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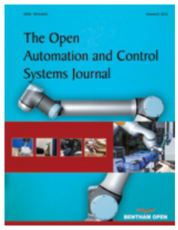
DOI: 10.2174/1874444301810010009, 2018, *10*, 9-14



RESEARCH ARTICLE

Conditions for Ensuring Aperiodic Transients in Automatic Control Systems with a PID Controller

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Received: April 18, 2018 Revised: June 5, 2018 Accepted: July 20, 2018

Abstract:

Research Problem:

The purpose of the study was to obtain the relatively simple conditions for ensuring aperiodic transients in remote control systems with a PID controller.

Research Questions:

1. Does the control loops model with the cubic characteristic equation leads the to the relatively simple conditions for ensuring aperiodic transients?

2. Does the simple terms derived by approximate formula for Q-factor in line with the terms of oscillability lack by using the certain inequality, which is correct for the cubic equation with the real roots only?

3. Does the simple regulators good in overdamping the transition oscillations?

4. Does the conditions for ensuring aperiodic transients in automatic control systems helpful for the quick robust PID tuning?

Literature Review:

The purpose of the literature review was to provide a brief historical background of the research task. The key research results are achieved in the quasi-optimal PID tuning field. The attempts of synthesis the relatively simple PID tunung analytical methods was undertaken for partial narrow tasks.

Methodology:

The case study is based on the qualitative analysis of cubic control loops characteristic equation. The results of qualitative analysis proved by the LabView simulation using the Control Design and Simulation Module.

Results and Conclusions:

Several examples of oscillation transients occurs at automatic control systems described by a mathematical model with a cubic characteristic equation were discussed in this paper. There were obtained matching sufficient conditions for aperiodic transient PID tuning based on the known condition of none complex conjugate transfer function poles and approximate formulas for finding the roots of cubic equations. Shown the condition of $\tau_0 > \frac{\tau}{\sqrt{G_0}}$ is the sufficient criterion for oscillation transition exception in the control process with the loop elements where is the real poles. The condition of $\tau_0 > \sqrt{\frac{2\tau\tau_0}{G_0}}$ is the sufficient criterion for oscillation exception in such process granting the complex-conjugate poles element. The aperiodic transition has been provided by using the PID regulator in the context of none of loops elements with the resonant behavior with the best time response.The achievement of

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relatively simple conditions for ensuring aperiodic transients in automatic control systems with a PID controller is extremely useful for many applications because most of the single-loop controllers used in practice are PID.

Keywords: Remote control system, Cubic characteristic equation, Aperiodic transition, Q-factor, PID controller, Robust PID tuning, Simple formulas.

1. INTRODUCTION

Using the simple regulators for robust control commonly cannot be successful. The need for qualitative transient analysis was first showed by I. A. Vyshnegradsky [1]. V. V. Solodovnikov proposed to diagnose the freedom from transient overshoot by frequency methods, which resulted as the necessary and sufficient conditions [2], and sufficient condition for aperiodic transient [3]. M. V. Meerov formulated necessary and sufficient conditions for the aperiodic stability in a few cases [4]. The key results of quasi-optimal PID tuning researches are given in [5 - 16]. The attempts of synthesis is relatively simple PID tunung analytical methods undertaken in [17 - 24]. Aperiodic transient even with the resonant amplitude-frequency response elements in closed feedback loop by using PID controller showed in [24]. The obtained results are based on [25], [26] formulas. The achievement of relatively simple conditions for ensuring aperiodic transients in automatic control systems with a PID controller will be extremely useful for many applications. So it is the main goal of the research presented in this paper.

