Research on Image Segmentation Algorithm Based on Entropy and PSO Algorithm

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Abstract: Because image segmentation is the base of image identification, analysis and interpretation, the image segmentation has been widely used in many fields. And PSO (Particle Swarm Optimization) algorithm is one of the most common image segmentation algorithms. However, it has the problems of premature convergence and local optimum. To solve the problems, ISABEP (Image Segmentation Algorithm Based on Entropy and PSO) algorithm has been proposed. Simulation demonstrates that the proposed algorithm has fast convergence speed.

1. INTRODUCTION

Under research of the image, people are only interested in some parts of the images, which is called target, and the rest is called background. According to the different characteristics of the image, image segmentation as a kind of target extraction technology, can divide the image into several areas, one or several of which are the targets [1]. The characteristics may be color, grayscale, texture, and so on. Meanwhile, the accuracy of image segmentation directly affects the effectiveness of the subsequent image processing, therefore, scholars and enterprises always pay close attention to the development and the use of image segmentation [2, 3]. To meet different applications, many image segmentation algorithms are proposed. However, the most of them are only applicable to certain applications. Besides, there isn't a unified selection standard, which brings many problems to hinder the use of image segmentation.

At present, image segmentation algorithms can be divided mainly into three class: threshold algorithm, edge detection algorithm, splitting algorithm and merging algorithm [4]. Among them, threshold algorithm is the most used. The basic idea of the threshold algorithm is that it firstly selects an image grey-scale value as the threshold, and then every pixel in the image will be compared with the threshold, finally, the image can be divided into two areas. The selection of threshold impacts on the effect of the segmentation significantly. Therefore, a lot of studies focus on how to select the effective and robust threshold. PSO (Particle Swarm Optimization) algorithm as one of most used threshold segmentation has many advantages, such as uncomplicated, easy to control and fast convergence [5]. Hence, it has been widely used. However, it has the problems of premature convergence and local optimum. To solve the problems, this paper proposes ISABEP (Image Segmentation Algorithm Based on

Entropy and PSO) algorithm. The core idea of ISABEP is the use of the PSOBM (PSO Based on Morlet) algorithm and MFE(Maximum Fuzzy Entropy) algorithm [6] to improve the performance of the image segmentation.

2. RELATED THEORIES

2.1. Concept of Image Segmentation

The concept of image segmentation is [7]: if R represents an image, the segmentation of R can be considered that R is divided into N non-empty subsets, R_1, R_2, L, R_N, $N \geq 1$, and all subsets must meet the following conditions:

Condition 1: $R = R_1 \cup R_2 \cup L \cup R_N$;

Condition 2: $R_i \cap R_j = \Phi, 1 \leq i, j \leq N, i \neq j$;

Condition 3: $P(R_i) = True, i = 1, 2, L, N$;

Condition 4: $P(R_i \cap R_j) = False, 1 \leq i, j \leq N, i \neq j$;

Condition 5: $R_i, i = 1, 2, L, N$ is a connected region.

Condition 1 means that after the image segmentation, union of all subsets is equal to the original image, and every pixel in the original image belongs to one of the subsets. Condition 2 means that after the image segmentation, a pixel in the original image can only belongs to one subset. Condition 3 means that every subset shouldn't be a null set. Condition 4 means that any two subsets shouldn't overlap. Condition 5 means that any two pixels in the same subset can be connected.

2.2 MFE Algorithm

Suppose that X is the collection of objects denoted by x, then the fuzzy set A in X is a set of ordered pairs: $A = \{(x_i, u_A(x_i)) | x_i \in X\}$, $u_A(x_i)$ is the membership function of x_i to A. Therefore, the fuzzy entropy $E(A)$ is [8]:

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$$E(A) = \frac{1}{n\ln 2}\sum_i S(u_A(x_i)), i=1,2,\text{L},n \qquad (1)$$

$S(u_A(x_i))$ is the entropy function, and the value is given as follow:

$$S(u_A(x_i)) = -u_A(x_i)\ln[u_A(x_i)]$$
$$-[1-u_A(x_i)]\ln[1-u_A(x_i)] \qquad (2)$$

