

A Novel Positioning Algorithm Optimized by Maximum Likelihood Estimation

Cuisong Chen and Jianqi Liu*

College of Information Engineering, Guangdong Mechanical & Electrical College, Guangzhou, 510515, China

Abstract: As for most location-aware applications need the precise location information, the wireless sensor networks (WSNs) are applied to positioning. But in some situation, Time Difference of Arrival (TDOA) positioning algorithm for WSNs may result in fuzzy positioning. This paper proposes a method to optimize fuzzy positioning of TODA through maximum likelihood estimation based on the identification of causes of fuzzy positioning resulted from TODA; the simulation experiment shows that TDOA positioning algorithm optimized by maximum likelihood estimation features high positioning accuracy and excellent environmental adaptability, thus constituting a practicable solution

Keywords: Wireless sensor networks, TDOA positioning, maximum likelihood estimation, fuzzy positioning.

INTRODUCTION

With the rapid development and increasing sophistication of communication technology, embedded computing technology, micro-processing technology and sensor technology, wireless sensor networks with sensing, computing and communication capabilities has gained more attention [1, 2]. As for most applications, data acquired through sensor would not come into question unless they incorporate location information. In recent years, the mushroom growth of satellite communication, mobile communication, wireless sensor networks and radio management etc. have turned wireless positioning technology into one of the major technologies of wireless communication, and enabled wireless positioning technology to be widely used in the areas of military affairs, production and life.

When it comes to general wireless positioning algorithm, a series of decentralized receivers are normally used to determine the time of arrival of transmitted signal (TOA) [3, 4] or the time difference of arrival (TDOA) [5] so as to estimate the location of source point. Comparatively speaking, TOA positioning algorithm requires accurate synchronization between transmitter and receiver clocks; from this point of view, TDOA positioning algorithm is easier to realize since it requires no rigorous clock synchronization. However, in view of the fact that TODA positioning algorithm for wireless sensor networks (WSN) may result in fuzzy positioning and null solution [6-8], this Paper proposes to optimize fuzzy positioning and null solution of TODA through maximum likelihood function based on the identification of causes of fuzzy positioning and null solution resulting from TODA positioning algorithm; the simulation experiment shows that

TDOA positioning algorithm optimized by maximum likelihood estimation features high positioning accuracy and excellent environmental adaptability, thus constituting a practicable solution.

2. DESCRIPTION OF PROBLEMS

The most fundamental TDOA positioning system is composed of one master and two secondary stations, assuming that the location of each station is (x_i, y_i) , where $i=0$ stands for master station; $i=1,2$ represents secondary station; the location of MS node is (x, y) ; r_i means the distance from mobile station to the master station and secondary stations; Δr_i refers to the difference between the distance from MS node to master station and that from MS node to secondary station, which can be expressed by the following equation:

$$\begin{cases} r_0^2 = (x - x_0)^2 + (y - y_0)^2 \\ r_i^2 = (x - x_i)^2 + (y - y_i)^2 \quad i=1,2 \\ \Delta r_i = r_i - r_0 \end{cases} \quad (1)$$

Through simplification and finishing, equation (1) is converted into:

$$x(x_0 - x_i) + y(y_0 - y_i) = k_i + r_0 \Delta r_i \quad (2)$$

Where, $k_i = \frac{1}{2} [\Delta r_i^2 + (x_0^2 + y_0^2) - (x_i^2 + y_i^2)] \quad i=1,2$

The two equations expressed by (1) compose a nonlinear equation set; therefore, a matrix expression is obtained when r_0 is taken as a known quantity:

$$AX = F \quad (3)$$

Where,

$$A = \begin{bmatrix} x_0 - x_1 & y_0 - y_1 \\ x_0 - x_2 & y_0 - y_2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, F = \begin{bmatrix} k_1 + r_0 \Delta r_1 \\ k_2 + r_0 \Delta r_2 \end{bmatrix}.$$

Where the three stations are not in the same line, if A is reversible:

$$\hat{X} = A^{-1}F \quad (4)$$

According to (3):

$$\begin{cases} \hat{x} = m_1 + n_1 r_0 \\ \hat{y} = m_2 + n_1 r_0 \end{cases} \quad (5)$$

$$\text{Where, } \begin{cases} m_i = a_{i1} k_1 + a_{i2} k_2 \\ n_i = a_{i1} \Delta r_1 + a_{i2} \Delta r_2 \end{cases} \quad i = 1, 2.$$

