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RESEARCH ARTICLE

Interval Observer Design for Discrete-Time Switched Systems by Linear Programming Method

Jun Huanga*, Xiang Maa and Guoguang Yina

School of Mechanical and Electrical Engineering, Soochow University, Suzhou, 215131, China

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Abstract:

Objective:

In this paper, the interval observer design problem for discrete-time switched systems is considered. The construction of the observer is designed and the sufficient conditions are derived.

Methods:

The tool that we employ is the linear programming method, and the multiple linear copositive Lyapunov function is used to analyze the stability of error systems.

Results:

The corresponding error systems are proved to be both positive and exponentially stable.

Conclusion:

As far as we know, it is the first piece of work to apply the LP method to design the interval observer for discrete-time switched system.

Keywords: Switched systems, Discrete time, Linear programming, Interval observer, (ADT), (MLCLF).

1. INTRODUCTION

As a kind of an important hybrid system, switched systems have been paid much attention in the past decades [1, 2]. Switched systems have a broad engineering background since they can be used to describe many kinds of practical systems, such as flight control systems, network control systems, power electronics, and so on. Generally, switched systems consist of some subsystems, which are controlled by switching laws. Under the given switching law, only one subsystem can be activated at each time. In order to study the stability of the switched system, some methods have been proposed, such as common Lyapunov function method [3, 4], multiple Lyapunov functions method [5, 6], Average Dwell Time (ADT) method [7 - 10], advanced Lyapunov function method [11 - 13]. As we know, the states cannot be obtained directly, thus the observer's design for switched systems is important. Due to the uncertainty of switching modes, the observer's design for switched systems is not an easy task [14 - 18].

On the other hand, the uncertainty always exists in real systems, it is not feasible for the exact state estimation of an uncertain system. In practice, it is very useful to estimate the upper and lower bound of the states [19]. After introducing the definition of interval observer for the biological system by [20], the investigation of interval observers became a hot topic in the field of control theory [21] and [22] both investigated the interval observers design problem for nonlinear

* Address correspondence to this author at the School of Mechanical and Electrical Engineering, Soochow University, Jixue Road No. 8, Suzhou, Jiangsu, China Tel: +86-18020275530; E-mails: cauchyhot@163.com; jhuang@suda.edu.cn

systems by Linear Programming (LP) method. Specifically [21], considered the uncertainty that is only contained in the state equation, while [22] studied the uncertainty that is contained in the state equation as well as the output equation. By using the coordinate transformation method [23], and [24] addressed the interval observers design problem for nonlinear systems. Under the time-varying transformation [23], established the interval observers frame for time-varying systems with the aid of the Jordan canonical form [24]. proved the existence of time-invariant interval observers for unobservable systems by the time-invariant transformation. Besides, there also exist many other works on the interval observers, such as LPV systems [25], singular systems [26], impulsive systems [27], PDE systems [28]. However, to the best knowledge of the authors, the interval observers for switched systems have been paid little attention [29 - 31].

Motivated by the above discussion, this paper investigates the interval observers design approach for discrete-time switched systems. Different from the mentioned works [29 - 31], the observer gains are constructed by Multiple Linear Copositive Lyapunov Function (MLCLF), and sufficient conditions for the existence of the interval observers are then given by LP forms. The remainder of the paper is organized as follows: Problem formulation as well as the framework of interval observers is given in Section 2. Section 3 presents the sufficient conditions, under which the designed observers are exponentially stable interval observers for the original systems. Section 4 simulates a numerical example to demonstrate the effectiveness of the designed interval observers.

Notations

– $||x||$: the Euclidean norm of the vector x;

– x^T: the transposition of the vector x;

– A^T: the transposition of the matrix A;

– $x > (\geq)\ 0$: its components are positive (nonnegative), *i.e.*, $x_i > (\geq)\ 0$:

– $A > (\geq)\ 0$: its components are positive (nonnegative), *i.e.*, $A_{ij} > (\geq)\ 0$:

– $\underline{\mu}(x)$: is the minimum value of the elements of x;

– $\overline{\mu}(x)$: is the maximum value of the elements of x;

– l: the identity matrix with appropriate dimensions.

