The Stick-Slip Vibration and Bifurcation of a Vibro-Impact System with Dry Friction

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Abstract: In this paper, the periodic motion, bifurcation and chatter of two-degree-of-freedom vibratory system with dry friction and clearance is studied. Slip, stick, and impact of system motions are analyzed. Numerical simulations are carried out. The results show that the system possesses rich dynamics characterized by periodic motion, stick-slip-impact motion, quasi-periodic motion and chaotic attractors, and the routes from periodic motions to chaos via Hof bifurcation or period-doubling bifurcation. Furthermore, it is found that there exists the chatter phenomena induced by dry friction in low frequency, and the windows of chaotic motion are broadened in the area of higher excitation frequencies as the dry friction increases.

Keywords: Chatter, dry friction, hopf bifurcation, stick-slip, vibro-Impact.

1. INTRODUCTION

The vibro-impact systems with dry friction have been considerable interest to researchers for a long time, for they occur frequently in everyday life as well as engineering systems such as rolling railway wheelset, gear transmission, disc brake system. In many applications, dry friction, clearance and impact factors often result in sudden change of the vector fields describing dynamic behaviors of mechanical systems [1]. Researches on the impact oscillator with dry friction have important significance in the optimization design of machinery with clearance, noise suppression and reliability etc. In the past several years, there exists a wide range of research devoted to analysis of non-smooth systems with dry friction, Feeny and Moon [2] investigated the geometry of chaotic attractors for dry friction oscillators experimentally and numerically, using three different friction law. The work of Galvanetto [3,4] deal with dynamics of a three blocks stick-slip system, as well as a one-dimensional map introduced for studying bifurcations in the four dimensional system. In the papers of Awrejcewicz and his collaborators [5-7], the Melnikov-Gruendler approach was used to analysis stick-slip chaos in one-degree-of-freedom and two-degree-of-freedom systems with friction. A chaotic threshold had been obtained for both smooth and stick-slip chaotic behaviors in these system. Thomsen and Fidlin [8] considered friction –induced vibrations of a mass-on-belt system, and they obtained approximate analytical expressions for the amplitudes and base frequencies of stick-slip and pure-slip oscillations.

It is important to note that many studies of dry friction system have been carried out, but few have considered impact oscillators with dry friction. Motion of the impact-dry-friction pair of bodies was analyzed by Peterka [9], with emphasis on the influence of dry friction on the system responds. He also performed several experimental tests to verify his solutions. Cone et al. [10] investigated pitchfork bifurcation, grazing bifurcation and Stick-slip vibration of an impact oscillator with addition of dry friction. In fact, the vibro-impact systems coupled with dry friction are closer to the actual applications. In this paper, we focus attention on the stick-slip vibration and bifurcations of two-degree-of-freedom vibro-impact system with dry friction. The influence of dry friction on dynamics of the vibro-impact system with dry friction is elucidated accordingly.

2. SYSTEM DESCRIPTION

Two-degree-of-freedom vibro-impact system with dry friction to be investigated, shown in Fig. (1), is composed of two mass blocks. The block M1 is connected to the block M2 via a linear spring of stiffness K1 and a damper of viscous damping coefficient C1. The block M2 is connected to a fixed support by a linear spring of stiffness K2 and a damper of viscous damping coefficient C2. Considered a dry friction Fig. (1). Modele of two-degree-of-freedom vibro-impact system with dry friction.
exerts on the contact surface of two blocks when there exists relative motion or a tendency towards relative motion. The dry friction follows Coulomb’s friction law. Both masses are subjected to harmonic excitations, namely $P_i \sin (\Omega t + \tau)$, where $P_i$ and $\Omega$ are the excitations’ strength and frequency. Displacements of the masses $M_1$ and $M_2$ are represented by $x_1$ and $x_2$, respectively, and the clearance between the two masses is denoted by $B$. Since the block $M_1$ contacts the block $M_2$ with friction, the block $M_1$ can move along, or rest on the block $M_2$ surface. The block $M_1$ impacts mutually with the block $M_2$ when $X_1(T) - X_2(T) = B$. The impact is described by the conservation law of momentum and a coefficient of restitution $r$, and it is assumed that the duration of impact is negligible compared to the period of the force.

