Random Response of Rotor System on the Filtered Gaussian White Noise Earthquake Ground Acceleration

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Abstract: With the pseudo-excitation method, the random response of a cantilever double-disc rotor system under the filtered Gaussian white noise earthquake ground accelerated random excitations is analyzed by computer numerical simulation. The results show that the random response is relatively strong during an earthquake of the horizontal direction. The cantilever disk vibration is the most strong. The inherent frequency of the site soil has a great effect on the random response on the horizontal direction, while the system frequency characteristics has a great effect on the random response of the vertical direction. The rotor imbalance random excitation intensity does not change the peak frequency of the response spectrum. The system random vibration response is large when the excitation intensity is in a high level state. It has a great impact on the low and high frequency of non-cantilever disc, the low frequency of the cantilever disc, the high-frequency of non-cantilever bearing vertical vibration and the high-frequency of cantilever bearing the intensity around the high-frequency random response.

Keywords: Earthquake, filtered Gaussian white noise, pseudo excitation method, random response, rotor system.

1. INTRODUCTION

The development of high performance and increased efficiency rotating machines have prompted the researchers worldwide to review and research rotating machines in a new light [1-7]. In the seismic design, it is a important to research the vibration characteristics of generator under the earthquake conditions. The random response analysis of complex rotor system is important to ensure that the system is operated safely and reliably. Due to the difficulties and complexity of such a problem there is limited research in this area and the numerical calculation for the problems are rare as well [8,9]. The method for the analysis of rotor system random response is the Monte Carlo method and the power spectrum algorithm. The researchers are often discouraged by the drawback of these methods as it involves complex calculations and the expensive computation time. The pseudo-excitation method which is proposed by Professor Lin Jiahao is an effective method to analysis linear random vibration system [10, 11]. This method transforms the stationary random excitation into steady state harmonic excitation. It transforms the non-stationary random excitation into the transient uncertainty excitation. This method has an advantage of being simple, efficient and accurate. With the pseudo-excitation method the cantilever double-disc rotor system under the filtered Gaussian white noise earthquake ground acceleration random excitations are analyzed by computational numerical simulation. It provides the basis for the rational selection and random control of the parameters of the rotor system.

2. SYSTEM MECHANICAL MODEL AND DIFFERENTIAL EQUATIONS OF MOTION

Mechanical model of the rotor system is set up in Fig. (1). We suppose that casing is an elastic ring of the linear radial contact stiffness. Stochastic differential equations of motion for the system under the effect of random excitation force is given by equation 1.

\[ M \ddot{u} + (C + G) \dot{u} + Ku = F \]  

Fig. (1). Mechanical model of system.

The displacement array, random excitation force array, the mass matrix, the gyroscopic matrix, the damping matrix and the stiffness matrix are given, respectively.

\[ u = [u_r, u_{rd}, u_{rn}]^T \]

and \[ u_r = [x_1, \theta_1, x_2, \theta_2, y_1, \theta_1, x_2, \theta_2]^T \].

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and is reaction where excitations are white noise earthquake, $\text{is}$

\[
T = F_1 + F_2,
\]

where $F_1$ includes gravity and the rotor imbalance random excitation force. $F_2$ is ground random earthquake excitation. $T$ $F_1$ can be expressed as

\[
F_1 = \begin{bmatrix} F_{1x}, 0, F_{2x}, 0, F_1, 0, F_2, 0, F_{3x}, F_{5x}, F_{7x} \end{bmatrix}^T
\]

\[
F_2 = m_1 e \omega^2 \cos(\omega t + \phi_1) - m_g
\]

\[
F_3 = m_1 e \omega^2 \sin(\omega t + \phi_2) F_2 = m_2 e \omega^2 \sin(\omega t + \phi_2)
\]

\[
F_{3x} = \begin{bmatrix} m_2 g - m_2 \omega^2 \end{bmatrix}^T F_{5x} = \begin{bmatrix} m_2 g - m_2 \omega^2 \end{bmatrix}^T F_{7x} = \begin{bmatrix} m_2 g, 0, -m_2 g, 0 \end{bmatrix}^T
\]

where $\phi_1$ and $\phi_2$ are random variables of uniformly distributed $[0, 2\pi]$.

