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Detecting a Large-Scale Upper-Atmospheric Plasma Inhomogeneity Using the Method of Multifrequency Radio Sounding in the Decametric Wavelength Range

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Abstract: We examine the problem of transionospheric sounding with satellite radio signals at frequencies that are close to the edge of the radio transparency frequency range of the ionosphere. We derive asymptotic formulas for the group delay time of the transionospheric radio signal and present an example of how they are implemented if there is a localized large-scale electron density inhomogeneity in the ionosphere. Finally, we suggest techniques for detecting large-scale ionospheric inhomogeneities, which are based on numerical-asymptotic synthesis of disturbed distance-frequency characteristics of decametric signals radiated from a low-orbiting or geostationary satellite.

Keywords: Ionosphere, ionospheric disturbances, transionospheric radio sounding.

1. INTRODUCTION

Recent investigations reveal that not only do plasma inhomogeneities in the Earth's upper atmosphere influence characteristics of radio wave propagation paths, but they can also serve as an indicator of coming geophysical or seismic events [1, 2]. Therefore, observations of the inhomogeneities dynamics and structure through the use of radio sounding of the ionosphere over a given geographical region are of utmost current importance.

A method for diagnosing the inhomogeneous near-Earth plasma structure, which involves multifrequency radio sounding from a spacecraft in geostationary or highly elliptical orbit, was suggested in [3,4]. In this method, pulsed transionospheric radio signals in the decametric wavelength range, recorded at several ground-based receiving sites, are analyzed. By studying the signal characteristics at different receiving sites, it is possible to monitor the ionosphere dynamics in the sounded region. An alternative method for diagnosing ionospheric inhomogeneities was discussed in [5-8]. It suggests sounding of the ionosphere with decametric signals from rapidly moving low-orbiting spacecraft. In this case, it is often enough to have just one ground-based observation site. It is significant to note that it is radio waves of the decametric range that are used in the two aforementioned diagnostic techniques. This is a highly important advantage over the other wavelength ranges, because ionospheric sounding is performed at the opacity threshold in this case, i.e. at frequencies approaching the critical frequencies of the F2 layer. This assures high sensitivity of the methods to near-Earth plasma inhomogeneities and furnishes a means of more accurately reconstructing the medium fine structure.

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Amongst the ionospheric inhomogeneities, isolated large-scale electron density inhomogeneities have been of specific recent interest. In particular, such inhomogeneities can arise in the neighborhood of the main electron density maximum of the ionosphere, over epicenters of coming seismic events [9-11]. In this context, analysis of transionospheric radio signals at operating frequencies approaching the limiting (the lowest possible) sounding frequency is of crucial importance. Specifically, the dynamics of a large-scale inhomogeneity can be monitored by recording and processing the distance-frequency characteristics (DFCs) of transionospheric signals in the range of operating frequencies adjacent to the limiting sounding frequency [7].

It should be pointed out that common to the aforementioned diagnostic methods is the fact that to determine parameters of a large-scale inhomogeneity requires direct numerical synthesis of disturbed DFCs along spacecraft-Earth paths. In real conditions, however, numerical modeling of DFCs for diagnosing the inhomogeneities represents a rather long, complicated procedure. This is especially true in regard to the procedure of tracing of radio rays to the observation site at different frequencies if there is a localized inhomogeneity in the ionosphere. Besides, direct numerical modeling of disturbed DFCs gives no way of determining analytical relationships between the DFC characteristics and inhomogeneity parameters. These difficulties can be obviated by using the numerical-asymptotic computing method when solving the problem of synthesizing disturbed DFCs.

2. METHOD FOR CALCULATING DISTURBED DISTANCE-FREQUENCY CHARACTERSTICS OF THE TRANSIONOSPHERIC SIGNAL

Let us consider a radio signal radiated from a spacecraft and propagating through an isotropic ionosphere. Let the electron density of the ionosphere be a function of distance

R and angle φ, where R is counted off from the Earth's center and φ - from the straight line passing through the observer point (Fig. **1**).

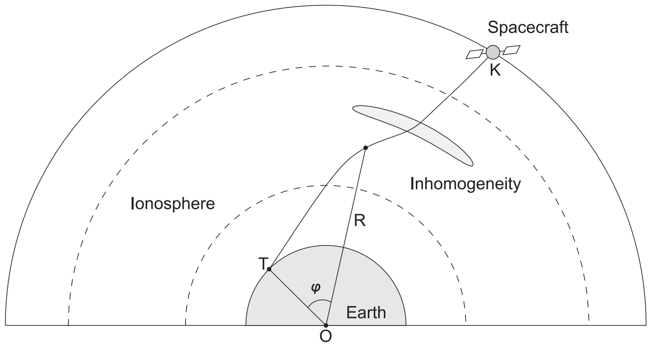

Fig. (1). Scheme illustrating transionospheric radio sounding: K and T correspond to the spacecraft and Earth-based observer, respectively.