Substituting (5) into r_0 expression brings about the following equation:

$$ar_0^2 + 2br_0 + c = 0 \quad (6)$$

$$\text{Where, } \begin{cases} a = n_1^2 + n_2^2 - 1 \\ b = (m_1 - x_0)n_1 + (m_2 - y_0)n_2 \\ c = (m_1 - x_0)^2 + (m_2 - y_0)^2 \end{cases}$$

When $b^2 < ac$ in equation (6) is true, the equation would have another different solution; by reason that r_0 must be an integer, it's possible to reject the negative root where one root is positive while the other is negative (positive root is a solution of non-fuzzy positioning); however, fuzzy positioning will arise where both roots are positive.

3. POSITIONING OPTIMIZED BY SIGNAL STRENGTH FACTOR

When $b^2 < ac$ is true as mentioned above, in view of the existence of fuzzy positioning point, the only way is to reject the data and take additional measurement in case of failure to obtain positioning point where $b^2 < ac$ is true; however, failure to position or fuzzy positioning may arise even in such a case. If so maximum likelihood function algorithm based wireless positioning algorithm could be a good solution.

Assuming the estimated likelihood function for MS node location is L_{in} :

$$L_{in} = \prod_{j \in H(i)}^N \left\{ \exp \left[-\frac{1}{2} \left(\frac{P_{ij} - \hat{P}_{ij}}{\sigma_{dB}} \right)^2 \right] \Delta P \right\} \quad (7)$$

Where, set $H(i)$ is the BS node that is located around MS node and is capable of communicating there-with. It is considered that such BS node can be neglected when the signal intensity value between MS node and BS node is below a certain predetermined threshold P_{thr} . This is because the BS node's being too far away from MS node may affect positioning accuracy. Substitute equations (2) and (4) into equation (7) and take the negative logarithm so that the available minimum value point of function is the coordinate of mobile node.

$$\{x, y\} = \arg_{x, y} \min [f(x_k, y_k)] \quad (8)$$

The function $f(x_k, y_k)$ here can be expressed as:

$$f(x_k, y_k) = \frac{b^2}{8} \sum_{j \in H(i)}^N \ln^2 \frac{\hat{d}_{ij}^2}{d_{ij}^2} \quad (9)$$

b means impact factor: $b = 10n / (\sigma_{dB} * \ln 10)$

Where the intensity of signals received by some points is extremely weak, the measured and calculated estimated distance is characterized by lower reliability. Such estimated distance can be taken as auxiliary parameter though its being substituted into the equation is adverse to the improvement of algorithm accuracy. Assuming the threshold of received signal is P_{thr} ; the BS node where the intensity of received signal is below such threshold will be taken into account in auxiliary operation. When likelihood function L_{out} is considered to be true, the effect of BS node on accuracy of the estimated location is as follows:

$$f(x_k, y_k) = \frac{b^2}{8} \sum_{j \in H(i)}^N \ln^2 \frac{\hat{d}_{ij}^2}{d_{ij}^2} - \sum_{j \in H(i)}^N \ln \left[Q \left(\frac{b}{2} \ln \frac{d_{thr}^2}{d_{ij}^2} \right) \right] \quad (10)$$

Q is the confidence interval of x that is consistent with the normal equilibrium value distribution. The final likelihood function is the sum of L_{in} and L_{out} . Take the minimum point of function as the coordinate of MS node. The coordinate of MS node can still be expressed by equation (8) though $f(x_k, y_k)$ turns into the following equation:

$$f(x_k, y_k) = \frac{b^2}{8} \sum_{j \in H(i)}^N \ln^2 \frac{\hat{d}_{ij}^2}{d_{ij}^2} - \sum_{j \in H(i)}^N \ln \left[Q \left(\frac{b}{2} \ln \frac{d_{thr}^2}{d_{ij}^2} \right) \right] \quad (11)$$

If the point where the minimum value of equation (11) can be obtained could be determined, the coordinate of such point should be the coordinate of MS node, in which case, to obtain the minimum value of function $f(x_k, y_k)$ would be the way to find the coordinates of mobile node. By conjugate gradient method, it's possible to obtain the optimal function