The $F_2$ can be expressed as

\[
F_2 = -\lambda E \ddot{Z}_g (t)
\]

(2)

where $Z(t)$ is the ground acceleration random earthquake, we suppose that $z(t)$ is the filtered Gaussian white noise earthquake ground acceleration random excitations process, the power spectral density is given by [12]

\[
S_w(\omega) = \frac{1 + 4\left(\frac{\xi_s \omega}{\omega_n}\right)^2}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + 4\left(\frac{\xi_s \omega}{\omega_n}\right)^2} \frac{1}{1 + \frac{\omega^2}{\omega_n^2}} S_0
\]

where $\omega_n$ and $\xi_s$ are the inherent angular frequency of site soil and damping ratio respectively. $\omega_n$ is the parameters of reaction bedrock spectral characteristics, $S_0$ is the bedrock acceleration power spectral density which is a spectral parameter of the reaction of ground motion intensity level. The stationary white noise earthquake excitation is given by $S_w(\omega) = \overline{S}_w$. For hard surface soil, the inherent angular frequency of the site and damping ratio are given by $\omega_n = 5\pi$, $\xi_s = 0.063$, $\omega_n = 8\pi$. Fig. (2) gives an example of the filtered Gaussian white noise random excitations earthquake spectrum. In Fig. (2), the peak frequency is 11 rad / s, the spectral peak is 1.438$\overline{S}_w$.

Analysis shows that the random response of rotor system is relatively strong during an earthquake of the horizontal direction, so the inertial force instruction vector can be derived as

\[
E = \begin{bmatrix} 100 0 0 0 0 1 0 1 0 1 0 1 0 \end{bmatrix}^T
\]

\[
M = \text{diag}(M_z, M_x, M_y, M_{5678})
\]

and $M_z = \text{diag}(m_3, m_4, m_3, m_4)$,
\[
M \frac{d^2 \ddot{u}}{dt^2} + (C + G) \frac{d\ddot{u}}{dt} + K\ddot{u} = \ddot{F}
\]

where the pseudo-excitation can be expressed as
\[
\ddot{F} = \ddot{F}_1 + \ddot{F}_2
\]

Solving virtual vibration differential equations, we can get virtual response
\[
\ddot{u} = \left[ \ddot{u}_1, \ddot{u}_{34}, \ddot{u}_{5678} \right]^T
\]
\[
\ddot{u}_i = \left[ \ddot{x}_1, \ddot{y}_1, \ddot{x}_2, \ddot{y}_2, \ddot{x}_3, \ddot{y}_3, \ddot{x}_4, \ddot{y}_4 \right]^T
\]
\[
\ddot{u}_{5678} = \left[ \ddot{x}_5, \ddot{y}_5, \ddot{x}_6, \ddot{y}_6, \ddot{x}_7, \ddot{y}_7, \ddot{x}_8, \ddot{y}_8 \right]^T
\]

The displacement power spectral matrix of random response is given by
\[
S_{\ddot{w}} = \ddot{u}^\ast \ddot{u}'
\]

where the superscript "\*" represents the complex conjugate numerical calculation.

Fig. (2). Gaussian filtered white noise earthquake spectrum.

4. NUMERICAL CALCULATION OF SYSTEM RANDOM RESPONSE

The standard values are inserted the in numerical calculation of random response, such as, the density of the material of the shaft is \( \rho = 7.8 \times 10^3 \text{ kg/m}^3 \), the elastic modulus is
\[ E = 210 \times 10^3 \text{ N/m}^2 \]
the deep groove ball bearing, \( \alpha = 0.36 \), the casing damping is \( C_{w} = C_{v} = 20 \text{ Ns/m} \) \((i = 5, 6, 7, 8)\), the casing internal damping is \( C_{\text{c}} = 20 \text{ Ns/m} \), the outside damping is \( C_{\text{e}} = C_{\text{c}} = 350 \text{ Ns/m} \) \((i = 1, 2)\), the casing stiffness is \( k_{w} = k_{v} = k_{p} = 10^8 \text{ N/m} \) \((i = 5, 6, 7, 8)\), the rolling stiffness is \( k_{jx} = k_{jy} = k_{j} = 10^7 \text{ N/m} \) \((i = 1, 2, 3)\) and \((k_{i} = k_{j} = k = 6 \times 10^7 \text{ N/m}) \((i = 3, 4)\), the stationary white noise earthquake excitation spectral density is \( S_{\text{g}} = 1.35 \times 10^{-6} \text{ m}^2/\text{s}^3 \). For hard surface soil the inherent angular frequency of the site soil and damping ratio are \( \omega_{g} = 5\pi \) and \( \xi_{g} = 0.63 \), the parameters of the bedrock spectral characteristics is \( \omega_{r} = 8\pi \) and the speed is \( n = 8000 \text{ r/min} \).