2. PROBLEM FORMULATION AND PRELIMINARY

Let us consider the following discrete-time switched system

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k), \\ y(k) = C_{\sigma(k)}x(k), \\ x^-(0) \leq x(0) \leq x^+(0), \end{cases} \tag{1}$$

where $x(k) \in R^n$, $u(k) \in R^m$ and $y(k) \in R^q$ are the state, control input and output of the system, repectively. σ(k) is the switching signal which takes value in a finite set $S = 1,2,..., N$.. For any σ (k) = $i \in S$, A_i, B_i and C_i are determined matrices with appropriate dimensions. The upper bound and lower bound of initial state x (0) are x^+ (0) and x^- (0) and x^- (0), and x^+ (0) and x^- (0) are both known.

Firstly, some general definitions of interval observer are introduced.

Definition 2.1 [*21*] *An interval observer for (1) is pair of upper and lower recovered states* $\{\hat{x}^+(k), \hat{x}^-(k)\}$, *which satisfy for any* $k > 0$

$$\hat{x}^-(k) \leq x(k) \leq \hat{x}^+(k),$$

under the initial condition

$$\hat{x}^-(0) \leq x(0) \leq \hat{x}^+(0).$$

Definition 2.2 *[17]* An interval observer for (1) is said to be exponentially convergent if there exist constants $\alpha_1 > 0,\ \alpha_2 > 0,\ 0 < \beta_1 < 1,\ 0 < \beta_2 < 1$ *such that for any* $k \geq 0$

$$||\hat{x}^+(k) - x(k)|| \leq \alpha_1 \beta_1^k ||\hat{x}^+(0) - x(0)||,$$

and

$$||x(k) - \hat{x}^-(k)|| \leq \alpha_2 \beta_2^k ||x(0) - \hat{x}^-(0)||.$$

Then, we design the following interval observer for the system (1):

$$\begin{cases} \hat{x}^+(k+1) = A_{\sigma(k)}\hat{x}^+(k) + B_{\sigma(k)}u(k) + L_{\sigma(k)}(y - C_{\sigma(k)}\hat{x}^+(k)), \\ \hat{x}^-(k+1) = A_{\sigma(k)}\hat{x}^-(k) + B_{\sigma(k)}u(k) + L_{\sigma(k)}(y - C_{\sigma(k)}\hat{x}^-(k)), \\ \hat{x}^-(0) = x^-(0), \hat{x}^+(0) = x^+(0), \end{cases} \quad (2)$$

where the observer gain $L_{\sigma(k)} \in R^{n\times q}$ will be determined later. Subtracting (2) from (1), we obtain the error system:

$$\begin{cases} e^+(k+1) = (A_{\sigma(k)} - L_{\sigma(k)}C_{\sigma(k)})e^+(k), \\ e^-(k+1) = (A_{\sigma(k)} - L_{\sigma(k)}C_{\sigma(k)})e^-(k), \\ e^+(0) \geq 0, e^-(0) \geq 0, \end{cases} \quad (3)$$

where $e^+(k) = \hat{x}^+(k) - x(k)$ and $e^-(k) = x(k) - \hat{x}^-(k)$. In what follows, we review some basics of positive system and more details can be referred to [17], [14], [33]. Consider the following switched system:

$$\begin{cases} x(k+1) = M_{\sigma(k)}x(k), \\ x(0) = x_0 \geq 0, \end{cases} \quad (4)$$

where σ(k) is defined above, and M_i is a constant matrix for any σ(k) = $i \in S$.

Definition 2.3 *[14]The system (4) is said to be positive if the corresponding trajectory* x(k) ≥ 0 *for any* k ≥ 0, $i \in S$.

.

Definition 2.4 *[14]* The matrix $M_{\sigma(k)}$ is said to be a Schur matrix if all its eigenvalues have a magnitude less than one for any $i \in S$.

Definition 2.5 *[17]The system (4) is exponentially stable if there exist constants* $C_1 > 0$ and $0 < c_2 < 1$ *such that* $||x(k)|| \leq c_1 c_2^k ||x_0||$ *for any* $k \geq 0$.

Definition 2.6 *[33]* Consider the time interval [K, K_2 where $K_1 \geq 0$. Let the switching number of σ(k) on [K_1, K_2) be N_σ (K_1, K_2). If the following inequality holds

$$N_\sigma(k_1, k_2) \leq N_0 + (k_2 - k_1)/\tau^*$$

where $N_0 \geq 0$, and $\tau^* > 0$ then τ^* is an ADT of the switching signal σ(K) and N_0 is the chatter bound.

