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Multi-user Detection Based on the Accelerated EM Algorithm

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Abstract: Multi-user detection (MUD) is one standard of 3G, which can effectively reduce the multiple access interference (MAI) and increase the system capacity. The Expectation-Maximization (EM) iterative algorithm is commonly used in recent years for missing data, which could be applied to MUD system. But the EM algorithm has a fatal weakness that its slow convergence speed. The new accelerated EM Algorithm is proposed in this paper, which is based on the EM Algorithm and Newton-Raphson algorithm, in order to improve the convergence speed. The proposed algorithm is applied to Multi-user Detection (MUD). The simulation results show that the performance of accelerated EM algorithm is almost as good as the standard EM algorithm in Gaussian noise. The proposed algorithm has better convergence speed than standard EM algorithm.

Keywords: Multi-user Detection (MUD), EM algorithm, Newton-Raphson algorithm, Accelerated EM algorithm.

1. INTRODUCTION

In the code division multiple access (CDMA) system, multi-user detection is served as one of the key technologies. What's more, it will play an important role in the 4G system. The Expectation-Maximization (EM) algorithm is a popular tool in statistics, which also can be effectively applied in MUD system. The limitation to the use of EM is, however, that quite often the E-step of the algorithm involves an analytically intractable, sometimes high dimensional integral [1]. But the slow convergence speed is a big shortcoming of EM algorithm itself which affects its application. How to improve the convergence speed? In many problems where Bayesian solutions iterative simulation or Newton type methods are used, the EM algorithm often comes in handy to provide these MLE or MAP estimates. In the last two decades or so a large body of methods has emerged based on iterative simulation techniques useful especially in computing Bayesian solutions to the kind of incomplete-data problems [2, 3].

The EM algorithm has been used in a lot of applications including system identification, array processing, medical imaging, and time series analysis. It works with a complete data specification, and iterates between estimating the log-likelihood of the complete data using the observed (incomplete) data and the current parameter estimates(E-step),and maximizing the estimated log-likelihood function to obtain the updated parameter estimates (M-step).The algorithm converges, under certain regularity conditions , to a stationary point of the observed log-likelihood function, where each iteration cycle increases the likelihood of the estimated parameters. But the EM algorithm has a fatal weakness that is slow convergence speed [4]. The disadvantage of EM is that convergent speed is very slow when missing data is large enough, in this paper, the focus research is to modify algorithm,and to improve convergence speed combine with some of accelerate algorithm.

Since the properties of the EM algorithm are to be contrasted with those of Newton-type methods, which are the main alternatives for the computation of MLE's, we give a brief review of the Newton-Raphson method and some variants in this paper.

Like many other methods for computing MLE's, the EM algorithm is a method for finding zeros of a function. In numerical analysis there are various techniques for finding zeros of a specified function, including the Newton-Raphson (NR) method, quasi-Newton methods, and modified Newton methods. In a statistical framework, the modified Newton methods include the scoring algorithm of Fisher and its modified version using the empirical information matrix in place of the expected information matrix.

In this paper, we describe the Newton-Raphson method to approximate the maximum likelihood estimate (MLE), when there are missing data and the observed data likelihood is not available in closed form. This method uses simulated missing data that are independent and identically distributed and independent of the observed data.

Many methods can be interpreted as iterative simulation analogs of the various versions of the EM algorithm and its extensions [5]. There are so many methods because each has its strength and weakness. The accelerated EM algorithm, which is one of the simpler extensions of the EM algorithm, only the Newton-Raphson (NR)-step is implemented after the M-step. There is, however, the shortage for EM algorithm, that the convergence rate of the two algorithms is linear. The convergence rate is very slow when the fraction of missing information is large. So this paper proposes the acceleration of EM Algorithm, which is based on EM Algorithm and Newton-Raphson algorithm, to improve the convergence rate. The new algorithm was named accelerated EM algorithm. Background noise chose to Gaussian noise, the accelerated EM algorithm is used to Multi-User Detection (MUD). The simulation results show that it not only has a good BER performance curve but also a faster convergence speed.

