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Modeling and Delay Propagation Analysis for Flight Operation Based on Time Interval Petri Net

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Abstract: A Time Interval Petri net (TIPN) is proposed and used to model flight operation. Based on the TIPN model, the flight delay propagation analysis algorithm is provided, which reduces the complexity of flight delay propagation analysis algorithm. Time interval constraints are attached to places and transitions in TIPN model, which could model the flight turnaround time and flight flying period. Meanwhile, firing rules for transition and transition sequence are defined based on the simplified model which is derived depending on the reasoning ability of linear logic. And also, the real time flight delay propagation analysis algorithm is proposed. TIPN model for flight operation considering different initial delay levels in source airport is established and the delay propagation analysis demonstrated that, the model and algorithm could rapidly predict flight operation state and delay level in downstream airports effectively.

Keywords: Flight, delay analysis, time Petri net, linear logic.

1. INTRODUCTION

Aircraft flying from one airport to another is called flight. Normally, one aircraft needs to undertake several flight tasks. In other words, one aircraft usually is designated to several flights. Thus, if delay is occurs to the aircraft of upstream flight and there is no other substitute aircraft available, it will cause propagation of flight delays. In this situation, lack of propagation prediction ability will undermine the effectiveness of the entire air transportation system, and lead to loss and inconveniences for airline passengers. The object of his paper is to provide an effective model and algorithm for the flight delay propagation. During the operation process of one flight, its state transit from one sate to another at discrete time, such as takeoff and landing. Thus, this process can be deemed as one kind of discrete event dynamic system (DEDS). Petri net has been broadly used on modeling, performance assessment, scheduling and control of DEDS. And it also can be used to model the flight operation process.

In this field, Ding has conducted deep research on modeling of flight operation procedure, and established the Petri net chain model for a single aircraft executing many flights, the color Petri net model for many flights among multiple airports [1,2]. However, some operation parameters, such as turn-around time, have not been represented in these models. Some mathematical analysis techniques of Petri net cannot be used to conduct flight delay propagation analysis. Therefore, it is hard to support the direct use of Petri net to describe and analyze the propagation of flight delays. In this aspect, Beatty proposed the delay adder to evaluate the delay status of downstream flights caused by flight crew or aircraft mechanical problems [3]. Paul analyzed the rule of flight delay propagation for consecutive flight and worked out a recursive model to mitigate flight delays [4]. Khaled revealed the reason of flight delay propagation and proposed the chain model for flight delay, and provided topological sorting algorithm to optimize the flight resources when flight delays happen [5]. Yao established directed acyclic graph for flight delay, and studied the effect from aircraft, crew and attendance to flight delay, and provided a quantitative analysis method of flight delay transmission as well as a warning graph package of flight delay prediction [6-8]. Chen viewed the real time prediction of flight delay as dynamic evaluation of the system status, considering different random factors during the flight operation, and the space model of flight delay status was also established in this research [9]. Bayesian network-based flight delay transmission model is investigated and verified with real time flight operation data [10, 11]. Generally, current methods of flight delay prediction are complicated, thus it is necessary to use those models to provide real time algorithm for flight delay prediction. In this paper, a Petri net model of flight operation is built and combining the advantage of linear logic reasoning, a real-time flight delay propagation analysis algorithm is proposed.

2. TIME INTERVAL PETRI NET

A new kind of time Petri net is defined to describe the flight operation process, which is characterized with time interval constraint on both transitions and places set.