The motion processes of the system, between consecutive impacts occurring at the stop, are considered. Between any two consecutive impacts, two types of steady motions existing in the system: one is the sliding motion in which the mass $M_1$ slides on the mass $M_2$ surface, and the velocity of two masses is unequal. While for the slip mode, we must judge if the velocity of two masses is equivalent after one time step. If the time point of the velocity of the mass $M_1$ equaling the mass $M_2$ to find, the system will enter the other sticking motion if the following relations are true:

$$\dot{x}_1 = 0, \quad f_0 \leq f, \quad \left(1\right)$$

where $f_0$ is the static friction force acting on the mass $M_1$ when, and $f_s$ is the maximum static friction. In the other hand, if $f_0 > f_s$, the mass $M_1$ still slides on the mass $M_2$ surface and the friction is reverse.

Between two consecutive impacts, the non-dimensional differential equations of sliding motion of the system are given by

$$\begin{align*}
\begin{bmatrix}
1 & 0 \\
-2\xi & 1 - 2\xi
\end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 2\xi & -2\xi \\
-2\xi & 2\xi(1+\mu_k) \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\
-1 & 1 + \mu_k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 1 - f_20 \\ f_20 \end{bmatrix} \sin(\omega t + \tau) + \begin{bmatrix} f_s \\ f_s \end{bmatrix} \text{sgn}(\dot{x}_1 - \dot{x}_2) (x_1 - x_2 < b) \\
&= \begin{bmatrix} 1 \\ -1 \end{bmatrix} (y > 0) \\
&= \begin{bmatrix} 0 \\ -1 \end{bmatrix} (y = 0) \\
&= \begin{bmatrix} -1 \\ 1 \end{bmatrix} (y < 0)
\end{align*}$$

$$\left(2\right)$$

where $\text{sgn}(y) = \begin{bmatrix} 1 \end{bmatrix} (y > 0)$

A dot (·) denotes differentiation with respect to the non-dimensional time $t$, and the non-dimensional quantities

$$\begin{align*}
\mu_n &= \frac{M_2}{M_1}, \quad \mu_k = \frac{K}{K_c}, \quad \mu_c = \frac{C_i}{C_c}, \quad \xi = \frac{C_i}{2\sqrt{M_1 K_1}}, \\
x_i &= \frac{X_i}{P_1 + P_2}, \quad f_{20} = \frac{P_1}{P_1 + P_2}, \quad f_s = \frac{F}{P_1 + P_2}, \\
\omega &= \sqrt{\frac{M_1}{K_1}} \frac{1}{t}, \quad T = \sqrt{\frac{K_1}{M_1}}, \quad b = \frac{BK_2}{P_1 + P_2}
\end{align*}$$

have been introduced.

When sticking occurs, it is assumed the maximum static friction is a set value so that $f_0 \leq f_s$. Their accelerations approach to be equivalent and the two-degree-of-freedom vibro system becomes a single-degree-of-freedom oscillator with the mass $(M_1+M_2)$. The change from free motion of both masses to their sticking represents a reduction in the degree of freedom of the system from 2 to 1. The non-dimensional differential equations of stick motion of the system are given by

$$\begin{align*}
(1 + \mu_n)\ddot{x}_1 + 2\xi\mu_k\dot{x}_1 + \mu_kx_1 = \sin(\omega t + \tau)
\end{align*}$$

$$\left(5\right)$$

According to $\dot{x}_1 = \dot{x}_2$, $\ddot{x}_1 = \ddot{x}_2$ and Eq.$(3)$, the static friction force acting on the mass $M_1$ is obtained as follow:

$$f_0 = \left[\mu_n(1 - f\xi) - f_{20}\right] \sin(\omega t + \tau) + 2\xi\mu_k\ddot{x}_1 + \mu_kx_1
\begin{array}{c}
\frac{1}{1 + \mu_n} \\
-(x_1 - x_2)
\end{array}$$

$$\left(6\right)$$

The mass $M_1$ rests on the mass $M_2$ surface until $f_0 > f_s$, and there is no impact because of dry friction.