This paper is organized as follows. In Section 2, we introduce notation and provide sufficient conditions for the EM algorithm and Newton-Raphson algorithm. The algorithm theories also were introduced in this chapter.

In Section 3, firstly, the concept of multi-user detection was introduced. And then, we present the Accelerated EM algorithm applied in MUD. In Section 4, we describe the simulation results and conclusions, and show that the proposed algorithm accelerates the convergence of the EM algorithm and simplifies the computations involved.

2. EM ALGORITHM AND NEWTON-RAPHSON ALGORITHM

2.1. EM Algorithm

This idea behind the EM algorithm being intuitive and natural, algorithms like the EM have been formulated and applied in a variety of problems even before the DLR paper. The situations where the EM algorithm can be applied include not only evidently incomplete-data situations, where there are missing data, truncated distributions, censored or grouped observations, but also a whole variety of situations where the incompleteness of the data is not all that natural or evident. These include statistical models such as random effects, mixtures, convolutions, log linear models, and latent class and latent variable structures. Hither to intractable ML estimation problems for these situations have been solved or complicated ML estimation procedures have been simplified using the EM algorithm.

In the generalized mixed effects model, the EM algorithm gives rise to difficulties because the E-step is intractable even under Gaussian assumptions of random effects. The EM iterative algorithm provides an attractive alternative to computing ML estimates. In this case where the information of priori estimated parameters is not given, and the observe data is incomplete, an easy-to-use iterative algorithm to compute the parameters of ML is proposed, and this algorithm guarantees convergence to get the maximum estimate. The principle of Expectation Maximization Algorithm: the observe data is *Y*, complete data is $X=(Y|Z)$, *Z* is missing data, θ is model parameter.

The MAP $P(\theta|Y)$ is very complex and difficult to calculate different statistical signal while not added missing data, but if added, it is easy to get the $P(\theta\,|\,y,z)$ [2]. Suppose there are two sample spaces *X* and *Y*, *X* is completely the data, *Y* is the observation data, $h:x\rightarrow y$ is a many-to-one mapping from *X* to *Y*, $g(\theta|y)$ means a posteriori distribution density function that base on observe data *Y*, called the observation posterior distribution; $f(\theta|y,z)$ means about the posteriori distribution function of θ while adding missing data; $k(\theta|y,z)$ means conditional distribution function of *Z* while θ and *Y* are given.

Simulating the missing data $Z=(z_1^i,z_2^i,...,z_m^i)$ from the conditional distribution $p(Z\,|\,\theta^i,Y)$. z_j^i is a vector of the missing data which was used to complete the observed data.

So $X=(Y,Z)$ is the complete-data, the missing value is replaced by z_j^i. The E-step may be difficult to implement because of difficulty in computing the expectation of log likelihood in an EM algorithm. Suppose θ^i is the parameter i-th iterations value, and the EM algorithm steps are:

$$\begin{aligned}Q(\theta,\theta^i)&=E[\log f(\theta\,|\,y,z)\,|\,y,\theta^i]\\&=\int\log f(\theta\,|\,y,z)k(z\,|\,y,\theta^i)\,dz\end{aligned}\quad. \tag{1}$$

M-step: Maximum $Q(\theta^{i+1},\theta^i)=\max Q(\theta,\theta^i)$,

$$\theta^{i+1}=\arg\max Q(\theta,\theta^i) \tag{2}$$

Repeat E-step and M-step iterations, and don't stop the loop until $\|\theta^{i+1}-\theta^i\|$ small enough.