Time Interval Petri net (*TIPN*) is defined by $TIPN = \{P, T, \boldsymbol{I}, \boldsymbol{O}, S, E, \eta, \gamma, \Phi_p, \varphi, \Phi_t, \boldsymbol{M}_0\}$,

In which,

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(1) $P=\{p_1,p_2,\ldots,p_n\}$ is the non-empty finite place set, $T=\{t_1,t_2,\ldots,t_m\}$ is the non-empty finite transition set, and $P\cap T=\phi$, $P\cup T\neq\phi$;

(2) ***I*** (***O***) is the model input (output) function ;

(3) $S=\{s_1,s_2,\ldots,s_n\}$ is finite state set;

(4) $E=\{e_1,e_2,\ldots,e_m\}$ is finite event set;

(5) $\eta:S\rightarrow P$ is the mapping function from state set to place set, which means that every place could describe one state;

(6) $\gamma:E\rightarrow T$ is the mapping function form event set to transition set, which means that every transition could describe one event;

(7) $A=\{a_1,a_2,\ldots,a_v\}$ is the flight set, v is number of flight;

(8) $\Phi_p:P\times F\rightarrow\{[a,b]|0\leq a\leq b\}$ is the mapping function for static time interval, in which, a and b are the planned start time and end time of aircraft token in place, respectively (in other words, it is also the start and end time of turn around);

(9) Mapping function φ: $P\times F\rightarrow\{z| z \epsilon R^+\}$ specifies one minimum turn round time for flight;

(10) $\Phi_t:T\rightarrow\{[a,b]|0\leq a\leq b\}$ is a static time interval mapping function, in which, a and b are the state enable and firing time for one transition, respectively that is the specified flight period.

(11) $\boldsymbol{M}_0$ is the initial marking.

The token color can be represented by π_k =<$rr(a)$> , where $rr(a)$ depicts the residual taxiing route of aircraft a from current location (which can be denoted by place sequence), k depicts the flight number.

Marking $\boldsymbol{M}(p_i)=\pi_k$ represents that there is one token in place p_i and the color of this token is π_k; $\boldsymbol{M}(p_i)$= <0> represents that there is no token in place p_i.

For $t_{i,m}\in T$, $p_i\in {}^{(p)}t_{i,m}$, $p_m\in t_{i,m}{}^{(p)}$, we define $\boldsymbol{I}(p_i, t_{im})=\mathrm{I_D}$, where $\mathrm{I_D}$ is a function which cannot change the token color; $\boldsymbol{O}(p_i, t_{im})$=UP, where UP is a function that can change the token color from <$rr(a)$> to <$rr'(a)$> ($rr'(a)$ is the result that the first element of $rr(a)$ is deleted).

For $t_{i,m}\in T$, the state enable conditions include: (1) $\boldsymbol{M}(p_i)=\pi_k$ =<$rr(a)$>;

(2) π_k=<$t_{i,}$ $t_{m,\ldots}$>.

The first condition represents that the airport denoted by place p_i is occupied by aircraft a and its planned flight route is π_k; the second condition represents that start of the following flight segment and is denoted by t_i;

The weak firing strategy is used in *TIPN*, which means that the state enabled transition cannot fire until $(b-a)$ unit time passes; the firing process is immediate. If the transition $t_{i,m}\in T$ is firing, in new marking $\boldsymbol{M}'$, we get:

(1) $\boldsymbol{M}'(p_i)$=<0>;

(2) $\boldsymbol{M}'(p_m)=\pi_k$ =<$rr'(a)$>.

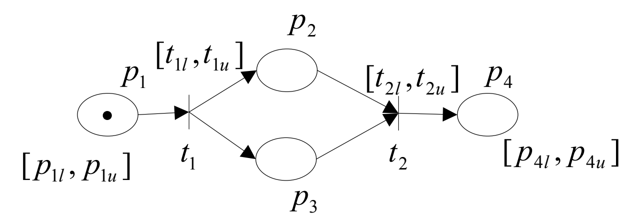


Fig. (1). One TIPN Model Example.

3. LINEAR INFERENCE OF TIPN

3.1. Description of TIPN Linear Logic

The connection symbols of linear logic are defined as follows:

(1) $\otimes$ means the "and" relationship between resources. For proposition A, $A\otimes A=2A$.

(2) $\oplus$ means the "or" relationship between resources. For proposition A, $A\oplus A=A$.