When the mass $M_1$ slips on the mass $M_2$ surface, the impact occurs with $x_1(t) - x_2(t) = b$. The conditions of conservation of momentum and the coefficient of restitution during the impact can be written as

$$\begin{align*}
\dot{x}_1 + \mu_n\dot{x}_2 = \dot{x}_1 + \mu_n\dot{x}_2, \\
x_1 - x_2 = b
\end{align*}$$

where the subscript minus sign denotes the states just before impact and the subscript plus sign denotes the states just after impact, $r$ is the coefficient of restitution.

Impacting systems are conveniently studied by using a map derived from the equations of motion. Each iterate of the map corresponds to the mass $M_1$ striking the mass $M_2$ once. Periodic-impact motions of the system are characterized by the symbol $n$-$p$, where $p$ denotes the number of impacts and $n$ denotes the number of excitation periods. Periodic non-sticking motion can be defined as $n$-$p$ in which two masses do not move in step during $n$ excitation periods and namely there is no sticking motion; $n$-$p$-II represents another type of motion: during $n$ excitation periods, when the velocity of the mass $M_1$ equaling the mass $M_2$, the mass $M_1$ stick to the mass $M_2$ until the static friction force overcome maximum static friction, the system enters slip mode once again. As a Poincare’ section associated with the state of the vibro-impact system, just immediately before impact, we chose the Poincare’ section

$$\sigma = \{ (x_1, \dot{x}_1, x_2, \dot{x}_2, \theta) \in R^4 \times S : x_1 - x_2 = b, \dot{x}_1 = \dot{x}_2 = \dot{x}_2 \}$$

$$\left(8\right)$$

to establish Poincare’ map of the vibro-impact system with dry friction.

3. DYNAMIC ANALYSIS

Numerical analyses are carried out for determining the dynamical responses of the vibro-impact system with dry friction.
friction. Due to the fact that the investigated has dry friction, the system exhibits periodic motion with slip and stick phases. The non-dimensional parameters are assumed to take the values

\[ \mu_n = 10, \mu_s = 5, \xi = 0.1, b = 0, f_{28} = 0, f_s = 0.15, R = 0.08. \] (9)

The forcing frequency \( \omega \) is taken as a control parameter in dynamic analysis. Bifurcation diagrams, phase plane portraits and time series are plotted to illustrate the influence of the parametric excitation on the dynamic behavior of the system. The bifurcation diagrams for the system are shown in Fig. (9), in which the quantity \( \omega t_f / 2\pi \) and the relative velocity \( \hat{x}_2 - \hat{x}_1 \) immediately before the impact are plotted versus the varying forcing frequency, where \( t_f \) denotes the time of sliding motion of two masses between successive impacts. By looking the bifurcation diagrams in Fig. (2), we have large windows of periodic n:1 motion for \( \omega \in [0.56, 10] \), and there exist narrow areas of chaotic motion between two adjacent n:1 motion. In the forcing frequency range interval \( \omega \in [1.12, 2.84] \), an impact occurs during cycle of forcing. Between two consequent impacts, non-stick motions occupy the majority of the parametric range and only in the forcing frequency range interval \( \omega \in [1.47, 1.6] \), slip-stick motions occur. We give an example of a period-1 sticking motion which exists at forcing frequency of \( \omega = 1.5 \). A time series of this periodic motion is shown in Fig. (3a), and a Phase portrait in Fig. (3b). Stable 1-1-I motion exists in the frequency interval \( \omega \in [1.12, 1.47] \& [1.6, 2.84] \), as seen in Fig. (2). A Phase portrait of 1-1-I motion is shown for \( \omega = 2.2 \) in Fig. (4b), and a time series in Fig. (4a). More interestingly, in the forcing frequency range interval \( \omega \in [0.55, 1.12] \), there exists the chatter induced by dry friction. As shown in Fig. (5) for \( \omega = 0.552 \), the times between consecutive impacts are getting shorter until leading into stick model, and the relative velocities of two mass are getting smaller until reach to zero in one periodic of chatter.