X is a gray-scale image which is made of $M*N$ pixels. And the greatest gray value of X is L. x_{mn} is the gray value of pixel coordinates (m, n). The fuzzy entropy of two-dimensional image given can be calculated as follows:

$$E(X) = \frac{1}{MN\ln 2}\sum_m\sum_n S(u_A(x_{mn})) \qquad (3)$$

In the gray-scale histogram, the horizontal axis is the gray value and the vertical axis is the number of pixels h(g), therefore, Eq. (3) can be written:

$$E(X) = \frac{1}{MN\ln 2}\sum_n S(u_A(g))h(g) \qquad (4)$$

If $u_1(g)$ and $u_2(g)$ are respectively the membership of target and the membership of background, according to S function belonging to fuzzy set theory, the values of $u_1(g)$ and $u_2(g)$ are:

$$u_1(g) = \begin{cases} 0, 0 \le g \le a \\ 2\left|\dfrac{g-a}{c-a}\right|^2, a \le g < 1 \\ 1-2\left|\dfrac{c-g}{c-a}\right|^2, t \le g < c \\ 1, c \le g < L \end{cases} \qquad (5)$$

$$u_2(g) = 1-u_1(g)$$

Where $[a,c]$ is the range of gray value, and $t=(a+c)/2$. The entropy of Area A and Area B is :

$$E(A) = \frac{1}{\ln 2}\sum_{g=0}^{t-1} S(u_1(g))h(g)$$
$$E(B) = \frac{1}{\ln 2}\sum_{g=t}^{L-1} S(u_2(g))h(g) \qquad (6)$$

The fuzzy entropy of image is:

$$E(X) = \frac{1}{Sum(X)}(E(A)+E(B)) \qquad (7)$$

Where Sum(X) is the sum of pixels of the image X.

2.3 PSOBM Algorithm

In 1995, PSO algorithm was proposed by Kennedy. PSO is an iterative algorithm, which stems from the simulation of birds flock's looking for foods [9, 10]. The workflow of PSO is that: firstly, it chooses a set of particles as the initial population, and every particle is a feasible solution of the optimization problem. Secondly, every particle moves towards the optimal particle. Lastly, through many iterations, the rest of the particle is chosen as the solution of the optimization problem.

The mathematical description of PSO is: global optimization problem can be defined as $(P):\min\{f(x):x\in\Omega\subseteq R^n\}$, $f:\Omega\subseteq R^n\to R^l$, where (P) is the set of feasible solutions. The number of (P) is called scale, and the member of (P) is called particle. The speed and position of the i-th particle are respectively $V_i=(v_{i1},v_{i2},\text{L},v_{in})^T\in\Omega$, $X_i=(x_{i1},x_{i2},\text{L},x_{in})^T\in\Omega$. In the movement of the particles, $P_i=(p_{i1},p_{i2},\text{L},p_{iM})$ is used to record the optimum position of the i-th particle. İndex number g is used to mark the optimum position of the whole particles. The updates of the speed and position of every particle separately follow Eq. (8) and Eq. (9):

$$x_{ij}(t+1) = x_{ij}(t)+v_{ij}(t+1) \qquad (8)$$

$$v_{ij}(t+1) = wv_{ij}(t)+c_1 rand_1(p_{ij}(t)-x_{ij}(t))$$
$$+c_2 rand_2(p_{gj}(t)-x_{ij}(t)) \qquad (9)$$

Where $i=1,2,\text{L},M$, $d=1,2,\text{L},n$, $rand_1$ and $rand_2$ are random numbers which follow $U(0,1)$. c_1 and c_2 are non-negative constant coefficients, and $c_1,c_2\in[0,2]$. w is the inertia coefficient.

Although PSO has many advantages, such as uncomplicated, easy to control and fast convergence, it still has obvious drawbacks of premature convergence and local optimum. To overcome the shortage of traditional PSO, people propose many improved algorithms. The essence of these algorithms is only the static adjustment of w or the change of the particle. However the PSOBM can not only dynamically adjust the value of w but also change some particles.

