

Equilibrium and Optimum City Size with the Presence of Pollution

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Abstract: This paper attempts to find the equilibrium and the optimum size of a monocentric open city that produces a constant return to scale good and a group of differentiated goods and services (increasing return to scale goods and services), where the production of goods and services causes negative externalities (pollution). The model in this paper also assumes the presence of positive externalities of agglomeration (agglomeration economies). As it is an open city, there will be too much agglomeration in the absence of any penalty on the polluters. The results of this paper show that if the city dwellers value environmental quality the market outcomes are sub-optimal. Therefore, a tax scheme is necessary for the correct level of agglomeration or the optimum size of the city. The optimal tax rate in the differentiated goods and services sector depends on the emission intensity of output, consumer preference for product diversity, demand for labour (or level of output), and the wage rate; and that in the constant return to scale good sector depends on the emission intensity of output, average labour productivity, and the wage rate.

Keywords: Equilibrium city size, optimum city size, city size and pollution.

INTRODUCTION

With increasing urbanization and growing concerns about environment, the concepts and the models of equilibrium, optimum, efficient, and sustainable city have been revised, re-visited, and extended in the recent years [1-4]. Different variations of the two well-known streams of city size models – the urban hierarchy model (also known as the central place theory) and the differentiated plane model (which predicts urban concentration at the points where the scarce transportation routes cross) – can be found in the literature [5]. Within these two streams, a number of well-regarded partial equilibrium models of city size were built to analyze the effects of some variables of interest on city size under restricted assumptions [6, 7]. This paper uses a partial equilibrium model to find equilibrium and optimum city size when consumers value both product diversity and environmental quality. The major contribution of this paper is that it demonstrates how environmental quality, when isomorphic to other goods and services in the utility function, and different output-emission relationships affect optimum city size. The findings of this paper are consistent with the findings of the well-cited papers by Verhoef and Nijkamp [8, 9]. This paper shows that with the presence of pollution optimum city size is not only smaller than the equilibrium city size; it is also smaller than the optimum city size found in Abdel-Rahman's model [10]. It proposes a tax scheme that is necessary for the correct level of agglomeration or the optimum size of the city. The optimal tax rate in this model depends on the emission intensity of output, consumer preference for product diversity, demand for labour (or level of output), average labour productivity, and the wage rate.

LITERATURE REVIEW

Early models of urban development and agglomeration include model of land use [11]; models of scale economies and optimal plant location [12, 13]; and land use model of a monocentric city [14]. Economies of scale and transportation costs are identified as determining factors of *central places* in the Central Place Theory [15, 16]. Henderson's [6] seminal work on urban systems also includes external economies of geographical concentration of firms and commuting costs in the model. External economies have been explicitly modelled with a distinctive spatial flavour in the urban-economic geography literature [17-19]. Krugman models [7, 20] include consumer preference for product diversity, economies of scale, transportation costs, population density, and the share of the population employed in manufacturing as determining factors of city size. A recent survey by Duranton and Puga [2] shows different types of externalities that have been modelled by researchers to explain urban agglomeration.

Glaeser *et al.*, [21], Miracky [22], and Henderson *et al.* [23] find that industrial environments of 10, 17 and even 30 years ago causally affect location decisions and patterns today. The consumption of public goods has also been shown to be a determining factor of consumer agglomeration [24, 25]. The Dixit-Stiglitz [26] model of monopolistic competition and product diversity in consumption has been used by many researchers, notably Abdel-Rahman and Fujita [12], to build models of equilibrium and optimum city size. In lieu of Dixit-Stiglitz constant elasticity of substitution (CES) model, variable elasticity of substitution (VES) model has also been used to examine the relationship between city size and other factors [27].

However, "the delicate interaction between endogenous technical development, location and environmental factors" was not addressed adequately in most of the aforementioned models [28]. The objectives of this paper are: (i) to determine the equilibrium city size with variable, variable

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coefficient, and fixed coefficient emissions in the production sector, (ii) to find out the optimum city size from the city planners' perspective, and (iii) to find a tax or a subsidy scheme to attain a city size that is socially optimal. In doing so, this paper will extend Abdel-Rahman's model [10] by assuming that production of goods and services in the city increases pollution which city dwellers dislike.