![Fig. (3). Phase plane portrait and time series of relative motion, 1-1-I motions \( \omega = 1.5 \).](image-url)

An observation of interest is the unusual routes to chaos. A transition to chaos for the forcing frequency range interval \( \omega \in [2.83, 6.7] \) of the system with Coulomb friction can be observed in the bifurcation diagram shown in Fig. (6). The transition from 1-I-I motion into chaos via double periodic bifurcation can be seen in Fig. (6a). With increasing the forcing frequency \( \omega \), 2-1-I motion occurs, and 4-2-I motion stabilizes, see Fig. (6b). The 4-2-I motion is represented by two fixed points in projected of Poincare map, as seen Fig. (7a, b). Instability of 4-2-I leads to Hopf bifurcation of the motion, so that the system exhibits quasi-periodic impact motion associated with 4-2-I motion. The quasi-periodic impact motion is represented by two attracting invariant circles in projected of Poincare map, as seen in Fig. (7c).
Further increase $\omega$, quasi-periodic motion leads into multiple periodic motion and into chaos via double periodic bifurcation followed. By looking the bifurcation diagrams in Fig. (6c), 3-1-I motion transit into 6-2-I motion via pitchfork bifurcation, and 6-2-I motion leads into Hopf bifurcation of the motion, which subsequently unstable and chaotic friction force increased. Moreover, the bifurcation point shifts to the left by the influence of the dry friction in the higher frequencies.

Fig. (4). Phase plane portrait and time series of relative motion, 1-I-I motions $\omega=2$.

Fig. (5). Phase plane portrait and time series of relative motion, chatter $\omega=0.552$.

Fig. (6). Different routes of $n$-I-I motions leading to chaos ($f_i=0.15$).
Fig. (7). Projected Poincaré section (a) and (b) 4-2-1 fixed points, $\omega = 4.801$; (b) attracting invariant circles associated with 4-2-1 fixed points, $\omega = 4.805$.

Owing to the existence of the non-smooth factors with friction and clearances, the dynamic behavior of the vibration system becomes complex. The bifurcation diagrams are plotted to illustrate the influence of the dry friction on the dynamic behavior of the system.

As shown in Fig. (8), the influence of the dry friction is not obvious in the lower excitation frequencies, only the velocity of impact get smaller with the friction increasing. But in the higher frequencies, less periodic motions occur whereas the chaotic motion area is broadened as the

Fig. (8). Bifurcation diagram under different dry friction forces.

CONCLUSION

In this paper, the dynamical of two-degree-of-freedom vibro-impact system with dry friction has been studied. Numerical simulations show that the following conclusions.

(1) The system possesses rich characterized by sticking-periodic, non-sticking-periodic, quasi-periodic and chaotic attractors. The system exhibits sticking motions but are present in small amounts in the frequency range, and with the friction crease, the proportion goes up. Furthermore, there exists the chatter induced by dry friction in low frequency.

(2) Different routes of n-1-1 motions to chaos are also carried out, and The Hof and period-doubling bifurcation for this oscillator are main routes leading to chaos.

(3) Fig. (8) shows the influence of the dry friction on the dynamic behavior of the system. With the friction increased, the influence of the dry friction is not obvious in the lower excitation frequencies, only the velocity of impact becomes smaller with the friction increasing. In the higher frequencies, less periodic motions occur whereas the windows of chaotic motion is broadened, and non-sticking periodic motions change into sticking periodic motions gradually.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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