2. CONDITIONS FOR ENSURING APERIODIC TRANSIENTS

Much of the PID control loops can be mathematically presented by model with the cubic characteristic equation $ax^3 + bx^2 + cx + d = 0$[24]. Let's find the terms of oscillability lack by using the certain inequality [27], which is correct for the cubic equation with the real roots only:

$$b^2c^2 + 18abcd > 4(b^3d + ac^3) + 27a^2d^2. \quad \mathbf{(1)}$$

This inequality leads to quadratic equation analysis by dividing its components by a^2d^2:

$$\alpha^2 + 18\alpha > 4\alpha^2\beta + 27, \quad \mathbf{(2)}$$

where $\alpha = \dfrac{bc}{ad}$, $\beta = \dfrac{bd}{c^2} + \dfrac{ac}{b^2}$. Hereout

$$\alpha^2(1-4\beta) + 18\alpha - 27 > 0. \quad \mathbf{(3)}$$

Perhaps it seems that the term (3) analysis (leads to the quadratic equation solving) is easy. However, in real control systems case coefficients a, b, c and d are depended by the multiple parameters of the remote control loop links. Thus, this analysis may be full of traps and pitfalls.

The analysis was carried out in terms of the scanning probe microscope automatic gain control loop. We will use the PID controller with the transfer function $R(p) = \dfrac{(1+p\tau_2)(1+p\tau_3)}{p\tau_1}$ [24]. Vibration of the Z-stage is described by the quadratic polynomial $p^2\tau^2 + p\dfrac{\tau}{q_0} + 1$. G_0 is the loop gain. So the resulting closed-loop characteristic polynomial could be written as:

$$p^3\tau^2\tau_1 + p^2\left(\frac{\tau\tau_1}{q_0} + G_0\tau_2\tau_3\right) + p\left(\tau_1 + G_0\tau_2 + G_0\tau_3\right) + G_0 = 0. \quad \mathbf{(4)}$$

There are some denominations, which have been used in characteristic equation (4): τ_1 – characteristic integrator (I regulator) time, τ_2 и τ_3 – characteristic times of the differential correction chains, – characteristic time, which determines the mechanical stage oscillations frequency, q_0 – oscillation process Q-factor.

Let's take $\tau_2 = \tau_3 = \tau_0$ and $G_0 = 1$ for the reckoning shortcut. Moreover, the time condition $\tau_1 >> \tau$ for I regulator must be taken into account. The term $\tau_0{}^2 >> \frac{\tau\tau_1}{q_0}$ is legitimate because the control process resonant frequencies are over 400 Hz and the Q-factor is about $q_0 \approx 5 \div 10$.

Thus the characteristic equation (4) can be simplified:

$$p^3\tau^2\tau_1 + p^2\tau_0^2 + p\tau_1 + 1 = 0. \quad \textbf{(5)}$$

Comparison of equation (5) and term (2) is $\alpha = \frac{\tau_0^2}{\tau^2}$, a $\beta = \frac{\tau_0^2}{\tau_1^2} + \frac{\tau_1^2\tau^2}{\tau_0^4} \approx \frac{y^2}{\alpha}$, where $y = \frac{\tau_1}{\tau_0} >> 1$.

There at the condition (2) is given by $\alpha^2 + (18 - 4y^2)\alpha - 27 > 0$. It can be simplified if $y > 10$ Then $4y^2 >> (400 > 18)$.

The roots of the equation $\alpha^2 - 4y^2\alpha - 27 = 0$ are $\alpha_{1,2} = 2y^2 \pm \sqrt{4y^4 + 27}$. Thus $\alpha = 4y^2$ or $\frac{\tau_0^2}{\tau^2} = 4\frac{\tau_1{}^2}{\tau_0{}^2}$, $\tau_0^4 = 4\tau_1^2\tau^2$ assuming that $y \geq 10$.

The resulting PID tuning rule is $\tau_0 > \sqrt{2\tau\tau_1}$. This term is completely in line with the results given by approximate formula for Q-factor $q \approx \frac{c\sqrt{ac}}{bc - ad}$ which had been used at the scanning probe microscope remote control system design implementation [28].

In fact, the oscillation transition and complex-conjugate poles may occur even in the context of none of loops elements with the resonant behavior. Let's take a look.