We will model a DFC within the geometrical optics approximation. According to [12], the group delay time τ of the radio signal along the spacecraft-Earth radio path in the two-dimensionally inhomogeneous ionosphere represents a path integral:

$$\tau = \frac{1}{c}\int^{S} \frac{dS}{\sqrt{\varepsilon(S)}} = \frac{1}{c}\int_0^{\varphi_K} \frac{R d\varphi}{\sqrt{\varepsilon}\sin\beta}, \tag{1}$$

where S is the radio signal propagation path, $\varepsilon(R,\varphi)$ is dielectric permittivity of the ionosphere, φ_K is the angular coordinate of the spacecraft, β is the refraction angle, and c is the speed of light.

Let $\varepsilon(R,\varphi)$ be represented as the sum of two terms:

$$\varepsilon = \varepsilon_0 + \varepsilon_1, \tag{2}$$

where $\varepsilon_0(R)$ is a regular component of the medium, and $\varepsilon_1(R,\varphi)$ is a small perturbation characterizing the ionospheric inhomogeneity:

$$\varepsilon_1(R,\varphi) << \varepsilon_0(R).$$

For asymptotic calculation of the group delay time variation associated with the inhomogeneity influence, we introduce in Eq. (1) small corrections $R_1(\varphi)$, $\Delta\tau$ and $\beta_1(\varphi)$ to the characteristics of the undisturbed transionospheric ray, namely the trajectory $R_0(\varphi)$, the group delay time τ_0, and the refraction angle $\beta_0(\varphi)$, respectively:

$$\begin{aligned} R &= R_0 + R_1, \\ \tau &= \tau_0 + \Delta\tau, \\ \beta &= \beta_0 + \beta_1. \end{aligned} \tag{3}$$

On taking a Taylor series expansion of the functions under the integral sign in Eq. (1), to a first approximation we obtain

$$\sin\beta = \sin\beta_0 + \beta_1\cos\beta_0,$$

$$\sqrt{\varepsilon} = \sqrt{\varepsilon_0}\left(1 + \frac{1}{2}\frac{\partial \ln\varepsilon_0}{\partial R_0}R_1 + \frac{\varepsilon_1}{2\varepsilon_0}\right). \tag{4}$$

Substituting Eqs. (2)-(4) into Eq. (1), for the group delay time variation $\Delta\tau$ we have

$$\Delta\tau = \frac{1}{2c}\int_0^{\varphi_K} \frac{1}{\sqrt{\varepsilon_0}\sin\beta_0} \times \left(2R_1 - R_1R_0\frac{\partial\ln\varepsilon_0}{\partial R_0} - R_0\frac{\varepsilon_1}{\varepsilon_0} - 2R_0\beta_1\cot\beta_0\right)d\varphi, \tag{5}$$

where integration is performed over the unperturbed path $R_0(\varphi)$ in a medium with dielectric permittivity $\varepsilon_0(R)$.

To simplify Eq. (5), we make use of the system of ray equations in a medium with dielectric permittivity $\varepsilon(R,\varphi)$ [13]:

$$\begin{cases} \dfrac{dR}{d\varphi} = R\cot\beta \\ \dfrac{d\beta}{d\varphi} = \dfrac{1}{2}\left(\cot\beta\dfrac{\partial\ln\varepsilon}{\partial\varphi} - R\dfrac{\partial\ln\varepsilon}{\partial R}\right) - 1 \end{cases} \tag{6}$$