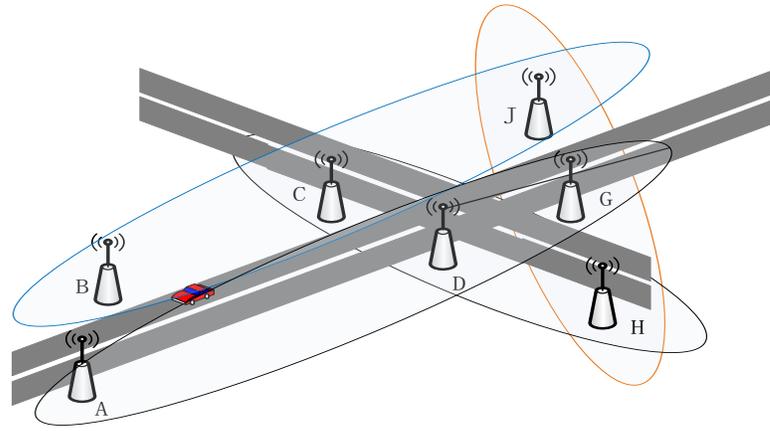


Fig. (1). Positioning scenario.

solution through the search in the direction in which the fastest decline of function is observed (i.e. negative gradient direction).

Step 1: It is assumed that the initial point is X_0 while the error accuracy is ε . The first search direction is:

$$S_0 = -\nabla f(x_0) \quad (12)$$

Step 2: Assuming that α_0 is an unknown value, and that

$$x_1 = x_0 + \alpha_0 S_0 \quad (13)$$

Obtain the α_0 value for minimum function:

$$f_{\min}(x_0 + \alpha_0 S_0) \quad (14)$$

Substitute the obtained α_0 into x_1 : If $f(x_1) < \varepsilon$, get out of operation and achieve the optimal solution; If not, proceed to the next step.

Step 3: Assuming that

$$\beta_k = \frac{\|\nabla f(x_{k+1})\|^2}{\|\nabla f(x_k)\|^2} \quad (15)$$

Search direction is:

$$S_{k+1} = -\nabla f(x_0) + \beta_k S_k \quad (16)$$

Assuming that α_k is an unknown value, and that

$$x_{k+1} = x_k + \alpha_k S_k \quad (17)$$

Obtain the α_k value for minimum function:

$$f_{\min}(x_0 + \alpha_0 S_0) \quad (18)$$

Substitute the obtained α_k into the equation to obtain x_{k+1} : If $f(x_1) < \varepsilon$, get out of operation and achieve the optimal solution; If not, go back to step 3. Get out of the loop

to achieve the optimum solution when k reaches the predetermined threshold.

4. EXPERIMENTS

According to Section 2, since the road is straight, the positioning stations deployed aside the road is stand in a straight line, as shown in Fig. (1) (A, D, G) and (B, C, G) is stand in a straight line aside the road, so the positioning use (A, D, G) as reference node will result in fuzzy positioning. Similarly, the node C, D, H is an inappropriate collection. In this situation, an easy way is that add a data field into data formation, the data field contain the road information, such road number, left side or right side. The number of the preference nodes selected from one side cannot over 2 nodes. For the red car as mobile node, we exclude G and J node, and use A, B, C, D node to determine the location of red car. However, a complex situation is J, G, and H. From the data field, we cannot acquire similar information. We should separate the unqualified node from the network topology. Put the coordinates of nodes J, G, and H into the formula $y = ax + b$, if three nodes lie in the same line, we separate one point from the reference collection.

In order to verify the optimized-TDOA performance by two methods, we transform the scenario of 7 RSE nodes into plane coordinates. We set the nodes (A, B, C, D, G, H, J) coordinates separately (0m, 15m), (5m, 25m), (20m, 25m), (20m, 15m), (30m, 15m), (20m, 0m), (50m, 25m), and put the positioning target (red car) on the road, the coordinates is (15, 20), as shown in Fig. (2). We use the basic TDOA algorithm, TDOA optimized by LM, TDOA optimized by LM and separating unqualified station nodes simultaneously to positioning the car, run 1000 time, the result is shown in Fig. (3). In order to compare positioning accuracy in simulation, we change the units from meter to centimeter. From the simulation result, we can see in basic TDOA algorithm the positioning points range from 1700cm to 2350cm, after optimized by signal strength factor, the range from 17500cm to 2300cm, separating the unqualified station nodes, the