Usually, PID tuning is done by the condition $\tau_1 >> \tau_0$. The term of oscillation inception following from the Q-factors approximate formula were obtained in [24]

$$G_0\tau_0^2 > \tau\tau_1(2 - \frac{1}{q_0}) + \tau^2. \quad \textbf{(6)}$$

Let's consider the two cases of this term satisfaction: 1) $q_0 \leq \frac{1}{2}$, 2) $q_0 > \frac{1}{2}$.

CASE 1. There are two elements with the real poles at their transfer functions for the automatic control gain loop in first case. The case of the real poles are equal ($q_0 = \frac{1}{2}$ [29]) is the most dangerous to the oscillation inception..

Substitution of $q_0 = \frac{1}{2}$ to the condition (6) leads to $G_0\tau_0^2 > \tau^2$. Therefore

$$\tau_0 > \frac{\tau}{\sqrt{G_0}}. \quad \textbf{(7)}$$

If $q_0 < \frac{1}{2}$, then the right hand side of the inequality (6) appears to be the negative component $\tau\tau_1(2 - \frac{1}{q_0}) < 0$ and the term (6) under condition (7) will be realized more.

CASE 2. There is the element with the complex-conjugate poles at the transfer function for the automatic control gain loop in the second case. Suppose the Q-factor of this poles is large enough $q_0 >> 1$. In that case the terms (1) can be represented as $G_0\tau_0^2 > 2\tau\tau_1 + \tau^2$. Practically handle to choose τ_1 under condition $2\tau_1 >> \tau$. Thus

$$G_0\tau_0^2 > 2\tau\tau_1 . \quad \textbf{(8)}$$

As $G_1 = 1$ it leads to the term of $\tau_0 > \sqrt{2\tau\tau_1}$, which is verified with the earlier obtained term by using of inequality (1).

If conditionq_0>> 1 is false, but the conditions $q_0 > \frac{1}{2}$ and $2\tau_1 >> \tau$ are true, then it can readily be assured that (8) achievement will result in the term (6) satisfaction.

There were the LabView simulation using the Control Design and Simulation Module to confirm the above findings with the following parameters: G_0= 1, τ = 10 ms, q_0= 0,5 (case 1) и q_0= 10 (case 2). The regulators for each case were tuned by terms (7) and (8): τ_0= 11 ms и τ_1 = 18 ms in case 1 and τ_0= 50 ms and τ_1 = 100 ms in case 2. There are the closed feedback loop transient at the Figs. (**1 and 2**) for the cases 1 and 2 correspondingly. There is the blue line at the Figs. (**1** and **2**) which represent the oscillation transient for the closed feedback loop with I regulator. There is the red line at the Figs. (**1** and **2**) which represent the aperiodic transient for the closed feedback loop with PID regulator tuned by obtained terms. Obviously, the I regulator usage leads to increasing the high frequency noise for the case 2. Also, the oscillations in transient have not been damped without PID regulator, only moved to the initial section.

