetrical

$$I_{XY} = I_{YZ} = I_{XZ} = 0, I_{YY} = I_{ZZ}$$

Constant aerodynamic coefficients for the most important forces and moments with respect to angle of attack and Mach number are taken into account. Flat-fire and small yaw trajectories are considered so the yaw angle ψ is small:

$$\sin(\psi) \approx \psi, \cos(\psi) \approx 1$$

The independent variable is changed from time t to dimensionless arclength s , measured in calibers of travel:

$$s = \frac{1}{d} \int_0^t V dt \quad (18)$$

This technique causes the equations determining the coupled pitching and yawing motion independent of the size of projectile, which turns out to be very convenient in the analysis of free-flight range data.

DIFFERENTIAL EQUATION OF MOTION

The differential equation governing the angular oscillatory motion for the complete linearized pitching and yawing motion of slightly symmetric projectiles [6] as a function of distance traveled s is shown below:

$$\zeta'' + (H - iP)\zeta' - (M + iPT)\zeta = iPG \quad (19)$$

where

$$H = \frac{\rho S d}{2m} C_L - \frac{\rho S d}{2m} C_D - K_y^{-2} \left(\frac{\rho S d}{2m} C_{MQ} \right) \quad (20)$$

$$P = \left(\frac{I_x}{I_y} \right) \left(\frac{pd}{V} \right) \quad (21)$$

$$M = K_y^{-2} \left(\frac{\rho S d}{2m} C_{MA} \right) \quad (22)$$

$$T = \left(\frac{\rho S d}{2m} C_L \right) + K_x^{-2} \left(\frac{\rho S d}{2m} C_{NPA} \right) \quad (23)$$

$$G = \frac{gd \cos \phi}{V^2} \quad (24)$$

$$\zeta = \alpha + i\beta \quad (25)$$

This differential equation contains all the significant aerodynamic forces and moments that affect the pitching and yawing motion of a spinning or non-spinning symmetric projectile body. The author's definition $\zeta = \alpha + i\beta$ was first chosen by Fowler *et al.* and was adopted by R. H. Kent. Gunners and engineers usually prefer this definition. The gunner observer looks downrange from a position located just behind the gun. Upward and to the right are always considered as positive directions. It is nature for the gunner to define the α axis as positive upward, the β axis as positive to right and the clockwise direction of all rotations as positive for right hand twist rifling.

STATIC AND DYNAMIC STABILITY CRITERIA

The solution of differential equation (19) tells us that the epicyclic frequencies depend only on the dimensionless roll rate P and overturning moment M , and are unaffected by any of other aerodynamic forces and moments.

From the definition of an unstable motion, we are led naturally to the concept of static stability:

$$(P^2 - 4M) > 0 \quad (26)$$

Classical exterior ballistics defines the static stability factor [5] S_g , as:

$$S_g = \frac{I_x^2 p^2}{2\rho I_y S d V^2 C_{MA}} \quad (27)$$

Eliminating P^2 between equations (26-27), we have:

$$4M(S_g - 1) > 0 \tag{28}$$

For statically unstable (spin-stabilized) projectile, $M > 0$ and equation (28) reduces to the classical static stability criterion:

$$S_g > 1 \tag{29}$$

Equation (26) is a more general result than the (29), because it shows that a statically stable missile ($M < 0$), is statically stable without spin.

Dynamic stability requires that both damping exponents be negative throughout the projectile's flight. For a non-spinning statically stable missile ($M < 0$) and P is either zero. For finned missiles, the pitch damping moment coefficient is usually negative. The lift and drag force coefficients are both positive, therefore H is nearly always greater than zero, and the dynamic stability is assured.

The dynamic stability factor S_d is defined as:

$$S_d = \frac{2T}{H} \tag{30}$$

and the expressions

$$H > 0 \tag{31}$$

$$\frac{1}{S_g} < S_d(2 - S_d) \tag{32}$$

are the generalized dynamic stability criteria for any spinning or non-spinning symmetric projectile flight body.