2.2. Newton-Raphson Algorithm

The Newton-Raphson method for solving the likelihood equation.

$$\mathrm{S}(\mathrm{y};\theta)=0 \tag{3}$$

approximates the gradient vector $\mathrm{S}(\mathrm{y};\theta)$ of the log likelihood function $\log L(\theta)$ by a linear Taylor series expansion about the current fit θ^i for θ [4-6]. Where $\mathrm{S}(\mathrm{y};\theta)=\partial\log L(\theta)/\partial\theta$. This gives

$$\mathrm{S}(\mathrm{y};\theta)\approx\mathrm{S}(\mathrm{y};\theta^i)\text{-}\mathrm{I}(\theta^i;y)(\theta\text{-}\theta^i) \tag{4}$$

$$\mathrm{I}(\theta^i;y)=\partial^2\log L(\theta)/\partial\theta\partial\theta^T \tag{5}$$

A new fit θ^{k+1} is obtained by taking it to be a zero of the right-hand side of (4). Hence

$$\theta^{i+1}=\theta^i+\mathrm{I}^{-1}(\theta^i;\mathrm{y})\mathrm{S}(\mathrm{y};\theta^i) \tag{6}$$

So we can get the Newton-Raphson-step:

$$\theta^{i+1}=\theta^i+(-\frac{\partial^2\log p(\theta\,|\,Y)}{\partial\theta^2}|_{\theta^i})^{-1}[\int\frac{\partial\log p}{\partial\theta}$$

$$\frac{(\theta \mid Y)}{} p(z \mid y,\theta^{i)} dz \mid_{\theta i}](\theta_{EM}-\theta^{i}) \qquad (7)$$

If the log likelihood function is concave and unimodal, then the sequence of iterates $\{\theta^i\}$ converges to the MLE of θ, in one step if the log likelihood function is a quadratic function of θ. When the log likelihood function is not concave, the Newton-Raphson algorithm is not guaranteed to converge from an arbitrary starting value. Under reasonable assumptions on $L(\theta)$ and a sufficiently accurate starting value, the sequence of iterates θ^i produced by the Newton-Raphson algorithm enjoys local quadratic convergence to a solution θ^* of (3). That is, given a norm $\|\cdot\|$ on Ω, there is a constant h such that if θ^0 is sufficiently close to θ^*, then holds for $i = 0,1, 2,\cdots$ Quadratic convergence is ultimately very fast, and it is regarded as the major strength of the Newton-Raphson algorithm. But there can be potentially severe problems with this algorithm in applications. Firstly, it requires at each iteration, the computation of the $d \times d$ information matrix $\mathrm{I}(\theta^i;y)$ (that is, the negative of the Hessian matrix) and the solution of a system of d linear equations. In general, this is achieved at a cost of $O(d^3)$ arithmetic operations. Thus the computation required for an iteration of the Newton-Raphson algorithm is likely to become expensive very rapidly as d becomes large.

One must allow for the storage of the Hessian or some set of factors of it. Furthermore, the Newton-Raphson algorithm in its basic form (6) requires for some problems an impractically accurate initial value for θ for the sequence of iterates $\{\theta^i\}$ to converge to a solution of (3). It has the tendency to head toward saddle points and local minima as often as toward local maxima. In some problems, however, Bohning and Lindsay (1988) show how the Newton-Raphson method can be modified to be monotonic.

Since the Newton-Raphson algorithm requires the evaluation of $\mathrm{I}(\theta^i;y)$ on each iteration i, it immediately provides an estimate of the covariance matrix of its limiting value θ^* (assuming it is the MLE), through the inverse of the observed information matrix $\mathrm{I}^{-1}(\theta^i;\mathrm{y})$. Also, if the starting value is a $\sqrt{n}$ -consistent estimator of θ, then the one-step iterate θ^1 is an asymptotically efficient estimator of θ.

3. EM ALGORITHM AND ACCELERATED EM ALGORITHM APPLIED IN MUD

3.1. Multi-user Detection

The EM algorithm has been used for estimating the channel parameters in CDMA systems by some investigators [7-9]. In the field of multi-user detection, apply the EM algorithm is based on: when the parameters are not easy to handle, or the formulate of ML estimation is unable to be gotten; and assuming some of the potential data expanded to receive data, get the complete set of data, use the EM iterative algorithm solute for maximum likelihood estimates. Several recent applications of EM algorithm to digital to communications consider related problems of parameter estimation for superimposed signals in noise.In this paper, by applying the EM algorithm, we decompose the K-dimensional maximization problem into K one-dimensional maximization problems, and thus striking reduces the computational complexity of the optimum detector. The resultant detector will have a computational complexity which is linear in K.