The following lemmas are essential to the main result of the paper.

Lemma 2.1 *[8] The system (4) is positive if and only if* $M_i \geq 0$ *for any* $i \in S$.

Lemma 2.2 *[20] If the system (4) is positive, then the following statements are equivalent:*

i. M_i a Schur matrix.
ii. There exists a vector $v > 0$ in R^n $(M_i - I)v > 0$.

3. MAIN RESULT

In this section, we design the observer gain L_i and derive sufficient conditions to make the error system (3) both positive and exponentially stable.

Theorem 3.1 If there exist constants $0 < \eta < 1,\ \rho > 1$ *and vectors* $v^{(i)} \in R^n > 0$, $v^{(j)} \in R^n > 0,\ z^{(i)} \in R^q,\ \forall i, j \in S,\ i \neq j$ such that

$$(A_i^T - \eta I)v^{(i)} + C_i^T z^{(i)} < 0, \tag{5}$$

$$\zeta^{(i)T} v^{(i)} (\zeta^{(i)T} v^{(i)} A_i + \zeta^{(i)} z^{(i)T} C_i) \geq 0, \tag{6}$$

$$v^{(i)} \leq \rho v^{(j)}, \tag{7}$$

where $\zeta^{(i)} \in R^n \neq 0$ is a pre-specified vector. Then the observer gain is designed by

$$L_i = -\frac{\zeta^{(i)} z^{(i)T}}{\zeta^{(i)T} v^{(i)}}, \tag{8}$$

and ADT satisfying

$$\tau^* \geq -\frac{\ln\rho}{\ln\eta}, \tag{9}$$

the error system (3) is both positive and exponentially stable.

proof By Definitions 2.1, we know that the upper error e^+ (K) and lower error e^- (k) should be positive. From Definition 2.3, the errors e^+ (k) and e^- (k) require exponential stability property. We will complete the proof by two steps:

(i) *Positivity.* Consider the upper error system, *i.e*,

$$\begin{cases} e^+(k+1) = (A_{\sigma(k)} - L_{\sigma(k)} C_{\sigma(k)}) e^+(k), \\ e^+(0) \geq 0. \end{cases} \tag{10}$$

It follows from (8) that

$$A_i - L_i C_i = A_i + \frac{\zeta^{(i)} z^{(i)T}}{\zeta^{(i)T} v^{(i)}} C_i, i \in S. \tag{11}$$

In view of (6), we can obtain

$$A_i + \frac{\zeta^{(i)} z^{(i)T}}{\zeta^{(i)T} v^{(i)}} C_i \geq 0. \tag{12}$$

Thus, $A_i - L_i C_i \geq 0$. From Lemma 2.1, the error system (10) is positive.

(*ii*) *Exponential stability.* Consider the switching sequence $\{K_p, p = 1, 2...\}$, and $0 < K_1 < K_2 < ....$ Suppose that $\sigma(K_l) = i \in S$, we choose the following MLCLF candidate

$$V_i(k) = (e^+(k))^T v^{(i)}. \tag{13}$$

Let $K \in [K_l, K_l + 1)$, we can compute the difference of V_i as

$$\begin{aligned}\Delta V_i &= V_i(k) - V_i(k-1) = (e^+(k))^T v^{(i)} - (e^+(k-1))^T v^{(i)} \\ &= (e^+(k-1))^T (A_i^T - C_i^T L_i^T) v^{(i)} - (e^+(k-1))^T v^{(i)} \\ &= (e^+(k-1))^T (A_i^T - C_i^T L_i^T - I) v^{(i)}.\end{aligned} \tag{14}$$

It follows from (8) that

$$\Delta V_i = (e^+(k-1))^T [(A_i^T - I) v^{(i)} + C_i^T z^{(i)}]. \tag{15}$$

By (5), we have

$$\Delta V_i \leq (\eta - 1)(e^+(k-1))^T v^{(i)} \leq (\eta - 1) V_i(k-1), \tag{16}$$

which implies that

$$V_i(k) \leq \eta V_i(k-1). \tag{17}$$

Consider the time interval $[K_l, K)$, it is deduced from (17) that

which implies that

$$V_i(k) \leq \eta^{(k-k_l)} V_i(k_l). \tag{18}$$

Since $e^+(K) \geq 0$ and (7), the following holds

$$(e^+(k_l))^T v^{(i)} \leq \rho (e^+(k_l))^T v^{(j)}, \forall i, j \in S, i \neq j. \tag{19}$$