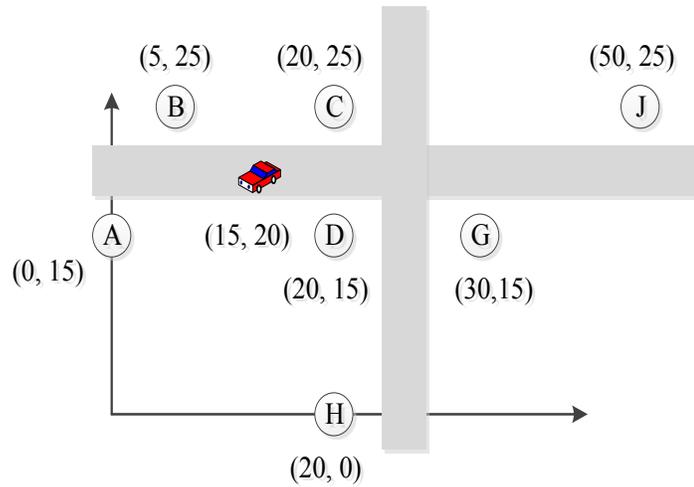


Fig. (2) Plane coordinates of positioning scenario.

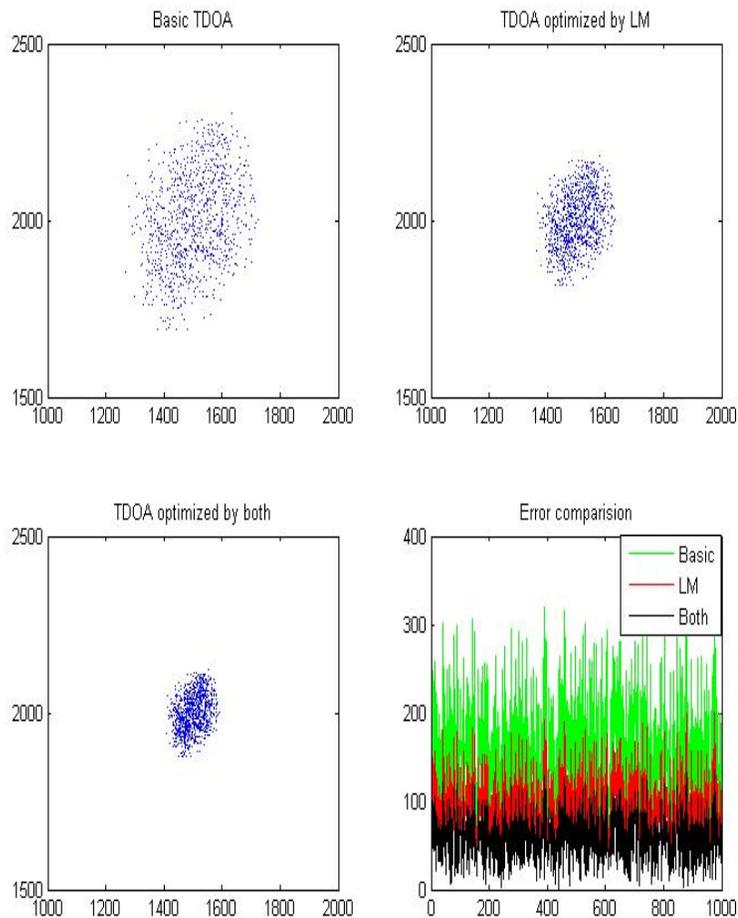


Fig. (3). Three algorithms comparison and error curve.

positioning become more accurate, the points range from 1800cm to 2200cm, and the points gather in point (1500cm, 2000cm), the error is also shown in Fig. (3).

CONCLUSION

TDOA maximum likelihood correction algorithm helps to overcome fuzzy positioning that is an extremely common

phenomenon observed during experiment based on actual data acquired. Maximum likelihood function based positioning method can function under any circumstances with good adaptability of algorithm in an experimental environment. Where a number of nodes are furnished, the positioning by the maximum likelihood estimation can enable the data collected from various beacon nodes to bear a part in the operation. This Paper intends to overcome fuzzy positioning and

null solution of TDOA through maximum likelihood estimation, while simulation data shows that this algorithm offers high positioning accuracy and excellent environmental adaptability.

CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

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