One of the major findings of this paper is that with the presence of pollution optimum city size is not only smaller than the equilibrium city size; it is also smaller than the optimum city size found in Abdel-Rahman's model [10].

THE MODEL

The basic model in this paper is adopted from Abdel-Rahman's [10] paper, "Product Differentiation, Monopolistic Competition, and City Size," which was recognized by the Nobel Committee while awarding the 2008 Nobel Prize to Paul Krugman for "his analysis of trade patterns and location of economic activity." The Dixit-Stiglitz model [26] of monopolistic competition and consumers' preferences for product variety are key determining forces of the equilibrium and the optimum city size in this model. The present model extends Abdel-Rahman's model in two ways - by incorporating environmental quality in the utility function and by assuming that the production of goods and services in the city directly contributes to higher level of emissions.

Consumers, in this model, derive utility from the consumption of a good x , which is available in the city and elsewhere, and from diverse goods and services (z_i) that are available only in the city. They also get utility from the consumption of land (h) and from environmental quality (e). The utility function is a constant elasticity of substitution (CES) utility function. The production sector in the city is characterized by a constant return to scale (CRS) sector (good x), and an increasing return to scale (IRS) sector (goods and services z_i), which has the general features of Dixit-Stiglitz [26] model of monopolistic competition. The production sector in this paper closely follows the production sector in Abdel-Rahman's [10] model with additional assumptions that both the CRS and the IRS sectors contribute to higher level of pollution. Different output-emission structures are assumed in three different versions of the pollution model in the next section.

Since the CRS good is available elsewhere, it does not attract people to the city. Differentiated goods attract households to the city, and firms cluster into the city to take advantage of increasing returns to scale in that sector. Production of goods increases emissions, but there are no taxes on emissions, therefore, firms are not expected to lower production. This reduces consumer utility from urban agglomeration as environmental quality is isomorphic to other goods in the utility function. This paper shows how the consumer demand for various goods change under these different circumstances, and proposes a tax scheme that is necessary to reduce the production of goods to the optimum level.

The basic assumptions of the present model are the same as in Abdel-Rahman's [10] model with a few additional assumptions about the nature of the production-emissions relationship and the structure of the utility function (as stated above). It is a representative agent model. The city is

assumed to be a monocentric open city. Price of x (P_x) is fixed ($=1$), and is determined by the national market. Production takes place in the central business district (CBD), and the rest of the city is for residential use. Land is owned by absentee land owners. The utility level of the city residents is equivalent to national utility level, U^* . The consumption of environmental quality is non-rival from household point of view.

One of the goals of this paper is to extend Abdel-Rahman's [10] results by including pollution into his model, therefore, this paper will solve for the major variables of his model using the same methodology. Unlike Abdel-Rahman, this paper will find the optimum city size by maximizing an arbitrary consumer's utility while keeping the utility of other consumers fixed at a certain level. An optimum tax (or subsidy) rate will be determined by equating the market outcomes with tax (or subsidy) to the optimum outcomes.

EQUILIBRIUM CITY SIZE WITH THE PRESENCE OF POLLUTION

Emission-output relationship can vary significantly between industries and even within the same industry due to the variations in technology use by different firms. Various types of emission intensity of output have been discussed in environmental literature. We will consider three possible cases in this section in three different versions of our pollution model to show their effects on city size. Model 1 will assume variable emissions across firms in all industries, hence no fixed emission-output relationship. Model 2 will assume same emission intensity of output across different industries. Model 3 will relax the assumption made in Model 2 and assume different emission intensity of output in different industries.

Model 1: Variable Emissions

This model assumes that there is no fixed relationship between emissions and the firms' output levels.