We define the complete data set $x_k\left(t\right)$:

$$x_k\left(t\right) = A_k(t)g_k(t)b_k(t) + n_k(t) \quad k = 1,2,...,K \qquad (8)$$

Where $n_k(t)$ is white Gaussian noise with zero mean and power spectral density σ_k^2 and

$$\sum_{k=1}^{K} \sigma_k^2 = \sigma^2 \qquad (9)$$

The k-th receive signal is:

$$r(t) = \sum_{k=1}^{K} A_k(t)g_k(t)b_k(t) + n(t). \qquad (10)$$

In the formula, A_k means the amplitude of the k-th signal; g_k means spread spectrum waveform of the k-th signal, and the value is ± 1; b_k means the k-th user data, the value is ± 1; $n(t)$ is background noise that is selected by different type of the noise [10-12].

Suppose y_k is the output of the k-th matched filter and its expression is:

$$y_k = \int_0^T r\left(t\right) g_k\left(t\right) dt, 1 \le k \le K. \qquad (11)$$

This result can be obtained by expansion:

$$y_k = \int_0^T \left(\sum_{k=1}^{K} A_k(t)g_k(t)b_k(t) + n(t) \right)$$
$$g_k\left(t\right) dt = A_k b_k + MAI_k + n_k \qquad (12)$$

It can be known clearly that the first item is the data that the k-th user wants to receive; the second item is multiple access interference (MAI) which is generated by other users; and the third is noise. In order to facilitate processing and analysis, formula (11) can be expressed as matrix form:

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \qquad (13)$$

$\mathbf{b} = [b_1, b_2 ... , b_K]^T$ is user data, $\mathbf{R}$ is symmetric correlation matrix of $K \times K$ order ($\rho_{i,k} = \rho_{k,i}$), $R_{jk} =$

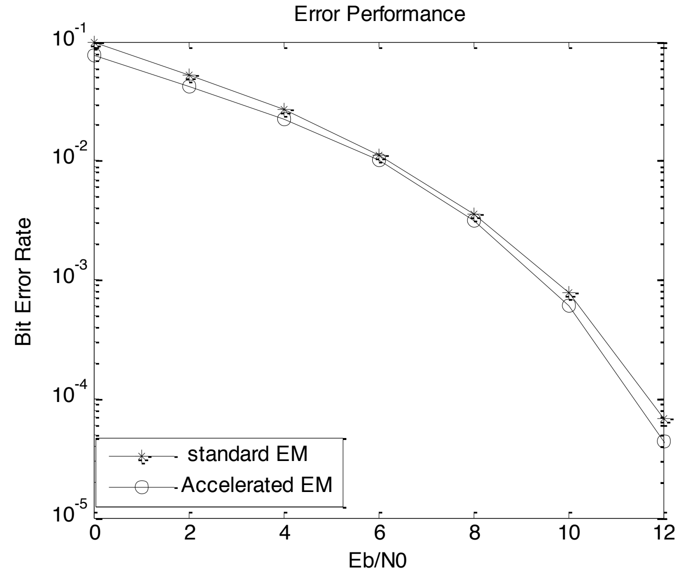

Fig. (1). The BER Curves of EM Algorithm and Accelerated EM Algorithm in Gaussian Noise.

$\left\langle s_j, s_k \right\rangle = \int s_k(t) s_j(t) dt$. **A** is the corresponding diagonal matrix $\mathbf{A} = diag(A_1, A_2 \ldots , A_K)$ of received signal power.

3.2. Multi-user Accelerated EM Detection

In Synchronous DS-CDMA system, the EM multi-user detection is where adding the expectation maximum function in the classic detector. Select here decor-relation detection, and get the initial estimate $\hat{b}$, then apply EM iterative algorithm, suppose the *k*-th user information bit estimate is b_k , b_k^i means the *i*-th iterative value of EM [13-16].