(3) $-\circ$ means the "infer" relationship. For proposition A and B, $A -\circ B$ means B can be inferred by A, while A doesn't exist.

(4) $|-$ means the "prove" relationship between resources, for example, $\Gamma |- \Delta$ means Δ can be proved by Γ , Δ and Γ are resource sets.

For any *TIPN* model, current markings can be described as $\mathbf{M}=\otimes P_k^{mk}$ by linear logic, in which, P_k is a place set; mk is the number of token in the model; the infer relationship between input and output place set can be described by transition t_j (or the symbol $-\circ$ in liner logic)

One *TIPN* model example is demonstrated in Fig. (**1**), and can be described with linear logic as follows:

t_1 : $p_1 -\circ p_2\otimes p_3\ \theta(t_1)$,

t_2 : $p_2\otimes p_3 -\circ p_4\ \theta(t_2)$;

$\mathbf{M}_0=p_1^1$, $\mathbf{M}_1=p_2^1\otimes p_3^1$, $\mathbf{M}_2=p_4^1$

3.2. Simplification Rules for Linear Regulation of TIPN

Firstly, three time calculation functions are defined as follows:

(1) Minimize function for single time interval $\alpha=[\tau_1, \tau_2]$ is expressed with M and M $(\alpha)=\tau_1$;

(2) Span function for single time interval $\alpha=[\tau_1, \tau_2]$ is expressed with D, and $\mathrm{D}(\alpha)=\tau_2-\tau_1$;

(3) Summation function for two time intervals $\alpha=[\tau_1, \tau_2]$, $\beta=[\tau_3, \tau_4]$ is expressed with L, and L $(\alpha, \beta)=\mathrm{D}(\alpha)+\mathrm{D}(\beta)$;

Secondly, we define simplification rules among *TIPN* transitions based on the linear logic, taking the *TIPN* model

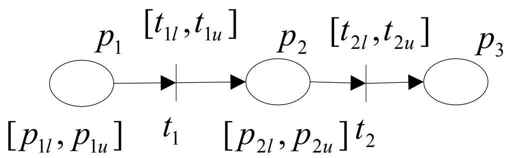


Fig. (2). The Model of the Order Triggering Relationship between Transitions.

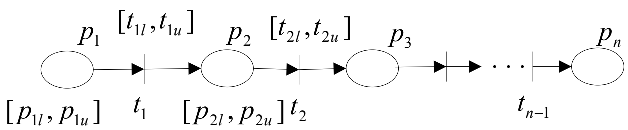


Fig. (3). The Model of Order Triggering Relationship between Transition Orders.

features and requirements of flight delay propagation analysis into account. What's more, we provide the time interval computation methods for *TIPN* transition and transition sequence.

Rule 1: For two transitions described by t_1: p_1-°p_2 $\theta(t_1)$ and t_2: p_2-°p_3 $\theta(t_2)$, if the firing of transition t_1 is based on the firing result of transition t_2, then, we call the relationship between transition t_1 and t_2 as orderly firing and meets the following rules:

$$t = t_1 \bullet t_2 : p_1 - \circ p_3, \theta(t).$$

which is demonstrated in Fig. (**2**).

In which, the symbol "$\bullet$" represent the sequence firing relationship for transition sequence, $\theta(t)=\theta(t_1)+\theta(t_2)$, $\theta(t_1)=\mathrm{L}(\Phi_p(p_1,a),\Phi_t(p_1,a))$, $\theta(t_2)=\mathrm{L}(\Phi_p(p_2,a), \Phi_t(p_2,a))$.

Rule 2: For transition sequence $r(a)= t_1t_2\ldots t_n$, if each two neighboring transitions have an orderly firing relationship, then, rule 1 is used recursively and the orderly firing relationship for transition sequence can be induced as follows:

$t=t_1t_2\ldots t_{n-1}$: p_1-°p_n $\theta(t)$

which is demonstrated in Fig. (**3**).