Household Sector

Each household maximizes a utility function:

$$U = x^\alpha h^\beta Q^\gamma e^\theta, \text{ where } Q = \left[\sum_{i=1}^n z_i (r)^\rho \right]^{\frac{1}{\rho}}; \alpha, \beta, \gamma, \theta, \rho > 0; \text{ and } \alpha + \beta + \gamma + \theta = 1$$

$$\text{subject to (s.t.) } y - tr = x + R(r)h(r) + \sum_{i=1}^n P_{z_i} z_i \text{ and } e \leq A - \sum_{k=1}^{n+1} (Y_e^k),$$

where

ρ = consumer's desire for product diversity

$R(r)$ = land rent per unit of land at distance r from the CBD

P_{z_i} = the price of differentiated good or service z_i

y = the household income (wage income)

tr = transportation cost of a household at distance r from the CBD

A = total environmental quality

Y_e^k = pollution by firm k (if firm k does not pollute, then $Y_e^k = 0$)

By rearranging the First Order Conditions (FOCs), we find the equilibrium solutions as

$$x(r) = \alpha(y - tr)$$

$$h(r) = \beta(y - tr)R(r)^{-1}$$

$$z_i(r) = [\gamma(y - tr)P_{z_i}^{-1}Q^{-\rho}]^{\frac{1}{1-\rho}}$$

$$e = \theta(y - tr)\lambda_1/\lambda_2,$$

where λ_1/λ_2 is the ratio of marginal utility of money to the marginal utility of environmental quality. These results show that the household consumption of x , h and z_i remain the same even after inclusion of pollution in the model. The reason is that the households cannot increase their utility by lowering the consumption of x or z_i , given the structure of the emission-output relationship.

From the above results the indirect utility function can be derived as

$$V = \alpha^\alpha \beta^\beta \gamma^\gamma \theta^\theta R(r)^{-\beta} I^{-\gamma} (y - tr) \left(\frac{\lambda_1}{\lambda_2} \right)^\theta, \quad (1)$$

where $I = \left(\sum_{i=1}^n P_{z_i}^{-\rho(1-\rho)} \right)^{\frac{1-\rho}{\rho}}$

By inverting equation (1) with respect to $R(r)$, the bid rent function can be obtained as

$$B(U^*, y - tr, I) = U^{*1/\beta} \alpha^{\alpha/\beta} \beta^{\beta/\beta} \gamma^{\gamma/\beta} \theta^{\theta/\beta} I^{\gamma/\beta} (y - tr)^{1/\beta} \left(\frac{\lambda_1}{\lambda_2} \right)^{\theta/\beta} \quad (2)$$

The bid rent function is the maximum rent a household at distance r from the CBD will pay per unit of land to attain U^* . At the equilibrium, the bid rent is equal to the market land rent for $0 \leq r \leq r_f$, where r_f is the urban fringe of the city. Setting $R(r) = B(\cdot)$ in (2), the demand for land can be derived as

$$h(U^*, y - tr, I) =$$

$$U^{*1/\beta} \alpha^{-\alpha/\beta} \beta^{-1} \gamma^{-\gamma/\beta} \theta^{-\theta/\beta} I^{\gamma/\beta} (y - tr)^{-(\alpha-\gamma)/\beta} \left(\frac{\lambda_1}{\lambda_2} \right)^{-\theta/\beta} \quad (3)$$

For simplicity, it is assumed that agricultural land rent (the opportunity cost of land) is zero. Therefore, the number of households, N , which will be accommodated in the city is given by

$$N(U^*, y, I) = \int_0^{r_f} \frac{2\pi r}{h(U^*, y - tr, I)} dr, \quad (4)$$

where $2\pi r$ is the amount of land available at distance r from the CBD in a circular city. $N(\cdot)$ is the number of households that will live in the city given that they will attain U^* . Integrating (4) and rearranging terms, the marginal cost of attaining a utility level U^* by a household is given by

$$y(U^*, I, N) = \left[\frac{NU^{*1/\beta} (\beta + 1) t^2 I^{\gamma/\beta}}{\alpha^{\alpha/\beta} \gamma^{\gamma/\beta} 2\pi \beta^2} \right]^{\frac{\beta}{1+\beta}} \left(\frac{\lambda_2}{\theta \lambda_1} \right)^{\frac{\theta}{1+\beta}}$$