In PSOBM, the value of w is:

$$\omega=\omega_0\times(1-(dist_i/dist_{\max})) \qquad (10)$$

Where ω_0 is a random number which follows $U(0.5,1)$, $dist_i$ is distance between the current particle and the global optimal value, $dist_i=(\sum_{j=1}^n(p_{gj}-x_{ij}))^{1/2}$, and $dist_{\max}=\max(dist_i)$, $i=1,2,\text{L},M$. Adjusting the value of w can make sure that the particle is always around the global optimal value, which can effectively avoid the occurrence of the premature convergence. To further improve the performance, the Morlet conversion is used for the particle with the probability $p_m\in[0,1]$ in PSOBM. The formula of the Morlet conversion is :

$$mut(x_{ij}(t)) = \begin{cases} x_{ij}(t)+\sigma\times(x_{\max}-x_{ij}(t)), \sigma>0 \\ x_{ij}(t)+\sigma\times(x_{ij}(t)-x_{\min}), \sigma\le 0 \end{cases} \qquad (11)$$

Where $mut(x_{ij}(t))$ is the new position of $x_{i,j}(t)$, x_{min} and x_{max} are respectively the minimum value of x and the maximum value of x, the value of σ is given as follows:

$$\sigma = \frac{1}{\sqrt{a}} e^{-(\phi/a)^2/2} \cos(5(\phi/a)) \tag{12}$$

Where, $\sigma = \frac{1}{\sqrt{a}} e^{-(\phi/a)^2/2} \cos(5(\phi/a))$, and the a is:

$$a = e^{-\ln(g)(1-t/t_{max})^{\xi wm} + \ln(g)} \tag{13}$$

Where g is the upper boundary of a, ξwm is the shape parameter of monotonic increasing equation, t is the current iterations, and t_{max} is the maximum current iterations.

3. ISABEP ALGORITHM

In order to further improve the image segmentation result, this paper proposes ISABEP algorithm. In ISABEP algorithm, the fuzzy entropy is introduced into PSOBE. According to the fuzzy theory, an image is equivalent to a fuzzy event, therefore, the image can be divided into two fuzzy sets d and b by threshold T. Suppose that $\mu_d(k)$ and $\mu_b(k)$ are respectively the membership function of d and the membership function of b. H_d and H_b are respectively the entropy of d and the entropy of b. Their value can be calculated by the following equations:

$$\mu_d(k) \begin{cases} 1, k \leq a; \\ 1 - \dfrac{(k-a)^2}{(c-a)\times(b-a)}, & a < k \leq b \\ \dfrac{(k-c)^2}{(c-a)\times(c-b)}, & b < k \leq c \\ 0, & c < k \end{cases} \tag{14}$$

$$\mu_b(k) \begin{cases} 0, k \leq a \\ \dfrac{(k-a)^2}{(c-a)\times(b-a)}, & a < k \leq b \\ 1 - \dfrac{(k-c)^2}{(c-a)\times(c-b)}, & b < k \leq c \\ 1, & c < k \end{cases} \tag{15}$$

$$H_d = \sum_{k=0}^{255} \frac{p_k \mu_d(k)}{p_d} \ln\left(\frac{p_k \mu_d(k)}{p_d} \right) \tag{16}$$

$$H_b = \sum_{k=0}^{255} \frac{p_k \mu_b(k)}{p_b} \ln\left(\frac{p_k \mu_b(k)}{p_b} \right) \tag{17}$$

Where $0 \leq a < b < c \leq 225$, $b = (a+c)/2$, $p_k = n_k/(m \times n)$, and n_k is the number of elements in D_K, therefore the total fuzzy entropy of the fuzzy event is:

$$H(a,c) = H_d + H_b \tag{18}$$

According to the entropy theory, when the total fuzzy entropy of the fuzzy event is the maximum, the target can be effectively separated from the background. Suppose that when $(a,c) = (a_{opt}, c_{opt})$, $H(a,c)$ reaches the maximum, so the optimal threshold is:

$$T_{opt} = b_{opt} = (a_{opt} + c_{opt})/2 \tag{19}$$

The details of ISABEP are described as follows.