By expanding the functions in system (6) into series in terms of a small parameter, it is possible to divide the initial system of equation into two systems, in accordance with the order of approximation:

$$\begin{cases} \dfrac{dR_0}{d\varphi} = R_0\cot\beta_0 \\ \dfrac{d\beta_0}{d\varphi} = -\dfrac{1}{2}\dfrac{\partial\ln\varepsilon_0}{\partial R_0}R_0 - 1 \end{cases} \tag{7}$$

$$\begin{cases} \dfrac{dR_1}{d\varphi} = R_1\cot\beta_0 - \dfrac{R_0\beta_1}{\sin^2\beta_0} \\ \dfrac{d\beta_1}{d\varphi} = \dfrac{1}{2}\left[\cot\beta_0\dfrac{\partial}{\partial\varphi}\left(\dfrac{\varepsilon_1}{\varepsilon_0}\right) - R_0\dfrac{\partial}{\partial R_0}\left(\dfrac{\varepsilon_1}{\varepsilon_0}\right)\right] \\ \qquad - \dfrac{1}{2}R_1\left(\dfrac{\partial\ln\varepsilon_0}{\partial R_0} + R_0\dfrac{\partial^2\ln\varepsilon_0}{\partial R_0^2}\right) \end{cases} \tag{8}$$

By taking the function $\beta_1(\varphi)$ from the first equation of system (8) and substituting it into Eq. (5), we obtain

$$\Delta\tau = \frac{1}{c}\int_0^{\varphi_K}\left(\frac{\cos\beta_0}{\sqrt{\varepsilon_0}}\frac{dR_1}{d\varphi} + \frac{1}{\sqrt{\varepsilon_0}\sin\beta_0}\Gamma(\varphi)\right)d\varphi, \tag{9}$$

where $\Gamma(\varphi) = \left(R_1\sin^2\beta_0 - \frac{1}{2}R_1R_0\frac{\partial\ln\varepsilon_0}{\partial R_0} - \frac{1}{2}R_0\frac{\varepsilon_1}{\varepsilon_0}\right)$.

Integrating the first term in Eq. (9) by parts yields

$$\int_0^{\varphi_K}\frac{\cos\beta_0}{\sqrt{\varepsilon_0}}\frac{dR_1}{d\varphi}d\varphi = -\int_0^{\varphi_K} R_1\frac{d}{d\varphi}\left(\frac{\cos\beta_0}{\sqrt{\varepsilon_0}}\right)d\varphi \tag{10}$$

and substituting Eq.(10) into Eq. (9), we finally obtain

$$\Delta\tau = \frac{1}{c}\int_0^{\phi_K}\left(\frac{2\sin\beta_0}{\sqrt{\varepsilon_0}}R_1\left(1+\frac{d\beta_0}{d\phi}\right)-\frac{R_0}{2\sin\beta_0}\frac{\varepsilon_1}{\varepsilon_0\sqrt{\varepsilon_0}}\right)d\phi\,. \quad (11)$$

Here it is taken into consideration that the variation of the trajectory $R_1(\varphi)$ along the spacecraft-Earth path satisfies the condition:

$$R_1(\varphi_k) = R_1(0) = 0. \quad (12)$$

As is evident from Eq. (11), for calculating the group delay time variation of the transionospheric radio signal, it is necessary to know the correction $R_1(\varphi)$ along the entire undisturbed trajectory. To determine $R_1(\varphi)$, we formulate the system of equations (8) in a general form:

$$\begin{cases}\dfrac{dR_1}{d\varphi} = a_{11}R_1 + a_{12}\beta_1 \\ \dfrac{d\beta_1}{d\varphi} = a_{21}R_1 + D_1\end{cases} \quad (13)$$

where

$$a_{11} = \cot\beta_0,\ a_{12} = -\frac{R_0}{\sin^2\beta_0},$$

$$a_{21} = -\frac{1}{2}\left(\frac{\partial\ln\varepsilon_0}{\partial R_0} + R_0\frac{\partial^2\ln\varepsilon_0}{\partial R_0^2}\right), \quad (14)$$

$$D_1 = \frac{1}{2}\left[\cot\beta_0\frac{\partial}{\partial\varphi}\left(\frac{\varepsilon_1}{\varepsilon_0}\right) - R_0\frac{\partial}{\partial R_0}\left(\frac{\varepsilon_1}{\varepsilon_0}\right)\right].$$

According to [14], the solution of the linear system (13) with variable coefficients is equivalent to the solution of the following inhomogeneous second-order differential equation:

$$\frac{d^2R_1}{d\varphi^2} - \left(\frac{a_{12}^{/}}{a_{12}} + a_{11}\right)\frac{dR_1}{d\varphi} - a_{12}\left[a_{21} + \left(\frac{a_{11}}{a_{12}}\right)^{/}\right]R_1 = a_{12}D_1, \quad (15)$$

where the prime sign denotes differentiation with respect to the variable φ.