Production Sector

Differentiated goods and services: Assuming a large number of firms each with a production function of

$$L_{z_i} = f + cz_i, \quad f, c > 0$$

where L_{z_i} = the amount of labour required to produce z_i

f = the fixed cost in terms of labour,

the marginal revenue is given by $MR_i = P_{z_i} \left(1 - \frac{1}{E} \right) = P_{z_i} \rho$,

where E = the elasticity of demand. The profit maximizing condition, $MR = MC$, implies that $P_{z_i} = \rho^{-1}wc$, where w = wage rate. By equating $TR = TC$, the output of any arbitrary firm i can be derived as $z_i = \rho fc^{-1}(1 - \rho)^{-1}$. Hence, the demand for labour by firm i is $L_{z_i} = f(1 - \rho)^{-1}$.

The symmetry of the problem ensures that at the equilibrium, the same quantity of each good or service will be produced. The reason is that all firms face the same cost and demand functions. Therefore, the total demand for labour by the differentiated goods and services sector is $nL_{z_i} = L^z = nf(1 - \rho)^{-1}$.

The CRS good: Assuming a production function of $x = aL_x$, where L_x is the amount of labour required to produce x , the firm's objective is to

$$\max \Pi = x - wL_x$$

$$L_x$$

$$\text{s.t. } x = aL_x$$

That gives $w = a$, where w represents the inverse demand for labour by industry x .

Equilibrium City Size

Assuming full employment in the city, the equilibrium population of the city is given by $N = L^x + L^z$, where L^x is the total labour force in industry x . Given that at equilibrium $w = y = w'$, w' can be written as

$$w'(U^*, n, L^*) = \left[\frac{U^{*1/\beta} t^2 (1 + \beta) c^{\gamma/\beta} n^{-(1-\rho)\gamma/\rho\beta}}{2\pi \alpha^{\alpha/\beta} \gamma^{\gamma/\beta} \beta \rho^{\gamma/\beta}} \right]^{\frac{\beta}{2\beta+\alpha}} \left[L^{*x} + fn(1 - \rho)^{-1} \right]^{\frac{\beta}{2\beta+\alpha}} \left(\frac{\lambda_2}{\theta \lambda_1} \right)^{\frac{\theta}{1+\beta}} = w'' \left(\frac{\lambda_2}{\theta \lambda_1} \right)^{\frac{\theta}{1+\beta}},$$

where L^{*x} is a given positive value of L^x . w' function represents the wage that L^z has to obtain in order to attain U^* in the city. w'' is the same as w' in absence of any pollution.

From the FOCs above, it can be showed that $\frac{\lambda_2}{\theta \lambda_1} = \frac{y - tr}{e}$.

Since $\frac{\theta}{1 + \beta} < 1$, $\left(\frac{\lambda_2}{\theta \lambda_1} \right)^{\frac{\theta}{1+\beta}}$ has only a scale effect on w'' .

Whether w' is larger or smaller than w'' that depends on whether $\left(\frac{\lambda_2}{\theta\lambda_1}\right)^{\frac{\theta}{1+\beta}}$ is greater or smaller than 1. However, from $\left(\frac{\lambda_2}{\theta\lambda_1}\right)^{\frac{\theta}{1+\beta}} = \left(\frac{y-tr}{e}\right)^{\frac{\theta}{1+\beta}}$, it can be said that for a given level of net income, if environmental quality improves, people will not need to spend as much to attain the same level of utility.

Equating $w' = y = a$, the equilibrium number of firms in the monopolistically competitive sector (or the city size) can be determined. In this model, if $\theta\lambda_1 = \lambda_2$ or $e = (y - tr)$, pollution does not have any effect on agglomeration or city size. Otherwise, depending on the value of the scale factor, there will be too much or too little agglomeration compared with the no-pollution outcome.