(1) Initialization. Suppose that N is the number of particle swarm, V is the speed matrix of particle swarm, X is the position matrix, and D is the dimension. İnitialization of N, V, X and D are done.

$$x_{ij} = x_{min} + (x_{max} - x_{min}) \times rand \tag{20}$$

$$X = \begin{vmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ M & M \\ x_{N1} & x_{N2} \end{vmatrix} \tag{21}$$

$$v_{ij} = -v_{max} + 2v_{max} \times rand \tag{22}$$

$$V = \begin{vmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \\ M & M \\ v_{N1} & v_{N2} \end{vmatrix} \tag{23}$$

Where rand is the random number, which follows $U(0,1)$, v_{max} is the maximum of v, x_{max} and x_{min} are respectively the maximum of x and the minimum of x. İn general, $x_{max} = L_{max}$, $x_{min} = L_{min} + 1$, L_{min} and L_{max} are respectively the minimum of the image and the maximum of the image.

(2) The fuzzy entropy is calculated by Eq. (**18**), then select the current optimal position of P_i and the optimal position of the total particle set P_g.

(3) According to the above formulas, the value of w is calculated by Eq. (**10**).

(4) The position and speed of every particle are calculated by Eq. (**8**) and Eq. (**9**).

(5) Select the particles by probability p_m, and the selected particle carries through the Morlet conversion on the basis of Eq. (**11**).

(6) If the iterations reach the maximum times, go to step 7. Otherwise, go to step 2.

(7) The threshold T_{opt} is calculated by the selection (a, c), and then T_{opt} is used to carry out the image segmentation.

4. SIMULATION RESULTS

To verify the effectiveness and efficiency of ISABEP proposed in this paper, we conduct extensive simulations for performance evaluation. We compare the performances of traditional PSO, PSOBM and ISABEP on the segmentation effectiveness. The images of Lena, Camel and Cinder are used in the experiment, as shown in Fig. (**1**). The three images have the different characteristics. Lena, Camel and Cinder are respectively multimodality, bimodality and unimodality, and the histograms of them are as shown in Fig. (**2**).


(a)   Lena

(b)   Camel

(c)   Cinder

Fig. (1). Original images.
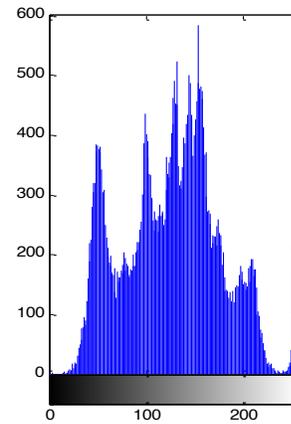

(a)   Lena
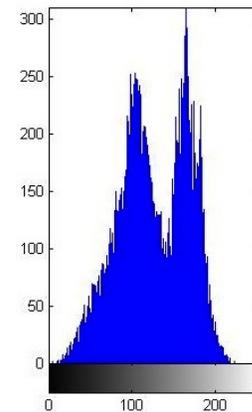
(b)   Camel
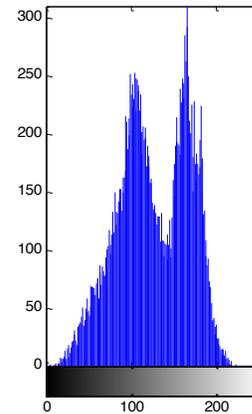
(c)   Cinder

Fig. (2). Histogram.