The solution to Eq. (15) in a general form is well known [15]:

$$R_1(\varphi) = C_1(\varphi)Y_1(\varphi) + C_2(\varphi)Y_2(\varphi), \quad (16)$$

where

$$C_1^{/} = \frac{Y_2 a_{12} D_1}{W},\ C_2^{/} = -\frac{Y_1 a_{12} D_1}{W}, \quad (17)$$

$$W = Y_1^{/}Y_2 - Y_2^{/}Y_1, \quad (18)$$

and the functions $Y_1(\varphi),\ Y_2(\varphi)$ are the solutions of the following homogeneous equation:

$$\frac{d^2Y_{1,2}}{d\varphi^2} - \left(\frac{a_{12}^{/}}{a_{12}} + a_{11}\right)\frac{dY_{1,2}}{d\varphi} - a_{12}\left[a_{21} + \left(\frac{a_{11}}{a_{12}}\right)^{/}\right]Y_{1,2} = 0\,. \quad (19)$$

On differentiating the Wronski determinant W and taking into consideration that $Y_1(\varphi),\ Y_2(\varphi)$ are the solutions of Eq. (19), we obtain

$$\frac{dW}{d\phi} = Y_1^{//}Y_2 - Y_2^{//}Y_1 = \left(\frac{a_{12}^{/}}{a_{12}} + a_{11}\right)W\,, \quad (20)$$

whence $W = Ba_{12}R_0$, with B the unknown constant. Then from Eq. (17) we have

$$C_1^{/} = \frac{Y_2D_1}{BR_0},\ C_2^{/} = -\frac{Y_1D_1}{BR_0}. \quad (21)$$

According to the theory of ordinary differential equations [14], the derivatives of the solutions for the system of unperturbed equations (7) with respect to the initial parameters of the problem represent the fundamental solutions for the homogeneous equation (19). Therefore we can choose the functions $Y_1(\varphi),\ Y_2(\varphi)$ in the following form:

$$Y_1 = \frac{\partial R_0}{\partial\beta_H}(\varphi)\,,\ Y_2 = \frac{\partial R_0}{\partial\beta_H}(\varphi_K - \varphi)\,, \quad (22)$$

where β_H is the initial angle of incidence of a radio wave from the source on the ionosphere.

Using the solutions to equation (19) with the conditions:

$$Y_1(0) = 0\,,\ \frac{dY_1}{d\varphi}(0) = -\frac{A}{\sin^2\beta_H}\,,$$

$$Y_2(\varphi_K) = 0\,,\ \frac{dY_2}{d\varphi}(\varphi_K) = \frac{R_K}{\sin^2\beta_H}\,, \quad (23)$$

with A the Earth's radius and R_K the spacecraft orbit height, it is possible to determine the constant B:

$$W(\varphi_K) = -\frac{R_K}{\sin^2\beta_H}Y_1(\varphi_K) = Ba_{12}(\varphi_K)R_K\,,$$

whence

$$B = \frac{Y_1(\varphi_K)}{R_K}.$$

On substituting Eq. (21) into Eq. (16) and taking into account the reciprocity of the transionospheric trajectory, we finally have:

$$R_1(\varphi) = -\frac{1}{Y_1(\varphi_K)} \times\left[Y_1(\varphi)\int_{\varphi}^{\varphi_K}D_1Y_2\frac{R_K}{R_0}d\varphi + Y_2(\varphi)\int_0^{\varphi}D_1Y_1\frac{R_K}{R_0}d\varphi\right]. \quad (24)$$

Using Eq. (24), one can calculate from Eq. (11) the group delay time variation of the radio signal at a specific sounding frequency. At the same time, a practical calculation of $R_1(\varphi)$ and $\Delta\tau$ by directly using Eq. (24) and Eq. (11) is rather complicated, because to integrate these expressions one needs to know the values of the integrands along the unperturbed trajectory $R_0(\varphi)$. This is particularly true in the case of modeling of DFCs where calculation of $R_1(\varphi)$ and $\Delta\tau$ is required for a set of sounding frequencies. Therefore, a preliminary transformation of the Eqs. (24) and (11) is needed for numerical-asymptotic synthesis of DFCs.