Model 2: Fixed Emission Intensity of Output in All Industries

This model assumes fixed coefficient emissions. In other words, emissions are proportional to the firms' output levels. Therefore, the environmental constraint is $e \leq A - k \sum_{i=1}^n z_i - kx$. Hence, the equilibrium consumer demands are

$$x(r) = \alpha(y - tr) \left(\frac{\lambda_1}{\lambda_1 + \lambda_2 k} \right) < x(r) \text{ from Model 1}$$

$$h(r) = \beta(y - tr)R(r)^{-1}$$

$$z_i(r) = [\gamma(y - tr)P_{ij}^{-1}Q^{-\rho}]^{\frac{1}{1-\rho}} [\lambda_1 P_{ij} / (\lambda_1 P_{ij} + \lambda_2 k)]^{\frac{1}{1-\rho}} < z_i(r) \text{ from Model 1}$$

$$e = \theta(y - tr)\lambda_1/\lambda_2$$

The significance of these results is that the city residents voluntarily reduce the consumption of x and z_i to reduce emissions as they are proportional to output levels of x and z_i . Such results can have important policy implications.

From these results, the consumption of land can be derived as

$$h(U^*, y - tr, J) = U^{*\beta} \alpha^{-\alpha/\beta} \beta^{-1} \gamma^{-\gamma/\beta} \theta^{-\theta/\beta} J^{\gamma/\beta} (y - tr)^{-(\alpha-\gamma)/\beta} \left(\frac{\lambda_1}{\lambda_2} \right)^{-\theta/\beta},$$

$$\text{where } J = \left[\sum_{i=1}^n \left(\frac{\lambda_1}{\lambda_1 P_{ij} + \lambda_2 k} \right)^{\frac{\rho}{1-\rho}} \right]^{\frac{1-\rho}{\rho}}$$

Therefore, the marginal cost of attaining a utility level U^* by a household is given by

$$y(U^*, J, N) = \left[\frac{NU^{*\beta} (\beta + 1)t^2 J^{\gamma/\beta}}{\alpha^{\alpha/\beta} \gamma^{\gamma/\beta} 2\pi\beta^2} \right]^{\frac{\beta}{1+\beta}} \left(\frac{\lambda_2}{\theta\lambda_1} \right)^{\frac{\theta}{1+\beta}}$$

Equilibrium City Size

Using the solutions from the production sector, it can be found that

$$I = \left[\sum_{i=1}^n P_{ij}^{-\rho(1-\rho)} \right]^{\frac{-(1-\rho)}{\rho}} = wc\rho^{-1}n^{-(1-\rho)/\rho}$$

$$J = \left[\sum_{i=1}^n \left(\frac{\lambda_1}{\lambda_1 P_{ij} + \lambda_2 k} \right)^{\frac{\rho}{1-\rho}} \right]^{\frac{-(1-\rho)}{\rho}} = \left[\frac{\lambda_1 \rho}{\lambda_1 wc + \rho\lambda_2 k} \right] n^{-(1-\rho)/\rho}$$

Therefore, y (or w') in this model is greater than, equal to, or less than y (or w') from Model 1, depending on

$$J^{\gamma/(1+\beta)} >, = \text{ or } < I^{\gamma/(1+\beta)}$$

$$\text{or, } \frac{\lambda_1 \rho}{\lambda_1 wc + \rho\lambda_2 k} >, = \text{ or } < wc\rho^{-1}$$

$$\text{or, } \rho^2 - (wc)^2 - \frac{\rho k wc \theta (y - tr)}{e} >, = \text{ or } < 0, \text{ which is quadratic in } \rho.$$

For different values of ρ or the consumers' desire for product diversity, the value of y (or w') will differ in these two models. The above condition can be re-written as $\left[\frac{\rho^2 - (wc)^2}{\rho wc \theta (y - tr)} \right] e >, = \text{ or } < k$. A smaller emission coefficient (k) means better environmental quality (e). Therefore, the households need to consume relatively smaller quantity of other goods to obtain a certain level of utility. When $k = 0$, the outcomes of this model are the same as the outcomes of Model 1.