COMPUTATIONAL SIMULATION

The constant dynamic flight model [10] uses mean values of the experimental aerodynamic coefficients variations (Table 2).

Table 2. Constant Aerodynamic Parameters for Dynamic Trajectory Flight of the Two Projectile Types

105mm Projectile	120mm Projectile
$C_D = 0.243, C_L = 1.76,$ $C_{LP} = -0.0108, C_{MQ} = -9.300,$ $C_{MA} = 3.76, C_{YPA} = 0.381$ $C_{NPA} = 0.215$	$C_D = 0.14, C_L = 2.76,$ $C_{MQ} = -22.300,$ $C_{MA} = -15.76$

Initial data for 105 mm dynamic trajectory model with constant aerodynamic coefficients are:

$$x = 0.0 \text{ m}, y = 0.0 \text{ m}, z = 0.0 \text{ m}, \varphi = 0.0^\circ, \theta = 45.0^\circ \text{ and } 70.0^\circ, \psi = 3.0^\circ, \tilde{u} = 494 \text{ m/s}, \tilde{v} = 0.0 \text{ m/s}, \tilde{w} = 0.0 \text{ m/s}, \tilde{p} = 1644 \text{ rad/s}, \tilde{q} = 0.0 \text{ rad/s and } \tilde{r} = 0.0 \text{ rad/s}.$$

The axial spin rate is calculated from

$$\tilde{p} = 2\pi V_T / \eta D \text{ (rad/s)} \tag{33}$$

where V_T is the total firing velocity (m/s), η the rifling twist rate at the gun muzzle (calibers per turn), and D the reference diameter of the projectile type (m).

In addition, the corresponding initial data for 120 mm are:

$$x = 0.0 \text{ m}, y = 0.0 \text{ m}, z = 0.0 \text{ m}, \varphi = 0.0^\circ, \theta = 45.0^\circ \text{ and } 85.0^\circ, \psi = 8.0^\circ, \tilde{u} = 318 \text{ m/s}, \tilde{v} = 0.0 \text{ m/s}, \tilde{w} = 0.0 \text{ m/s}, \tilde{p} = 0.0 \text{ rad/s}, \tilde{q} = 1.795 \text{ rad/s and } \tilde{r} = 0.0 \text{ rad/s}.$$

The density and pressure are calculated as function of altitude from the simple exponent model atmosphere, and gravity acceleration is taken into account with the constant value $g = 9.80665 \text{ m/s}^2$.

The flight dynamic models of 105 mm HE M1 and 120 mm HE mortar projectile types involves the solution of the set of the twelve first order ordinary differentials, equations (3)-(6), which are solved simultaneously by resorting to numerical integration using a 4th order Runge-Kutta method, and regard to the 6-D nominal atmospheric projectile flight.

RESULTS AND DISCUSSION

The flight path trajectories motion with constant aerodynamic coefficients of the 105 mm projectile with initial firing velocity of 494 m/sec, initial yaw angle 3 deg and rifling twist rate 1 turn in 18 calibers (1/18) at 45° and 70° are illustrated in (Fig. 2).

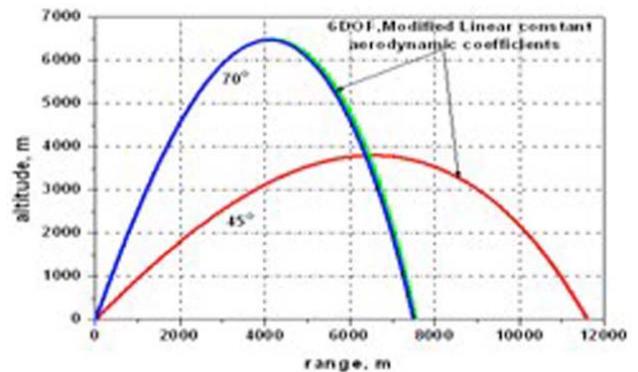


Fig. (2). Impact points and flight path trajectories of 6-DOF and Modified Linear with constant aerodynamic coefficients for 105 mm projectile.