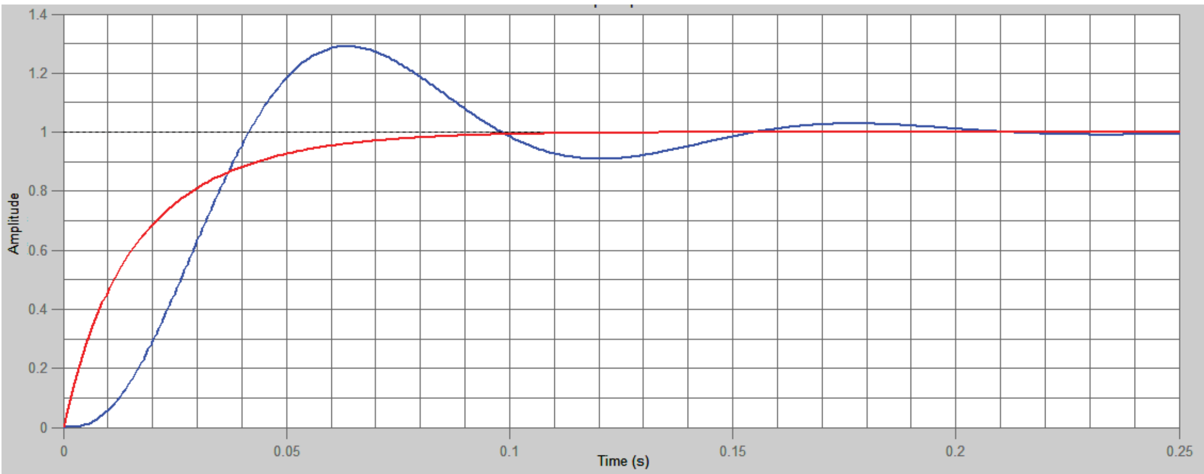

Fig. (1). The case 1 transients: The oscillation transient for the closed feedback loop with I regulator's characteristic time τ_1 (blue) and the aperiodic transient for the closed feedback loop with PID regulator tuned by obtained terms (red).

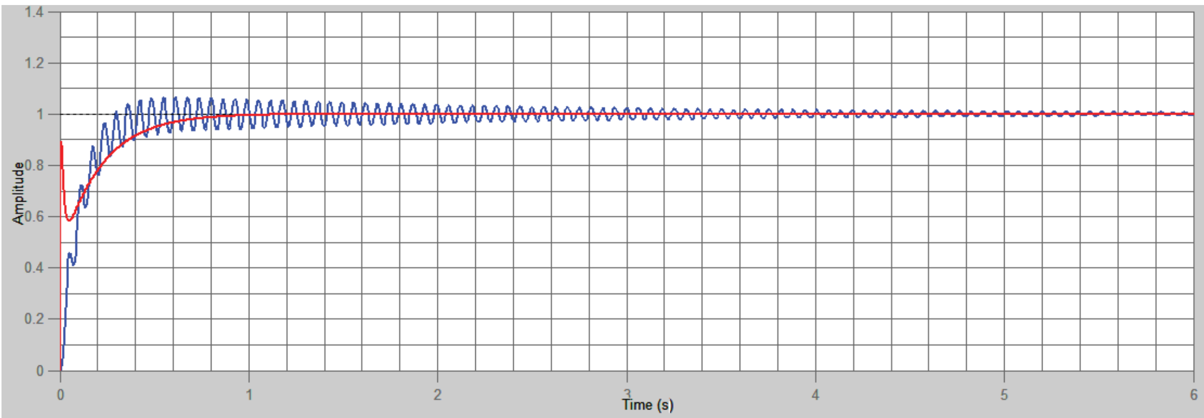

Fig. (2). The case 2 transients: The oscillation transient for the closed feedback loop with I regulator's characteristic time τ_1 (blue) and the aperiodic transient for the closed feedback loop with PID regulator tuned by obtained terms (red).

Thus the condition of $\tau_0 > \frac{\tau}{\sqrt{G_0}}$ is the sufficient criterion for oscillation transition exception in the control process with the loop elements where is the real poles. The condition of $\tau_0 > \sqrt{\frac{2\tau\tau_0}{G_0}}$ is the sufficient criterion for oscillation exception in such process granting the complex-conjugate poles element.

CONCLUSION

Several examples of oscillation transients occur at automatic control systems described by a mathematical model with a cubic characteristic equation were discussed in this paper. There were obtained matching sufficient conditions for aperiodic transient PID tuning based on the known condition of none complex conjugate transfer function poles and approximate formulas for finding the roots of cubic equations. The aperiodic transition has been provided by using the PID regulator in the context of none of loops elements with the resonant behavior with the best time response. The correctness of the obtained terms was confirmed by LabView simulation.

CONSENT FOR PUBLICATION

Not applicable.

CONFLICT OF INTEREST

The authors declare no conflict of interest, financial or otherwise.

ACKNOWLEDGEMENTS

Declared none.

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