Model 3: Variable Emission Intensity of Output Across Firms in All Industries

In this model, emissions are proportional to the firms' output levels, but these proportions vary across firms. Because of variable coefficient emissions, the environmental constraint becomes $e \leq A - \sum_{i=1}^n k_i z_i - lx$. Hence, the equilibrium consumer demands are

$$x(r) = \alpha(y - tr) \left(\frac{\lambda_1}{\lambda_1 + \lambda_2 l} \right) < x(r) \text{ from Model 1}$$

$$h(r) = \beta(y - tr)R(r)^{-1}$$

$$z_i(r) = [\gamma(y - tr)P_{ij}^{-1}Q^{-\rho}]^{\frac{1}{1-\rho}} [\lambda_1 P_{ij} / (\lambda_1 P_{ij} + \lambda_2 k_i)]^{\frac{1}{1-\rho}} < z_i(r) \text{ from Model 1}$$

$$e = \theta(y - tr)\lambda_1/\lambda_2$$

This modification of the model does not change the basic results found in Model 2. Instead of one, j number of CRS goods could be assumed in all models. That would make the algebra complicated without adding much to the models. Households could also be assumed as polluters.

OPTIMUM CITY SIZE WITH THE PRESENCE OF POLLUTION

We will find the optimum city size for the variable coefficient emissions model (Model 3). The equilibrium outcomes in Model 3 are basically the same as those in Model 2. However, x and z_i are smaller in Model 3 than those in Model 1. Any limit to production because of high level of pollution (emission) is not considered in our model. Therefore, without any government control or tax on emissions, a polluting firm's equilibrium output level will remain unchanged.

The city planners can find optimum quantities of x , z_i , h , and e by maximizing an arbitrary consumer's utility, U^1 , given $U^m = U^{m*}$, for $m=2, 3, \dots, N$ subject to the constraints. Therefore, the Lagrange function \mathfrak{S} is

$$\begin{aligned} \mathfrak{S} = & U^1 + \sum_{m=2}^N \alpha^m (U^m - U^{m*}) \\ & + \sum_{m=1}^N \sigma^m \left(y - tr - x - R(r)h(r) - \sum_i^n P_{z_i} z_i \right) \\ & + \sum_{m=1}^N \beta^m \left(A - \sum_{i=1}^n k_i z_i - lx - e \right) \\ & - \sum_{k=1}^n \gamma^k \left(f - cz_i - L_{z_i} \right) - \mu (x - aL_x) \end{aligned}$$

This is a representative agent model. Therefore, the above Lagrange function can be re-written by using $\sum_{m=1}^N \sigma^m = N\sigma = \lambda_1$, etc. as

$$\begin{aligned} \mathfrak{S} = & U^1 + \alpha_1 (U^m - U^{m*}) + \lambda_1 \left(y - tr - x - R(r)h(r) - \sum_i^n P_{z_i} z_i \right) \\ & + \lambda_2 \left(A - \sum_{i=1}^n k_i z_i - lx - e \right) - \sum_{k=1}^n \gamma^k \left(f - cz_i - L_{z_i} \right) - \mu (x - aL_x) \end{aligned}$$

By re-arranging the FOCs, the optimum solutions can be derived as

$$x(r)^* = \alpha(y - tr) \left(\frac{\lambda_1}{\lambda_1 + \lambda_2 l + \mu} \right) < x(r)^{market}$$

$$h(r)^* = \beta(y - tr)R(r)^{-1} = h(r)$$

$$z_i(r)^* = [\gamma(y - tr)P_{z_i}^{-1}Q^{-\rho}]^{\frac{1}{1-\rho}} [\lambda_1 P_{z_i} /$$

$$(\lambda_1 P_{z_i} + \lambda_2 k_i + c \sum_{k=1}^n \gamma^k)]^{\frac{1}{1-\rho}} < z_i(r)^{market}$$

$$e^* = \theta(y - tr) \lambda_1 / \lambda_2 = e,$$

The above results show that the market outcomes for x and z_i are larger than the optimum outcomes in the absence of any policy actions by the city planners. This unambiguous result is obtained due to particular assumptions about the utility function, production functions, emission-structure, and the nature of the city.¹

¹Optimum city size without any pollution: If production of goods and services does not cause pollution in the city, or if the city dwellers are not concerned about pollution, the optimization problem is symmetric to the above problem. Solving the city planners' problem, the optimum solutions can be derived as

$$x(r)^* = \alpha(y - tr) \left(\frac{\lambda_1}{\lambda_1 + \mu} \right) < x(r)^{market(withoutpollution)}$$

OPTIMUM POLLUTION TAX

As optimum city size with the presence of pollution is smaller than the equilibrium city size, a tax scheme can be devised which, by reducing emissions, will reduce the equilibrium size of the city to its optimum level. Assume a flat tax rate on emissions and a lump-sum transfer to the households. Under this assumption, a firm will pay taxes based on its level of emission. Depending on the output-emission relationship, tax volume will be determined by a firm's output level. Therefore, big emitters will pay proportionately more taxes. Adoption of cleaner technology will reduce the tax bill for a firm.