Suppose that $\sigma(K_{l-1}) = j$, combining (18) with (19) yields

$$V_i(k) \leq \rho \eta^{(k-k_l)} V_j(k_l). \tag{20}$$

By Definition 2.6, we have $l = N_\sigma \leq N_0 + k/\tau^*$. It follows from (18)-(20) that

$$\begin{aligned}V_i(k) &\leq \rho \eta^{(k-k_l)} V_{\sigma(k_{l-1})}(k_l) \leq \rho \eta^{(k-k_{l-1})} V_{\sigma(k_{l-1})}(k_{l-1}) \\ &\leq \cdots \leq \rho^l \eta^k V_{\sigma(0)}(0) \\ &\leq \exp\{N_\sigma \ln\rho\} \exp\{k \ln\eta\} V_{\sigma(0)}(0) \\ &\leq \exp\{(N_0 + \tfrac{k}{\tau^*}) \ln\rho\} \exp\{k \ln\eta\} V_{\sigma(0)}(0) \\ &\leq c_1 c_2^k V_{\sigma(0)}(0),\end{aligned} \tag{21}$$

where $c_1 = \exp\{N_0 \ln\rho\}$, $c_2 = \exp\{\ln\eta + \frac{\ln\rho}{\tau^*}\}$. Since $\rho > 1$ and (9), we have $c_2 < 1$. (21) is equivalent to

$$(e^+(k))^T v^{(i)} \leq c_1 c_2^k V_{\sigma(0)}(0). \tag{22}$$

In view of the positivity of $e^+(k)$, the following two inequalities hold

$$\begin{aligned}(e^+(k))^T v^{(i)} &= \textstyle\sum_{m=1}^n e_m^+(k) v_m^{(i)} \geq \underline{\mu}(v^{(i)}) \sum_{m=1}^n e_m^+(k) \\ &\geq \underline{\mu}(v^{(i)}) ||e^+(k)||,\end{aligned} \tag{23}$$

and

$$(e^+(0))^T v^{(\sigma(0))} \quad = \sum_{m=1}^{n} e_m^+(0) v_m^{(\sigma(0))} \leq \bar{\mu}(v^{(\sigma(0))}) \sum_{m=1}^{n} e_m^+(0) \leq \sqrt{2}\mu(v^{(\sigma(0))}) ||e_m^+(0)||. \tag{24}$$

Let $c_3 = \frac{\sqrt{2}\bar{\mu}(v^{(\sigma(0))})}{\underline{\mu}(v^{(i)})} c_1$ it follows from (22)-(24) that

$$||e^+(k)|| \leq c_3 c_2^k ||e^+(0)||. \tag{25}$$

Thus we can conclude that the upper error system (10) is both positive and exponentially stable. Then we consider the lower error system, *i.e.*,

$$\begin{cases} e^-(k+1) = (A_{\sigma(k)} - L_{\sigma(k)} C_{\sigma(k)}) e^-(k), \\ e^-(0) \geq 0. \end{cases} \tag{26}$$

Since the lower error system has the same construction as that of the upper error system, consequently we can use the similar method to prove that the positivity and exponential stability of the lower error system. By using (6) and (8), we can prove that the lower system is positive. If we choose the MLCLF candidate as $V_i(k) = (e^-(k))^T v^{(i)}$, then we can prove that e^- (K) is exponentially stable by (5) and (7)-(9). The proof is completed.

Remark 3.1 By solving (5)-(7), we can obtain $v^{(i)}$ and $Z^{(i)}$. By the $v^{(i)}$, $Z^{(i)}$, and $\xi^{(i)}$, we can determine the gain . Note that the dominator ξ(i)T$_V$(i) is a scalar, the expression of L_i is valid.

Remark 3.2 *Compared with current works [12], [10], [7], the sufficient conditions in this paper are derived by LP forms, which can be easily solved by Matlab.*

For the purpose of numerical computation, the following Corollary is necessary.