First we consider Eq. (24). As follows from Eq. (24), to calculate the variation of the transionospheric trajectory along the transmitter-receiver path, it is necessary to determine, first of all, the value of the fundamental solution $Y_1(\varphi_K)$. The system of equations for determining the function $Y_1(\varphi)$ can be obtained by differentiating the system of unperturbed ray equations (7) with respect to the initial parameter β_H:

$$\begin{cases} \dfrac{dY_1}{d\varphi} = Y_1 \cot\beta_0 - \dfrac{R_0}{\sin^2\beta_0} L_1 \\ \dfrac{dL_1}{d\varphi} = -\dfrac{1}{2} Y_1 \left(\dfrac{\partial \ln\varepsilon_0}{\partial R_0} + R_0 \dfrac{\partial^2 \ln\varepsilon_0}{\partial R_0^2} \right) \end{cases} \tag{25}$$

where

$$L_1 = \frac{\partial\beta_0}{\partial\beta_H}(\varphi).$$

By combining Eq. (7) and Eq. (25) we obtain a system of equations for simultaneous calculation of the function $Y_1(\varphi)$ and the unperturbed trajectory $R_0(\varphi)$:

$$\begin{cases} \dfrac{dR_0}{d\varphi} = R_0 \cot\beta_0 \\ \dfrac{d\beta_0}{d\varphi} = -\dfrac{1}{2} \dfrac{\partial \ln\varepsilon_0}{\partial R_0} R_0 - 1 \\ \dfrac{dY_1}{d\varphi} = Y_1 \cot\beta_0 - \dfrac{R_0}{\sin^2\beta_0} L_1 \\ \dfrac{dL_1}{d\varphi} = -\dfrac{1}{2} Y_1 \left(\dfrac{\partial \ln\varepsilon_0}{\partial R_0} + R_0 \dfrac{\partial^2 \ln\varepsilon_0}{\partial R_0^2} \right) \end{cases} \tag{26}$$

To model DFC, it is necessary to solve system (26) with the following initial conditions: $R_0(0,f) = A$, $\beta_0(0,f) = \beta_H^0(f)$, $Y_1(0,f) = 0$ and $L_1(0,f) = 1$ for a number of sounding frequencies *f*. The quantity β_H^0 (which may be different at different frequencies) is the initial angle of incidence on the ionosphere of the ray coming out from the radio source and arriving at the receiver point. This quantity should be determined by means of tracing of rays in the undisturbed ionosphere. Let us note that to get a ray trajectory connecting the source and receiver points is much easier in the case of the undisturbed ionosphere than in the case of an ionosphere with localized inhomogeneities. Regarding the direct calculations of $R_1(\varphi)$ and $\Delta\tau$ based on Eqs. (24) and (11), it is convenient to transform them into appropriate differential equations.

To do this, let Eq. (24) be represented as the sum of two terms:

$$R_1 = R_{11} + R_{12}, \tag{27}$$

where

$$R_{11}(\varphi) = -\frac{Y_1(\varphi)}{Y_1(\varphi_k)} \int_{\varphi}^{\varphi_K} D_1(\varphi) Y_2(\varphi) \frac{R_K}{R_0(\varphi)} d\varphi, \tag{28}$$

$$R_{12}(\varphi) = -\frac{Y_2(\varphi)}{Y_1(\varphi_k)} \int_0^{\varphi} D_1(\varphi) Y_1(\varphi) \frac{R_K}{R_0(\varphi)} d\varphi. \tag{29}$$

By introducing the functions

$$P_1 = \frac{R_{11}(\varphi) Y_1(\varphi_k)}{Y_1(\varphi)}, \tag{30}$$

$$P_2 = \frac{R_{12}(\varphi) Y_1(\varphi_k)}{Y_2(\varphi)} \tag{31}$$

into Eqs. (28) and (29), respectively, and substituting them into Eq. (27), we have:

$$R_1 = \frac{1}{Y_1(\varphi_K)} (P_1 Y_1 + P_2 Y_2). \tag{32}$$

Differential equations for the functions P_1 and P_2 can be obtained by taking the derivatives of the integrals in Eqs. (28) and (29) with respect to the variable lower or upper limit, respectively:

$$\frac{dP_1(\varphi)}{d\varphi} = D_1(\varphi) Y_2(\varphi) \frac{R_K}{R_0(\varphi)}, \tag{33}$$

$$\frac{dP_2(\varphi)}{d\varphi} = -D_1(\varphi) Y_1(\varphi) \frac{R_K}{R_0(\varphi)}. \tag{34}$$