At 45° the 6-DOF model for 105 mm M1 projectile, fired at sea-level neglecting wind conditions, gives a predicted range to impact of approximately 11,600 m and a maximum height at almost 3,600 m. From the results of the modified linear model, the maximum range and the maximum height are almost the same, as shown in (Fig. 2). Also at 70° , the predicted level-ground range of 6-DOF model is 7,500 m with maximum height at about 6,450 m and the modified linear trajectory simulation gives the same values.

The mortar projectile of 120 mm diameter is also examined for its atmospheric constant flight trajectories predictions at pitch angles of 45° and 85° , with initial firing velocity of 318 m/s, initial yaw angle 8° and pitch rate 1.795 rad/s, as shown in (Fig. 3).

At 45° , the 120 mm mortar projectile, fired at no wind conditions, the 6-DOF trajectory gives a range to impact at 7,000 m with a maximum height at almost 2050 m. At 85° , the predicted level-ground range is approximately 1,230 m and the height is 3,950 m. For the same initial pitch angles, the modified linear model's results have satisfactory agreement. (Fig. 3).

In (Fig. 4), static stability factor for the 105 mm HE M1 projectile trajectory motion with constant aerodynamic coef-

ficients is calculated at pitch angles 45° and 70°, respectively. After the damping of the initial transient motion, at apogee, the stability factor for 45 degrees has increased from 3.1 at muzzle to 23 and then decreased to value of 8 at final impact point. The corresponding flight behavior at 70 degrees initial pitch angle shows that the transient motion damps out quickly, where the gyroscopic static stability factor has increased from 3.1 to 121 and then decreased to value of almost 6.6 at the impact area.

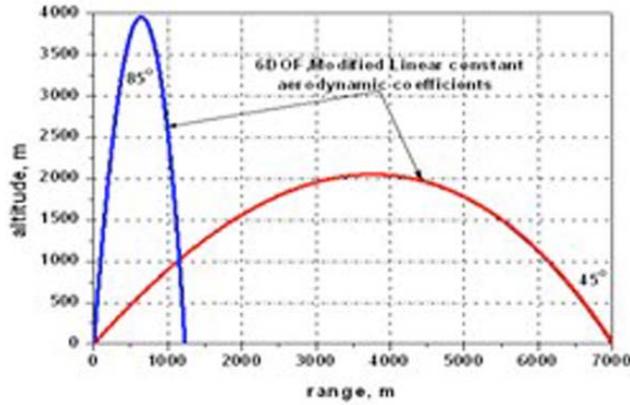


Fig. (3). Flight path trajectories of 6-DOF and Modified Linear with constant aerodynamic coefficients for 120 mm at quadrant elevation angles of 45° and 85°.

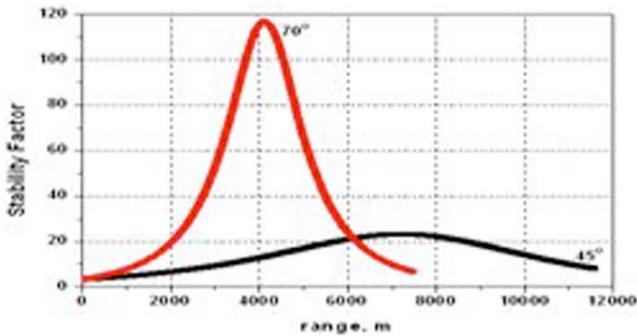


Fig. (4). Comparative static stability variation with constant aerodynamic model at low and high quadrant angles for the 105 mm projectile.

In (Fig. 5), at 45 and 70 degrees the Mach number was 1.45 at the muzzle, then decrease to 0.6 and 0.3 at the summit of the trajectory and then decrease to values of 0.8 and 0.9 at the impact area, respectively. The constant value of dynamic stability for 105 mm is 1.095 and lies within the interval $(0 < S_d < 2)$. So the spin-stabilized 105 mm projectile is gyroscopically and dynamically stable.