Households maximize utility subject to

$$y - tr + S = x + R(r)h(r) + \sum_{i=1}^n P_{z_i} z_i$$

and $e \leq A - \sum_{i=1}^n k_i z_i - lx$, where $S =$ lump-sum transfer

The FOCs can be re-arranged to get

$$x(r) = \alpha(y - tr + S) \left(\frac{\lambda_1}{\lambda_1 + \lambda_2 l} \right)$$

$$h(r) = \beta(y - tr + S)R(r)^{-1}$$

$$z_i(r) = [\gamma(y - tr + S)P_{z_i}^{-1}Q^{-\rho}]^{\frac{1}{1-\rho}} [\lambda_1 P_{z_i} / (\lambda_1 P_{z_i} + \lambda_2 k_i)]^{\frac{1}{1-\rho}}$$

$$e = \theta(y - tr + S) \lambda_1 / \lambda_2$$

Differentiated Goods and Services Sector

Given the production function of differentiated goods in this model, the profit maximizing condition, $MR = MC$, implies that $P_{z_i} = \rho^{-1}(wc + k_i t)$, where $t =$ tax rate. By equating $TR = P_{z_i} z_i = \rho^{-1}(wc + k_i t)z_i$ and $TC = wL_{z_i} = w(f + cz_i)$, the output of any arbitrary firm i can be derived as

$z_i = \rho f c^{-1} \left[(1 - \rho) + \frac{k_i t}{cw} \right]^{-1}$. Hence, the demand for labour of

firm i is $L_{z_i} = f \left[1 + \rho \left\{ (1 - \rho) + \frac{k_i t}{cw} \right\}^{-1} \right]$.

The total demand for labour by the differentiated goods sector is, therefore,

$$L^z = \sum_{i=1}^n L_{z_i} = fn + f\rho \sum_{i=1}^n \left[(1 - \rho) + \frac{k_i t}{cw} \right]^{-1}$$

By comparing this result with the result in the previous section, it is evident that with any tax on pollution, post-tax industry demand for labour, L^z , is smaller than pre-tax

$$h(r)^* = \beta(y - tr)R(r)^{-1} = h(r)$$

$$z_i(r)^* = [\gamma(y - tr)P_{z_i}^{-1}Q^{-\rho}]^{\frac{1}{1-\rho}} [\lambda_1 P_{z_i} / (\lambda_1 P_{z_i} + c \sum_{k=1}^n \gamma^k)]^{\frac{1}{1-\rho}} < z_i(r)^{market(withoutpollution)}$$

Comparing these optimum outcomes and the market outcomes with the presence of pollution, no unambiguous conclusion can be drawn. Whether an optimum size city with no pollution is smaller or larger than the equilibrium size of the city with the presence of pollution that depends on several parameter values of the model, namely the values of $l, \mu, \lambda_2, c, \gamma^k$ and k_i .

industry demand for labour. Therefore, the after-tax total production of z_i is smaller.

Solving from the above equations, the tax rate can be derived as

$$t = \frac{1}{k_i z_i} [wf\rho - z_i(1-\rho)cw], \quad (A)$$

which is positive for all $z_i < \frac{f\rho}{(1-\rho)c}$. It shows that the

volume of tax ($tk_i z_i$) in industry z depends on consumer preference for product diversity, demand for labour (or level of output), and the wage rate.

The CRS Good Sector

With a tax on emissions, a firm's objective function is

$$\max \Pi = x - wL_x - lt$$

L_x

s.t. $x = aL_x$, where lx = total emissions to produce x .