Then the full system of equations for calculating the variation of the transionospheric trajectory can be written as:

$$\begin{cases} \dfrac{dR_0}{d\varphi} = R_0 \cot\beta_0 \\ \dfrac{d\beta_0}{d\varphi} = -\dfrac{1}{2} \dfrac{\partial \ln\varepsilon_0}{\partial R_0} R_0 - 1 \\ \dfrac{dY_1}{d\varphi} = Y_1 \cot\beta_0 - \dfrac{R_0}{\sin^2\beta_0} L_1 \\ \dfrac{dL_1}{d\varphi} = -\dfrac{1}{2} Y_1 \left(\dfrac{\partial \ln\varepsilon_0}{\partial R_0} + R_0 \dfrac{\partial^2 \ln\varepsilon_0}{\partial R_0^2} \right) \\ \dfrac{dY_2}{d\varphi} = -Y_2 \cot\beta_0 + \dfrac{R_0}{\sin^2\beta_0} L_2 \\ \dfrac{dL_2}{d\varphi} = \dfrac{1}{2} Y_2 \left(\dfrac{\partial \ln\varepsilon_0}{\partial R_0} + R_0 \dfrac{\partial^2 \ln\varepsilon_0}{\partial R_0^2} \right) \\ \dfrac{dP_1}{d\varphi} = D_1 Y_2 \dfrac{R_k}{R_0} \\ \dfrac{dP_2}{d\varphi} = -D_1 Y_1 \dfrac{R_k}{R_0} \end{cases} \tag{35}$$

where $L_2 = \dfrac{\partial\beta_0}{\partial\beta_H}(\varphi_K - \varphi)$. In this case, the initial conditions have the form: $R_0(0) = A$, $\beta_0(0) = \beta_H^0$, $Y_1(0) = 0$,

$L_1(0)=1$, $Y_2(\varphi_K)=0$, $L_2(\varphi_K)=1$, $P_1(\varphi_K)=0$, $P_2(0)=0$. By transforming the integral in Eq. (11) into the differential equation, in view of Eq. (32), we obtain

$$\frac{d\Delta\tau}{d\varphi}=\frac{1}{c}\left(\frac{2\sin\beta_0}{\sqrt{\varepsilon_0}}\frac{1}{Y_1(\varphi_K)}(P_1Y_1+P_2Y_2)\right.\\ \left.\times\left(1+\frac{d\beta_0}{d\varphi}\right)-\frac{R_0}{2\sin\beta_0}\frac{\varepsilon_1}{\varepsilon_0\sqrt{\varepsilon_0}}\right) \tag{36}$$

The corresponding initial condition for Eq. (36) is $\Delta\tau(0)=0$. By combining Eq. (35) and Eq. (36), we obtain a system of equations for joint calculation of transionospheric trajectories and group delay time variations of the radio signal along the spacecraft-Earth path. We can use Eqs. (35) and (36), based on the unperturbed ray tracing for determining values of β_H^0 at various sounding frequencies, for computing disturbance introduced into a DFC by the ionospheric inhomogeneity.

For the particular case of a low-orbiting spacecraft being within the field of vision of the ground-based observer, Eqs. (35) and (36) can be simplified. By introducing new variables $dx=Ad\varphi$ and $R_0=A+z_0$, then letting $A\to\infty$ and making use of the Snell law, we get the following system of equations for the plane case:

$$\begin{cases}\dfrac{dz_0}{dx}=\cot\beta_0\\ \dfrac{d\beta_0}{dx}=-\dfrac{1}{2}\dfrac{\partial\ln\varepsilon_0}{\partial z_0}\\ \dfrac{dY_1}{dx}=-\dfrac{1}{\sin^2\beta_0}L_1\\ \dfrac{dL_1}{dx}=-\dfrac{1}{2}\dfrac{\partial^2\ln\varepsilon_0}{\partial z_0^2}Y_1\\ \dfrac{dY_2}{dx}=\dfrac{1}{\sin^2\beta_0}L_2\\ \dfrac{dL_2}{dx}=\dfrac{1}{2}\dfrac{\partial^2\ln\varepsilon_0}{\partial z_0^2}Y_2\\ \dfrac{dP_1}{dx}=D_1Y_2\\ \dfrac{dP_2}{dx}=-D_1Y_1\\ \dfrac{d\Delta\tau}{dx}=\dfrac{1}{c}\left(\dfrac{2\sin\beta_0}{Y_1(x_K)\sqrt{\varepsilon_0}}(P_1Y_1+P_2Y_2)\dfrac{d\beta_0}{dx}-\dfrac{1}{2\sin\beta_H^0}\dfrac{\varepsilon_1}{\varepsilon_0}\right)\end{cases} \tag{37}$$