The simulation parameters are set up as follows:

N = 10
D = 2
tmax = 100
wmax = 0.9
wmin = 0.4
c1 = c2 = 1.49
pm = 0.5
g = 1000
$\xi_{wm} = 2$
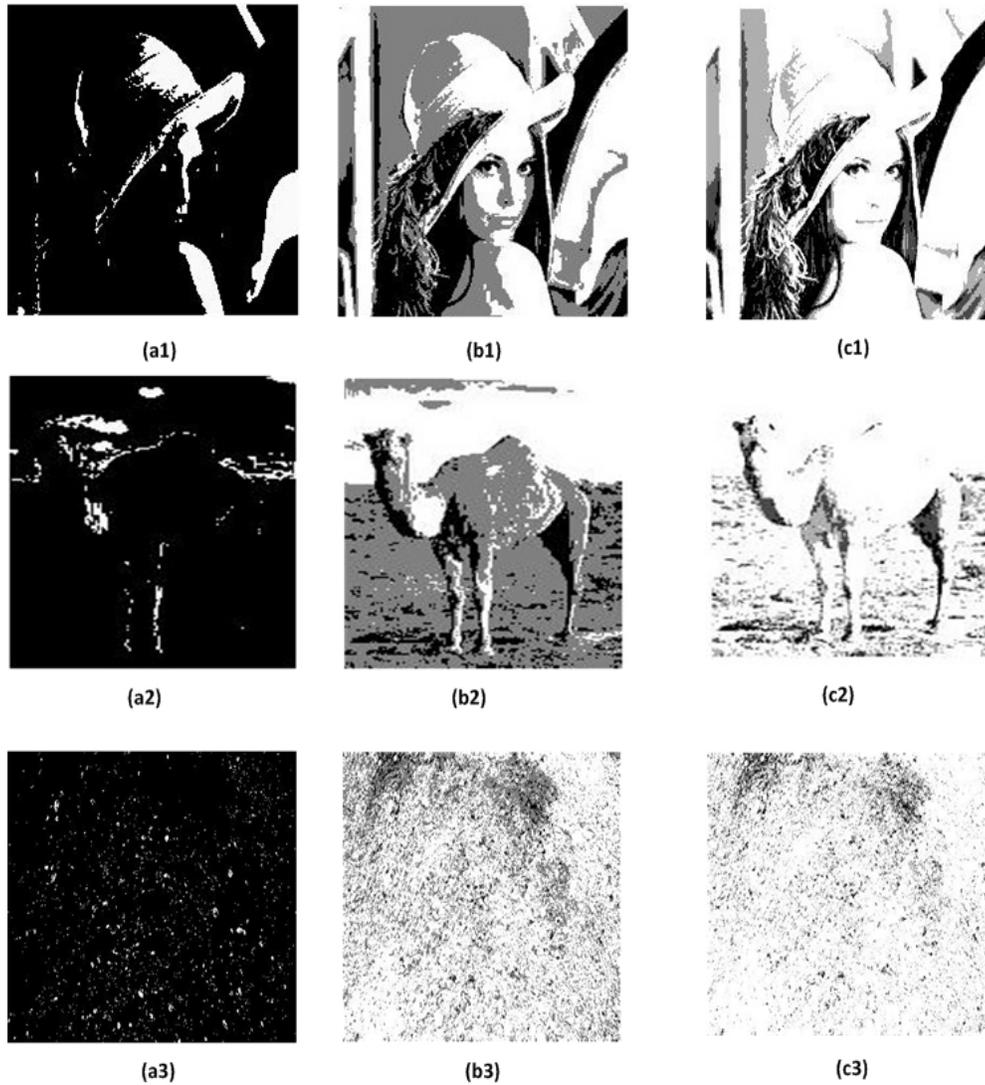
Fig. (3). Segmentation effectiveness.

Fig. (**3**) gives the segmentation effectiveness. a1~a3 is the segmentation effectiveness of PSO, b1~b3 is the segmentation effectiveness of PSOBM, and c1~3 is the segmentation effectiveness of ISABEP. From Fig. (**3**), we can see that the performance of ISABEP is the best.

CONCLUSION

To solve the premature convergence and local optimum of PSO, we propose ISABEP. ISABEP uses PSOBM algorithm and MFE algorithm to improve the performance of the image segmentation. And simulations have verified the effectiveness of ISABEP.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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