As in ref. [7] the expectation complete likelihood function of the EM algorithm is

$$Q(b_k \mid b_k^i) = \frac{A_k^2}{2\sigma^2}(-(b_k)^2 + 2b_k \frac{1}{A_k}(y_k - \sum_{j \neq k} R_{kj} A_j b_j)) \tag{14}$$

In above formula

$$b_j = E\{b_j \mid r(t), b_k = b_k^i\} . \tag{15}$$

Using Bayes' formula: $b_j = \tanh(\frac{A_j}{\sigma^2}(y_j - R_{jk} A_k b_k^i))$, maximize expectation:

$$\widehat{b}_k^{i+1} = \arg\max Q(b_k \mid b_k^i) . \tag{16}$$

Because of $\hat{b} = \{\pm 1\}$ we obtain the simple formula $\widehat{b}_k^{i+1} = \mathrm{sgn}(y_k - \sum_{j \neq k} R_{kj} A_j b_j)$.

Compared with the EM algorithm, the accelerated EM algorithm just modified the result of the E-step of the EM algorithm, which added a NR-step after E-step.

The result $\widehat{b}_k^{i+1}$ is improved by Newton-Raphson algorithm. We get NR-step:

$$Norm = \frac{\| b_k^{i+1} - b_k^i \|}{b_k^i} \tag{17}$$

We will define normal deviation ratio is $Norm = \frac{\| b_k^{i+1} - b_k^i \|}{b_k^i}$, while we analysis iterative convergence.

The accelerated EM algorithm concrete steps:

Step 1: Obtain the initial estimate value $b_k^0 = \hat{b}$.

Step 2: Computed b_j and $\widehat{b}_k^{i+1}$, b_k^{i+1} .

Step 3: Through Newton-Raphson algorithm, computed b_k^{i+1} .

Step 4: Judge $\| b_k^{i+1} - b_k^i \| < \delta$, the permanent δ can be set by requirement.

Step 5: If "no", $i++$ and $b_k^i = b_k^{i+1}$, return to Step 2; If "yes", $b_{opt} = b_k^{i+1}$ and b_{opt} is the final optimum value of iteration.

4. SIMULATION RESULTS AND CONCLUSIONS

In the DS-CDMA system, applying the standard EM algorithm and accelerated EM algorithm, and we select 5 users, 1000 information bits, 31-bit gold spread-spectrum code, and the user power partial value is 12.

Background noise chose to Gaussian noise, the standard EM algorithm and accelerated EM algorithm are used to MUD.Compared the standard EM algorithm with the accelerated algorithm, simulation results show that: The performance of accelerated EM algorithm is almost as well as the EM algorithm in Gaussian noise (Fig. **1**). Contrast Fig. (**2**) shows the accelerated EM algorithm has faster convergence

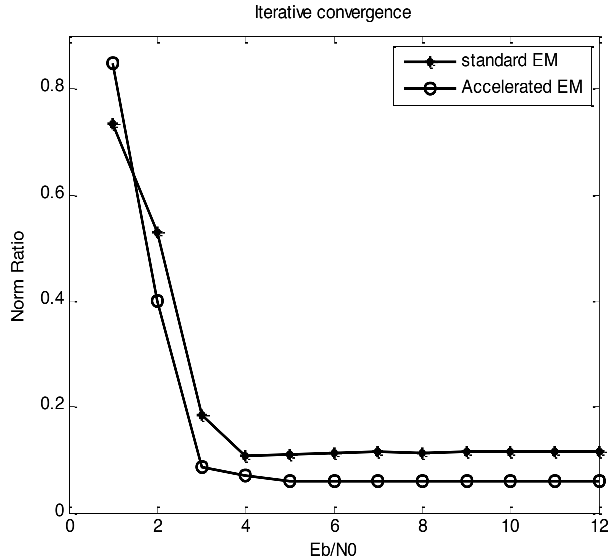

Fig. (2). The Iterative Convergence of EM Algorithm and Accelerated EM Algorithm in Gaussian Noise.

speed than the standard EM algorithm (Fig. **2**). So, the MUD based on the improved EM iterative algorithm in Gaussian Noise will achieve a stable convergence, while convergence speed can be improved.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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