The corresponding initial conditions have the form: $z_0(0)=0$, $\beta_0(0)=\beta_H^0$, $Y_1(0)=0$, $L_1(0)=1$, $Y_2(x_K)=0$, $L_2(x_K)=1$, $P_1(x_K)=0$, $P_2(0)=0$, $\Delta\tau(0)=0$. Here the following designations are introduced: x_K is the horizontal distance of the spacecraft and z_K is its altitude, $z(x_K)=z_K$. The observer is at the point with the coordinates $z=0,\ x=0$. When calculating a DFC with the use of Eq. (37), one should apply the results of the preliminary ray tracing for determining the quantity β_H^0 in the undisturbed medium, which is performed when solving system (26). In the plane case, system (26) has the following form:

$$\begin{cases}\dfrac{dz_0}{dx}=\cot\beta_0\\ \dfrac{d\beta_0}{dx}=-\dfrac{1}{2}\dfrac{\partial\ln\varepsilon_0}{\partial z_0}\\ \dfrac{dY_1}{dx}=-\dfrac{1}{\sin^2\beta_0}L_1\\ \dfrac{dL_1}{dx}=-\dfrac{1}{2}\dfrac{\partial^2\ln\varepsilon_0}{\partial z_0^2}Y_1\end{cases} \tag{38}$$

3. EXAMPLE OF NUMERICAL-ASYMPTOTIC SYNTHESIS OF A DISTURBED DFC FOR DETECTING A LARGE-SCALE INHOMOGENEITY

Based on asymptotically derived Eqs. (37) and (38), we have carried out preliminary modeling of the disturbance introduced by the inhomogeneity into the DFC for different parameters of the inhomogeneity as well as for several positions of the low-orbiting spacecraft with respect to the ground-based observation site.

To model the undisturbed ionospheric electron density profile, we used the two-layer Gaussian model [1]. In this case, dielectric permittivity of the undisturbed ionosphere is a function of height and has the form:

$$\varepsilon_0(z)=1-\frac{f_{kp}^2}{f^2}\exp\left(-\left(\frac{z-z_m}{h_m}\right)^2\right)\\ -\frac{f_{kpE}^2}{f^2}\exp\left(-\left(\frac{z-z_{mE}}{h_{mE}}\right)^2\right), \tag{39}$$

where z_m and z_{mE} are the F2 and E electron density maximum heights, respectively, h_m and h_{mE} characterize the thicknesses of the layers, and the quantities $f_{êð}$ and $f_{êðÅ}$ characterize their critical frequencies.

We specified the model characterizing a localized inhomogeneity in the following form:

$$\varepsilon_1(x,z)=-\gamma\frac{f_{kp}^2}{f^2}\left(1-th\left[\frac{(x-x_c)^2}{b}+\frac{(z-z_c)^2}{a}-r\right]\right), \tag{40}$$

where x_c and z_c are the coordinates of the disturbance center, and parameters a, b and r are related to the scales and contrast of the inhomogeneity, and the parameter γ specifies its intensity.

As an example, Fig. (**2**) presents the results from calculating the disturbed portions of the DFC for two positions of the spacecraft with respect to the observer position: x_K = 1000 km (Fig. **2a**), and x_K = 1200 km (Fig. **2b**). We took the following values for the undisturbed ionosphere parameters: z_m = 300 km, z_{mE} = 125 km, h_m = 100 km, h_{mE} = 25 km, $f_{êð}$ = 8 MHz, and $f_{êðÅ}$ = 3 MHz. The inhomogeneity intensity was γ = 0.02. The parameters

$a = 25$, $b = 50$ and $r = 100$ effectively characterize the inhomogeneity size along the vertical and horizontal, which were taken to be 100 km and 140 km, respectively. The coordinates of the inhomogeneity center were $x_c = 330$ km and $z_c = 360$ km. The spacecraft was placed at an altitude of $z_k = 1000$ km. It is apparent from Fig. (**2**) that the group delay time variation decreases with the increasing operating frequency. This is due to the weakening of the inhomogeneity effect on the transionospheric trajectory of the wave in a high frequency range. Furthermore, one can see maxima of $c\Delta\tau$ on the plots, which correspond to the trajectories passing near the ionospheric inhomogeneity edge.