In which,

$$\theta(t) = \sum_{i=1}^{n-1} \theta(t_i)a \text{ , } \theta(t_i) = \mathrm{L}(\Phi_p(p_i,a),\Phi_t(t_i,a)).$$

4. THE FLIGHT DELAY PROPAGATION ANALYSIS ALGORITHM BASED ON TIPN MODEL

The sub-model of flight path should be extracted according to the specific flight before analyzing the flight delay propagation using *TIPN* model.

The sub-model for one flight delay propagation can be extracted from *TIPN* model based on its flight routes. The sub-model has the following characteristics:

(1) There is only one start and end place in the sub-model, which have different names and represents the occupation state of flight for the source airport;

(2) There is no cycle in sub-model. The above two features are an aid to model the flight's flying process and the final state, which is corresponds to flight reaching the destination airport, which can be induced by the simplification rules in section 3.2.

Alogrithm1: flight delay propagation analysis algorithm based on *TIPN* model

Input: *TIPN* model, flight route $r(a)$, flight delay in one airport $X(p_i,a)$, delay redundancy calculator $\Delta(p_k)$;

Output: flight delay state and level in the next following airport which corresponding to the place in *TIPN* model.

Step 1: construct the *TIPN* model corresponding to the considered problem

Step 2: get the sub-model relevant to the flight route $r(a)$, using place representing the airports which will be occupied by flight in its routes;

Step 3: For each place p_k, simplify the sub-model recursively using rules in section 2.2; Note: for place p_k after the place p_i in flight route $r(a)$, $\Phi_p(p_k,a)$ is $[0,\phi(p_k,a)]$;

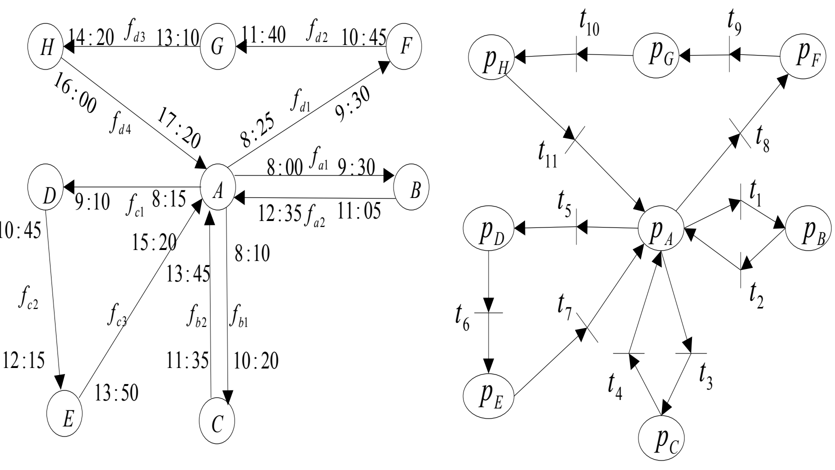
Step 4: For the time interval $[0,\phi(p_k,a)]$ from step 3, the time interval for place p_k can be denoted by interval $[\tau(p_k),\tau(p_k)+\phi(p_k,a)]$, in which $\tau(p_k)$ = $M(\Phi_p(p_i,a))+X(p_i,a)+\theta(t)$.

Step 5: For the time interval from step 4, if $\tau(p_k)+\phi(p_k,a) > D(\Phi_p(p_i,a)) + \Delta(p_k)$, then the flight in airport represented by place p_k will be delayed, and the corresponding delay time is $\tau(p_k)+\phi(p_k) - D(\Phi_p(p_i))$; or else, the flight in this airport will not be delayed.

Step 6: For other place after the place p_k in the following consecutive flight route, step3-step5 can be used in the same way.