4.1. Random Response of System Under the Filtered Gaussian White Noise Earthquake Spectrum

Not considering rotor imbalance random excitation (the disc eccentricity is \( \epsilon_{1} = \epsilon_{2} = \epsilon = 0 \)) the system under power spectral density of random response of Gaussian filtered white noise earthquake spectrum excitation is shown in Fig. (3). The first circular frequency of a spectral peak \((11 \text{ rad/s})\) is the peak frequency of filtered Gaussian white noise earthquake spectrum. It reflects the inherent frequency of the site soil, the second circular frequency of a spectral peak \((72 \text{ rad/s})\) is the cantilever disk vertical critical angular velocity, it reflects system frequency characteristics. In Fig. (3), it is clear that the inherent frequency of the site soil has a great effect on the random response in the horizontal direction (y direction), but less impact on the vertical direction (x direction). The system frequency characteristic has a great effect on the random response in the vertical direction, but it has less impact in the horizontal direction. Because of the constraint of cantilever disk 2, the response is relatively weak, so the random vibration response of disk 2 is the most strongly. Due to the filtering effect of surface soil, the vibration energy of medium and high end seismic waves have been filtered out. The vibrational energy of random response is mainly concentrated in the low frequency band. The high frequency band of the vibrational energy is relatively small.

4.2. Random Response of System Under Rotor Imbalance Random Excitation and Filtered Gaussian White Noise Earthquake Spectrum Excitation

Power spectral density of random response about disc and bearing under rotor imbalance random excitation and filtered Gaussian white noise earthquake spectrum excitation is shown in Fig. (4). In Fig. (4), system has a certain amount of vibration energy in low, medium and high-end under rotor imbalance random excitation. When the eccentric moment is relatively small \((\epsilon < 0.005 \text{ mm})\) rotor imbalance random excitation has little effect on the random response, but when the eccentric moment is relatively large \((\epsilon > 0.005 \text{ mm})\) the effect cannot be ignored. The rotor imbalance random excitation is stronger, random response is more intense, but it does not change the peak frequency of the response spectrum. The specific circumstances are as follows,
(1) The rotor imbalance random excitation is stronger. The intensity of the low and high frequency of non-cantilever disc 1 has improved. The intensity of the low frequency random response of cantilever disc 2 has improved but intensity of the high frequency random response has changed little.

Fig. (3). Random response of the system under Gaussian filtered white noise.

Fig. (4). Rotor imbalance random excitation effect on random response.
(2) With the rotor imbalance random excitation enhanced for cantilever disc bearing 3, the intensity of vertical high frequency random response increases rapidly. But the intensity of vertical low frequency random response exhibits little change. The intensity of the horizontal low frequency random response is improved, but the intensity of high frequency random response exhibits little change.

(3) The rotor imbalance random excitation is stronger and the high frequency random response of cantilever bearing 4 is more intense. The intensity of the low frequency random response exhibits little change.

Integrating the above analysis the main conclusion we got is that

a. Under the rotor unbalance random excitation, the system has certain vibrational energy, irrespective of high, medium or low frequency range.

b. The rotor unbalance random excitation intensity doesn’t change the peak frequency to make the strength of the random response increase. Especially when eccentric torque is equal to or more than 0.005 Nm \((\varepsilon \geq 0.005 \text{ mm})\), the rotor unbalance random excitation on the influence of random response cannot be ignored.

c. The rotor unbalance random excitation has much influence on the high and low frequency of disk 1, the low frequency of disc 2, the vertical vibration high frequency of bearing 3 and the high frequency random response strength of bearing 4.

5. CONCLUSION

It is appropriate that the method of pseudo-excitation is applied in analysis of rotor system in rotor imbalance random excitation and the filtered Gaussian white noise earthquake ground acceleration. Some of the main conclusion, which got through the numerical simulation, is that

(1) As the cantilever disk is constrained relatively weak, so the random vibration response is the most intense.

(2) The inherent frequency of the site soil has a great effect on the horizontal direction of the random response. The system frequency characteristic has a great effect on the vertical direction of the random response.

(3) The intensity of the rotor imbalance random excitation does not change the frequency peak of response spectrum. When the excitation intensity is high, the system random vibration response is large. It has a great impact on the low and high frequency of non-cantilever disc, the low frequency of the cantilever disc, the high-frequency of non-cantilever bearing vertical vibration and the high-frequency of cantilever bearing with the intensity around the high-frequency random response.

CONFLICT OF INTEREST

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