Corollary 3.1 If there exist constants $0 < \eta < 1,\ \rho > 1$ *and vectors* $v^{(i)} \in R^n > 0$, $v^{(j)} \in R^n > 0,\ z^{(i)} \in R^q,\ \forall i,j \in S,\ i \neq j$ such that

$$(A_i^T - \eta I) v^{(i)} + C_i^T z^{(i)} < 0, \tag{27}$$

$$\zeta^{(i)T} v^{(i)} > 0, \tag{28}$$

$$\zeta^{(i)T} v^{(i)} A_i + \zeta^{(i)} z^{(i)T} C_i \geq 0, \tag{29}$$

$$v^{(i)} \leq \rho v^{(j)}, \tag{30}$$

$$(A_i^T - \eta I) v^{(i)} + C_i^T z^{(i)} < 0, \tag{31}$$

$$\zeta^{(i)T} v^{(i)} < 0, \tag{32}$$

$$\zeta^{(i)T} v^{(i)} A_i + \zeta^{(i)} z^{(i)T} C_i \leq 0, \tag{33}$$

$$v^{(i)} \leq \rho v^{(j)}. \tag{34}$$

Then the observer gain is given by (8) and ADT satisfying (9), the error system (3) is positive and exponentially stable.

Proof In fact, we just need argue the condition (5) of Theorem 3.1 in two cases.

i. If $\zeta^{(i)T}v^{(i)} > 0$, it follows from (5) that $\zeta^{(i)T}v^{(i)}A_i + \zeta^{(i)}z^{(i)T}C_i \geq 0$ and (27)-(30) are equivalent to (5)-(7).
ii. If $\zeta^{(i)T}v^{(i)} < 0$, it is deduced from (5) that $\zeta^{(i)T}v^{(i)}A_i + \zeta^{(i)}z^{(i)T}C_i \leq 0$ and (31)-(34) are equivalent to (5)-(7).

Thus, combining (i) with (ii), we complete the proof.

Remark 3.3 It is worth pointing out that conditions (5)-(7) can not be solved by Linprog in Matlab since the bilinear term $(\zeta^{(i)T}v^{(i)})^2$ is contained. Conditions (27)-(30) or (31)-(34) are standard LP forms, and Corollary 3.1 is important to practical computation.

Remark 3.4 *This paper mainly focuses on switched systems without uncertainties or disturbances. All the system matrices are determined. The aim is to design exponentially stable interval observers for discrete-time switched systems. In the future, we will consider the interval observers design for uncertain switched systems.*

4. NUMERICAL EXAMPLE

Consider the system (1) with two modes, where

$$A_1 = \begin{bmatrix} -0.1 & -0.3 \\ -0.2 & -0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.1 & -0.2 \\ -0.3 & 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2 & 0.2 \\ 0.3 & 0.2 \end{bmatrix},$$

$$C_1 = [0.30.3], \quad C_2 = [0.20.2].$$

Selecting $\zeta^{(1)} = \zeta^{(2)} = [1; 1]$, $\eta = 0.5$, $\rho = 1.5$. By Corollary 3.1, we obtain

$$v^{(1)} = \begin{bmatrix} 128.4588 \\ 130.5851 \end{bmatrix}, \quad z^{(1)} = 304.1366,$$

$$v^{(2)} = \begin{bmatrix} 114.2839 \\ 137.8171 \end{bmatrix}, \quad z^{(2)} = 389.1809.$$

Then the observer gain can be designed as

$$L_1 = \begin{bmatrix} -1.1741 \\ -1.1741 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -1.5437 \\ -1.5437 \end{bmatrix}.$$

In the simulation, the inputs and initial conditions of systems (1) and (2) are chosen as:

$$u = \begin{bmatrix} \sin t \\ \sin 2t \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

$$x_0^+ = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad x_0^- = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The time response of switching signal σ(t) is depicted in Fig. (**1**). The results of simulation of the interval observer are given in Figs. (**2** and **3**). It is shown that the upper and lower estimations can converge to the original state after about 1s. From the simulation result, we can conclude that the proposed method is effective.