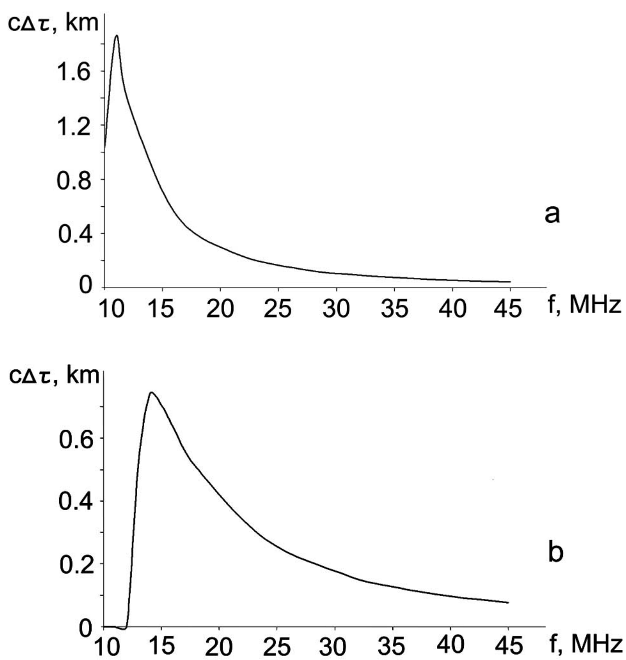

Fig. (2). Examples of numerical-asymptotic synthesis of the disturbance introduced into the distance-frequency characteristic by a large-scale plasma inhomogeneity. The plots correspond to different horizontal distances between the low-orbiting satellite and the ground-based observer: **a**) $x_K = 1000$ km, **b**) $x_K = 1200$ km.

Using the numerical-asymptotic method of modeling, it is possible to compute a series of disturbed DFCs for different positions of the spacecraft with respect to the ground-based observation site, thereby performing the multifrequency scanning of the inhomogeneity with many transionospheric rays. The large-scale inhomogeneity parameters can be reconstructed from the DFC disturbance in two stages. Initially, in order to determine the location and size of the inhomogeneity, the approach suggested in [5, 7] can be used. Namely, first we collect those DFCs (corresponding to various positions of the spacecraft with respect to the observer) that display significant variations of the signal group delay time within some limited frequency range. After that, based on analysis of the collected DFCs, we identify a disturbed region in the ionosphere where the inhomogeneity may reside. The boundary of this region is formed by the transionospheric rays with frequencies corresponding to the limiting frequencies of the DFC disturbance. These trajectories are calculated in the unperturbed ionosphere. Overlapping the disturbed regions, constructed for the set of DFCs corresponding to different positions of the spacecraft, makes it possible to determine the location and size of a large-scale solitary inhomogeneity. For a more accurate determination of the inhomogeneity localization region, additional receiving sites can be used. Overlapping disturbed regions, constructed for different ground-based stations, permits a more accurate identification of the ionospheric region where the disturbance is localized.

Considering now that the geometrical parameters of the inhomogeneity are known, Eqs. (37) and (38) can be used to calculate its intensity. In this case, we use, as $\Delta\tau$, the value of the group delay time variation at a specific sounding frequency that we choose from the range of frequencies at which the DFC is disturbed. It is important to point out that the suggested approximate method of synthesizing disturbed DFCs for diagnosing large-scale localized inhomogeneities can also be used in the case where the ionosonde is aboard a geostationary satellite. By keeping track of deformations of the DFCs recorded at different receiving sites on the Earth, it is possible to monitor the parameters of the inhomogeneity through monitoring its movements in the transionospheric scanning cone [3,4].

It should be noted that the suggested method of synthesizing DFCs for diagnosing the large-scale inhomogeneities fails to give satisfactory results if the transionospheric sounding is carried out at the lowest possible frequencies. In this case, large-scale plasma inhomogeneities can give rise to transionospheric trajectories sliding along the main ionospheric maximum, thereby producing a considerable increase in variations of the trajectory parameters. Under such conditions, the perturbation theory method in its simplest form is inapplicable for calculating deflections of the ray trajectory from its unperturbed path. This dictates a need for relevant generalizations of the asymptotic solution derived in this paper.

4. CONCLUSIONS

We have obtained approximate analytical formulas for calculating the deformation of the distance-frequency characteristic of the transionospheric decametric radio signal under the influence of a localized large-scale plasma inhomogeneity. We have suggested techniques for detecting such a large-scale inhomogeneity, based on numerical-asymptotic synthesis of disturbed DFCs of decametric radio signals radiated from a low-orbiting or geostationary spacecraft. Using numerical modeling, we have demonstrated the possibility of determining the size and intensity of the large-scale inhomogeneity from disturbed portions of a series of DFCs.

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