COMPLEXITY ANALYSIS

The flight delay propagation analysis is operated in real time, it is necessary to analyze the algorithm complexity. First, assuming that the number of source airport where initial flight delay exist is s in the current state, and the number of airports for each flight in next flight segment is n, the time complexity for deriving the sub-model which contributes to the flight delay analysis from *TIPN* can be described as $O(xD)$; Second, Assuming that the number of airports for each flight in next flight segment is n, then the time complexity of flight delay analysis is $O(n(n+1)/2)$, and also, the time complexity of flight delays analysis in all sub-models can be represented by $O(sn^2)$. Thus, the time complexity of the algorithm is $O(s(n^2+n))$. Generally, the number of flights of one aircraft is relatively small, which means n is small; the number of flights influenced by the delay from source airport is small, which means s is not very big. So, time



(1) Flight informaiton graph for four aircrafts (2) *TIPN* model for flight operation

Fig. (4). Flight Information and Its *TIPN* Model.

Table 1. Place Properties for Model in Fig. (4b).

Model Elements	Meaning	Φ_p	$\varphi(P)$
p_A	Flight occupying airport A	$\Phi_p(p_A,a_1)$=[8:00,8:00] $\Phi_p(p_A,b_1)$=[8:10,8:10] $\Phi_p(p_A,c_1)$=[8:15,8:15] $\Phi_p(p_A,d_1)$=[8:25,8:25]	$\varphi_p(p_A,a_1)$=55 $\varphi_p(p_A,b_1)$=55 $\varphi_p(p_A,c_1)$=55 $\varphi_p(p_A,d_1)$=55
P_B	Flight occupying airport B	$\Phi_p(p_B,a_1)$=[9:30,11:05]	$\varphi_p(p_B,a_1)$=55
P_C	Flight occupying airport C	$\Phi_p(p_C,b_1)$=[10:20,11:35]	$\varphi_p(p_C,b_1)$=55
P_D	Flight occupying airport D	$\Phi_p(p_D,c_1)$=[9:10,10:45]	$\varphi_p(p_D,c_1)$=55
P_E	Flight occupying airport E	$\Phi_p(p_E,c_1)$=[12:15,13:50]	$\varphi_p(p_E,c_1)$=55
P_F	Flight occupying airport F	$\Phi_p(p_F,d_1)$=[9:30,10:45]	$\varphi_p(p_F,d_1)$=55
P_G	Flight occupying airport G	$\Phi_p(p_G,d_1)$=[11:40,13:10]	$\varphi_p(p_G,d_1)$=55
P_H	Flight occupying airport H	$\Phi_p(p_H,d_1)$=[14:20,16:00]	$\varphi_p(p_H,d_1)$=55

complexity of the algorithm can be accepted and it can be used for real time flight delay analysis.

5. CASE STUDY

Take the example in reference [2] as an example. The flight operation process of four aircrafts from source airport A is modeled. In which, aircraft *a* and *b* return to the source airport after finishing two flights, aircraft *c* returns to the source airport after finishing three flights. According to the advisory of civil aviation flight statistical method, the flight taking off during the interval $[T_s, T_s+\Delta(p_k)]$ is accepted and is normal, in which, T_s is the planned departure time, $\Delta(p_k)$ is the redundant operator dependent on individual airport. In this example, we assume $\Delta(P_A)$= $\Delta(P_B)$= $\Delta(P_E)$= $\Delta(P_H)$=25min, $\Delta(P_C)$= $\Delta(P_D)$= $\Delta(P_F)$= $\Delta(P_G)$=15min. The flight turnaround time is $T_t = 55\text{min}$ and is assumed the same in each airport. Flight information graph and its *TIPN* model are demonstrated in Fig. (**4**).

Place and transition properties of *TIPN* model are demonstrated in Table **1** and Table **2**, respectively.

Mapping function $\Phi_{\mathrm{p}}: P\times F \rightarrow \{[a,b] \mid 0 \le a \le b \le \infty\}$ in *TIPN* could describe the planned turnaround time in one airport. For simplification, we assume the start and end time for flight in source airport is the planned arrival and departure time. Mapping function $\varphi: P\times F \rightarrow \{z \mid z \in R^+\}$ could describe the smallest turnaround time.