On the other hand the 120 mm mortar projectile has uncanted fins, and do not roll or spin at any point along the trajectory. Because of that, the static stability is zero, and we examined only the dynamic stability for 120 mm projectile.

In the modified linear trajectory of the 120 mm mortar projectile, the constant value of dynamic stability is 0.735 at pitch angles of 45 and 85 degrees. If the dynamic stability factor S_d , lies within the interval $(0 < S_d < 2)$, a statically stable projectile is always dynamically stable, regardless of spin. The mortar projectile of 120 mm belongs in the above interval and $H > 0$, so is dynamically stable.

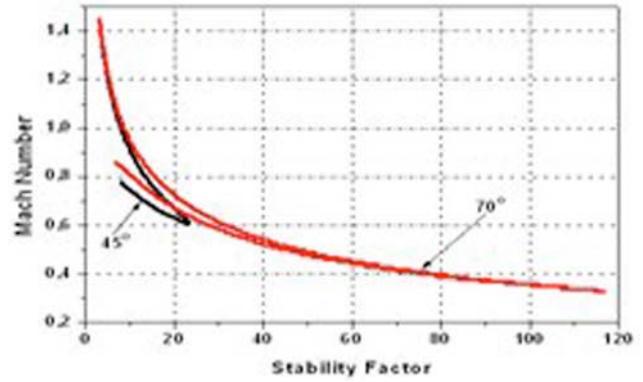


Fig. (5). Comparative stability factor versus Mach number with constant aerodynamic model at low and high quadrant angles for the 105 mm projectile.

CONCLUSION

A six degrees of freedom (6-DOF) simulation flight dynamics model is applied for the accurate prediction of short and long-range trajectories for spin and fin-stabilized projectiles. The modified projectile linear theory trajectory reported here should prove useful to estimate flight trajectory's phenomena as static and dynamic stability.

A LIST OF SYMBOLS

- C_{D0} = zero-yaw drag aerodynamic coefficient
- C_{D2} = yaw drag aerodynamic coefficient
- C_L = lift aerodynamic coefficient
- C_{LP} = roll damping aerodynamic coefficient
- C_{MQ} = pitch damping aerodynamic coefficient
- C_{MA} = overturning moment coefficient
- C_{YPA} = Magnus moment coefficient
- C_{NPA} = Magnus force aerodynamic coefficient
- S_g = static stability factor
- S_d = dynamic stability factor
- $\bar{x}_{if}, \bar{y}_{if}, \bar{z}_{if}$ = projectile position, m
- $\tilde{u}_{NRF}, \tilde{v}_{NRF}, \tilde{w}_{NRF}$ = projectile velocity components expressed in no-roll-frame, m/s
- \tilde{P}_{NRF} = projectile roll rate, rad/s
- $\tilde{q}_{NRF}, \tilde{r}_{NRF}$ = projectiles pitch and yaw rates expressed in no-roll-frame, rad/s
- $\bar{\theta}, \bar{\psi}$ = projectiles pitch and yaw angles, deg
- $\bar{\phi}$ = projectile roll angle, deg
- I = projectile inertia matrix
- I_{XX}, I_{YY}, I_{ZZ} = diagonal components of the inertia matrix
- I_{XY}, I_{YZ}, I_{XZ} = off-diagonal components of the inertia matrix

V_T	= total aerodynamic velocity, m/s
ρ	= atmospheric density, kg/m ³
D	= projectile reference diameter, m
S_{ref}	= projectile reference area ($\pi D^2/4$), m ²
m	= mass of projectile, kg
t	= time, s
α, β	= aerodynamic angles of attack and sideslip, deg
$R_{\ominus MAC}$	= distance from the projectile center of mass to the center of pressure, m
$R_{\ominus MAX}$	= distance from the projectile center of mass to the Magnus center of pressure, m
g	= gravity acceleration, m/s ²

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