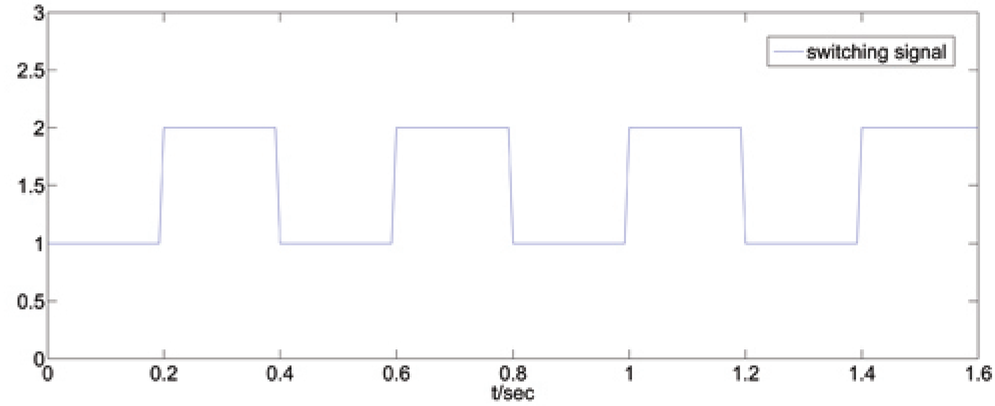

Fig. (1). Simulation of switching signal σ(t)

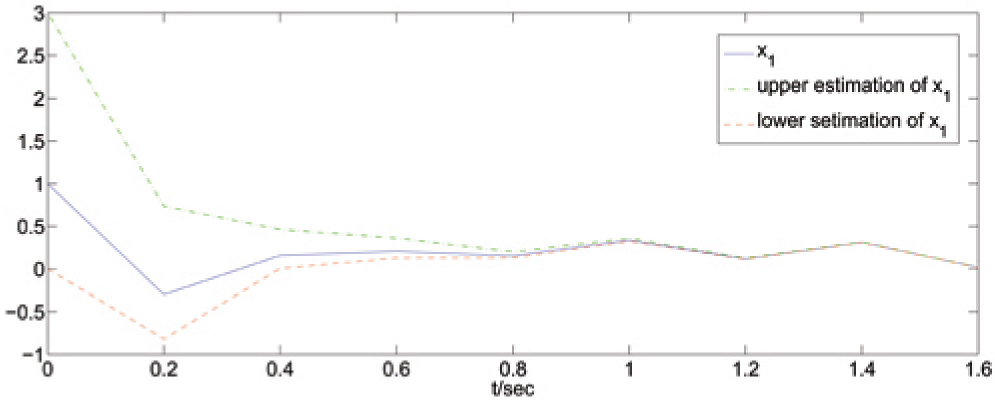

Fig. (2). Evolution of the real state x_1 (K) and the estimations $\hat{x}_1^+(k)$, $\hat{x}_1^-(k)$.

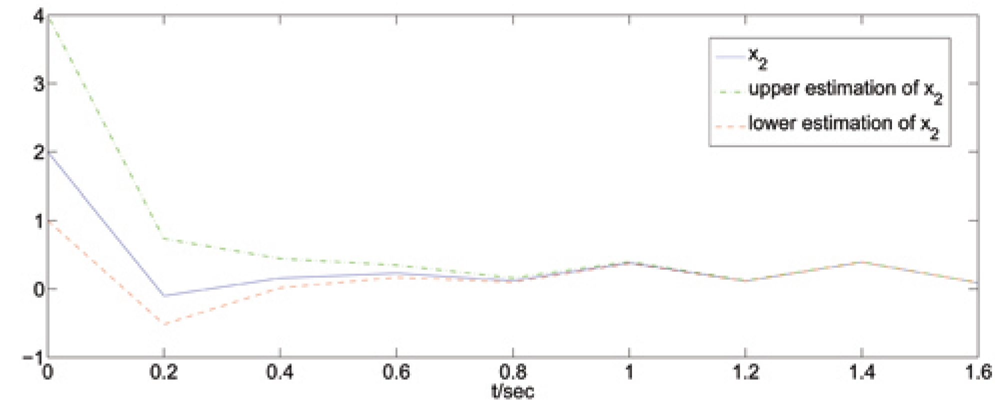

Fig. (3). Evolution of the real state x_2 (K) and the estimations $\hat{x}_2^+(k)$, $\hat{x}_2^-(k)$

CONCLUSION

In this paper, the interval observer design problem for discrete-time switched systems is considered. The construction of the observer is designed and the sufficient conditions by the LP forms are also given. As far as we know, it is the first piece of work to apply the LP method to design the interval observer for discrete-time switched system. A numerical example is given to show the validity of the proposed method.

CONSENT FOR PUBLICATION

Not applicable.

CONFLICT OF INTEREST

The authors declare no conflict of interest, financial or otherwise.

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