Function $\Phi_{\mathrm{t}}: T\times F \rightarrow \{[a,b] \mid 0 \le a < b \le \infty\}$ could describe the interval for the operation of one specific flight. Additionally, according to the flight plan, the flight route can be described by transition $\pi_a =< t_1 t_2 >$ 、 $\pi_b =< t_3 t_4 >$ 、 $\pi_c =< t_5 t_6 t_7 >$ 、 $\pi_d =< t_8 t_9 t_{10} t_{11} >$.

Table 2. Transition Properties for Model in Fig. (4b).

Model Elements	Meaning	Φ_t
t_1	Flight a1 from airport A to B	$\Phi_t(t_1,a_1)$=[8:00,9:30]
t_2	Flight a2 from airport B to A	$\Phi_t(t_2,a_2)$=[11:05,12:35]
t_3	Flight b1 from airport A to C	$\Phi_t(t_3,b_1)$=[8:10,10:20]
t_4	Flight b2 from airport C to A	$\Phi_t(t_4,b_2)$=[11:35,13:45]
t_5	Flight C1 from airport A to D	$\Phi_t(t_5,c_1)$=[8:15,9:10]
t_6	Flight C2 from airport D to E	$\Phi_t(t_6,c_2)$=[10:45,12:15]
t_7	Flight C3 from airport E to A	$\Phi_t(t_7,c_3)$=[13:50,15:20]
t_8	Flight d1 from airport A to F	$\Phi_t(t_8,d_1)$=[8:25,9:30]
t_9	Flight d2 from airport F to G	$\Phi_t(t_9,d_2)$=[10:45,11:40]
t_{10}	Flight d3 from airport G to H	$\Phi_t(t_{10},d_3)$=[13:10,14:20]
t_{11}	Flight d4 from airport H to A	$\Phi_t(t_{11},d_4)$=[16:00,17:20]

Table 3. The Flight Delay Propagation on Downstream Airport under Different Delay Level (unit: min).

Airport		T_{id}			
		60	**90**	**150**	**180**
A	Delay	1	1	1	1
	T_d	60	90	150	180
B	Delay	0	1	1	1
	T_d	20	50	110	140
C	Delay	1	1	1	1
	T_d	40	70	120	160
D	Delay	1	1	1	1
	T_d	20	50	110	140
E	Delay	0	0	1	1
	T_d	0	10	70	100
F	Delay	1	1	1	1
	T_d	40	70	130	160
G	Delay	0	1	1	1
	T_d	5	35	95	125
H	Delay	0	0	1	1
	T_d	0	0	50	80
Number of airport influenced by fight delay		4	6	8	8

Assume one flight from airport A is delayed, and the delay time is not mitigated by the acceleration in the consecutive flight segment, the propagation of flight delay is analyzed and demonstrated in Table **3** by *TIPN* model and the corresponding algorithm under different flight delay levels.

Obviously, the affected downstream airports is limited when the initial delay time T_{id} is short, and the number of affected downstream airports grows when T_{id} becomes longer. However, the initial delay will affect all downstream airports when T_{id} reaches a threshold value, while the delay time in each airport will decrease.

From the analysis result in Table **3**, we could come to the conclusion that if the initial flight delay T_{id} in source airport is relatively small, the flight delay will be propagated to downstream airports, and vice versa. However, when the initial flight delay T_{id} reaches a threshold value, then all downstream airports will be influenced by flight delays, but the flight delay in each airport will become smaller after every airport.

CONCLUSION

Time Petri net model for flight operation and the corresponding linear logic-based simplification method for it are proposed. The real time algorithm for flight delay propagation analysis is provided which supports the analysis of the influence of delay in downstream airports. The proposed model and algorithm in this paper will contribute to flight operation scheduling, flight delay alerting and decision-making. As flight operations are highly affected by uncertain environment, it is necessary to make the flight delay propagation analysis algorithm more feasible and real time.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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