From the FOCs, it can be derived that $w = a(1-lt)$, where w represents the inverse demand for labour by industry x . Solving for t , the tax rate can be derived as

$$t = \frac{(a-w)}{al} = \frac{1}{l} - \frac{w}{al} \quad (B)$$

It shows that the volume of tax (tl) in industry x depends on average labour productivity and wage rate. Equating the market outcomes with tax to the optimum outcomes, the optimum tax rate can be determined. From (A) and (B), it can be found that

$$\frac{1}{l} - \frac{w}{al} = \frac{1}{k_i z_i} [wf\rho - z_i(1-\rho)cw]$$

By re-arranging this equation, the production of differentiated goods and services (z_i), a determinant of city size, can be derived as a function of l , a , f , c , w , ρ and k_i :

$$z_i^* = \left[\frac{k_i}{l} \left(1 - \frac{w}{a}\right) + (1-\rho)cw \right]^{-1} wf\rho \quad (C)$$

Equation (C) shows that the optimum production of differentiated goods and services (z_i) depends on consumers' desire for product diversity (ρ), fixed cost in terms of labour to produce z_i (f), urban wage rate (w), productivity of workers (a , $\frac{1}{c}$) and emission intensity of output (l , k_i) in both constant return to scale (x) and differentiated goods and services (z_i) sectors. As expected, increase in consumers' desire for product diversity, more fixed cost in terms of labour and higher marginal productivity of workers in z_i ($\frac{1}{c}$) will increase the city size. Any increase in urban wage rate and the emission intensity of output in industry x (this is an industry that is available both in the city and elsewhere) will also increase the production of z_i or increase the city size. Any increase in the emission intensity of output in industry z (this is an industry that is only available in the city) will have negative effect on city size. This is a very desirable outcome.

The above result also shows that any improvement in the average productivity of workers in industry x will have negative effect on city size. This is useful information for the policy makers who are trying to limit the city size.

CONCLUSIONS

The models in this paper use partial equilibrium framework to define both the equilibrium and the optimum city size. Therefore, many determinants of a city size are not considered in these models. However, such partial equilibrium models are useful, because of their suggestive set of results.

In a city where consumers are indifferent about environmental quality, one would expect too little agglomeration, so the city planners may have to give subsidies to firms and/or to households to attract them to the city. If there are only negative externalities from productive activities, the city planners have to impose taxes on firms to lower production. In the presence of both positive and negative externalities of urban agglomeration, whichever dominates determines if there should be a tax or a subsidy scheme.

Whether pollution can work as a strong deterrent to consumer agglomeration - or not - is an empirical question; although it is recognized that the *urban lights* are strong pull-factors for consumer agglomeration. This paper finds that with the presence of pollution the market outcomes are sub-optimal, when consumers value environmental quality. One of the findings of this paper is that utility maximizing city dwellers voluntarily reduce their consumption of goods and services in order to reduce emissions when emissions are proportional to output levels. If cleaner technology is adopted by firms, or environmental quality in the city improves, city dwellers can attain same level of utility by consuming relatively less amount of goods and services. This is a significant result for policy makers and environmental groups.

When the emission-output relationship is variable or random, the effect of pollution on equilibrium city size depends on marginal utility of income, marginal utility of environmental quality, and consumer preferences for consumption of land in the city and environmental quality. City residents may not necessarily attain the same level of utility by reducing their consumption of goods and services, when no clear emission-output relationship exists.

Due to the variations in emission intensity of output within and between industries, output tax and emission tax may yield different outcomes unless all of these possible variations are considered by policy makers. Policy makers can design emission taxes to encourage the adoption of cleaner technology. The results in this paper suggest that a tax scheme is necessary to ensure the correct level of agglomeration. The optimal tax rate in the differentiated goods and services sector depends on the emission intensity of output, consumer preference for product diversity, demand for labour (or level of output), and the wage rate; and that in the constant return to scale good sector depends on the emission intensity of output, average labour productivity, and the wage rate. Therefore, these variables

are also the key determinants of optimum urban agglomeration or optimum city size.

The models in this paper can be extended to address various sustainability issues, or to build sustainable city models. I will extend the present analysis with a calibration exercise using real-world values for key parameters in such a model in a future study.

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CONFLICT OF INTEREST

None declared.

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