Send Orders for Reprints to reprints@benthamscience.ae

*The Open Automation and Control Systems Journal*, 2015, *7*, 2083-2089



Open Access

# Parameter Estimation of Fractional Low Order Time-frequency Autoregressive Based on Infinite Variance Analysis

Cao Ying*, Yuan Qingshan and Zeng Lili

*Northeast Petroleum University, Daqing, Heilongjiang 066004, China*

**Abstract:** The Parameter model analysis algorithms include autoregressive (AR) model, moving average (MA) and autoregressive moving average (ARMA) model The existing TFAR model is improved, the new fractional low order time-frequency autoregressive(FLO-TFAR) model and the concept of generalized TF-Yule-Walker equation are proposed, fractional low-order covariance is instead of autocorrelation in the model, The parameter estimation of the model is derived, spectrum estimation algorithm based on the FLO-TFAR model is presented, and the steps of the algorithm are summarized. The detailed comparison of the FLO-TFAR $S\alpha S$ model based on fractional low order moment(FLOM) and the Gaussian TFAR model based on autocorrelation. Simulation show that the proposed FLO-TFAR algorithm can carry out high-resolution spectrum estimation, provided better performance than the TFAR algorithm, and is robust.

**Keywords:** Stable distribution, time-frequency autoregressive, time-frequency spectrum, generalized TF-Yule-Walker equation d index.



## 1. INTRODUCTION

AR model parameter estimation is the most simple, and the MA and ARMA model can be represented by an infinite order AR model, hence, the AR model has been widely used in stationary random signal modeling, such as radar and communication, etc. A lot of random signals is non-stationary process in signal processing, therefore, non-stationary process TF-ARMA model concept is proposed in [1, 2], Michael put forward the time-frequency AR(TFAR) non-stationary process model in the literature [3], and the parameter estimation method based on the TF - Yule - Walker equation is given, the TFAR model can accomplish the signal's high resolution time-frequency spectrum estimation without the cross terms. The improved TFAR model methods have been proposed, such as the LS-TFAR TFAR model parameter estimation algorithm and ML-TFAR model algorithm [4], vector time-frequency AR model algorithm (VTFAR) [5], in some special occasions, such as biomedical signal, meteorological data, stock price etc, random signal or noise process often have strong pulse characteristics, the variance of the process is no finite, they can be described by $\alpha$ stable distribution [6-8].

The larger error is produced if the AR model based on Gaussian is used in $\alpha$ stable distribution environment, therefore, the AR $S\alpha S$ parameter estimation is proposed based on the fractional lower order moment (FLOM) in the literature [9, 10], and the corresponding $\alpha$ spectrum estimation which can realize the frequency spectrum estimation under $\alpha$ stable distribution environment is proposed. The new improved AR model and ARMA model are put forward using the fractional lower order covariance (FLOC) replace FLOM in [11, 12], realize the frequency spectrum estimation of the higher precision and resolution.

The $\alpha$ spectrum estimation only can realize the frequency estimation of the stationary $S\alpha S$ process, and in view of the time-varying non-stationary process, TFAR non-stationary Gaussian excitation linear AR model method in literature [9-12] will no longer be applicable, hence, traditional Cohen class time-frequency distribution is improved based on the fractional lower order moment, a fractional lower order Cohen class time-frequency distribution is got [13, 14], use fractional low-order covariance to replace correlation in the model, and put forward the non-stationary process fractional lower order time-frequency autoregressive (FLO-TFAR) model, the generalized TF-Yule-Walker equation is defined to compute the parameter estimation of the FLO-TFAR model. The FLO-TFAR model time-frequency spectrum estimation is defined, it can realize model time-frequency distribution of the observation signals. Computer simulation shows that the proposed FLO-TFAR model can realize linear approximation of non-stationary $S\alpha S$ process, and can realize high-resolution frequency estimation, it has better performance than the existing TFAR model algorithm and fractional lower order Cohen class time-frequency distribution and has a certain toughness.

## 2. STABLE DISTRIBUTION

### A. $\alpha$ Stable Distribution

$\alpha$ stable distribution is a kind of generalized Gaussian distribution, the process is not limited in variance and their

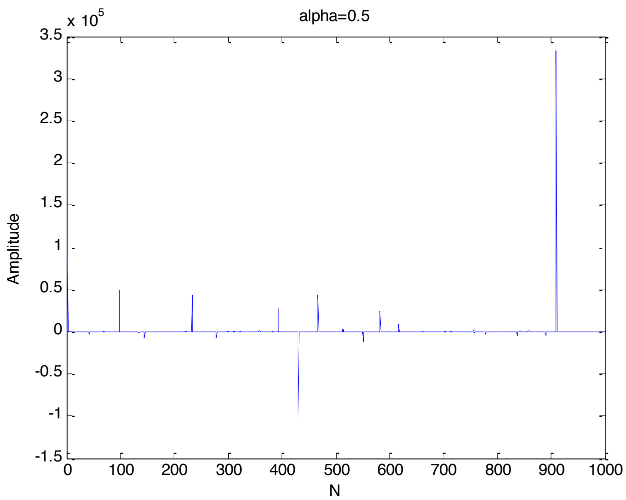
*Address correspondence to this author at the Northeast Petroleum University, Daqing, Heilongjiang 066004, China; Tel: 13333305201;
E-mail: Caoying0909@163.com

1874-4443/15 2015 Bentham Open

probability density function has a serious tail, its characteristic function can be described as [6-8].

$$\phi(t)=\exp\left\{j\mu t-\gamma|t|^{\alpha}[1+j\beta sign(t)\omega(\tau,\alpha)]\right\} \quad (1)$$

The time-domain waveform of $S\alpha S$ stable distribution are shown in (Fig. **1**), its robability density function(PDF) are shown in (Fig. **2**).




$\alpha = 0.5$

**Fig. (1).** Time-domain waveform of $S\alpha S$ stable distribution under $\alpha$ = 0.5, 1.0, 1.5 and 2.0

(Fig. **3**) is variance waveforms of $S\alpha S$ stable distribution with successively increase of sample numbers with $\alpha$ = 0.5, 1.0, 1.5 and 2.0. The result show that variance is not limited when the values of $\alpha$ belong to $0<\alpha<2$, variance is convergent when $\alpha$ = 2 (Gaussian distribution), where $\gamma=2\sigma^2$=2 ($\sigma=1$).

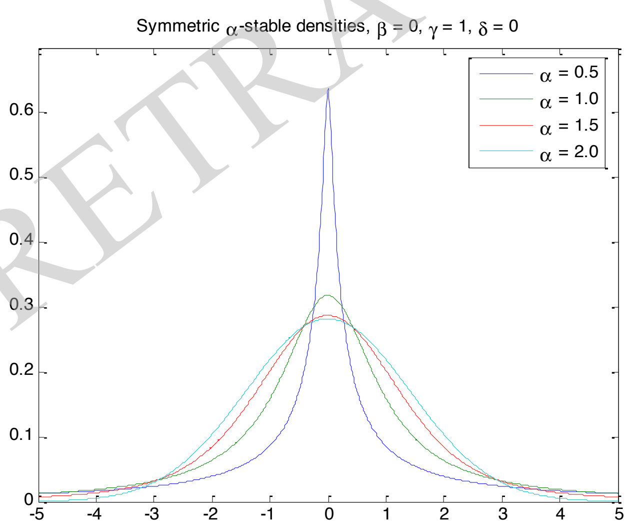



**Fig. (2).** PDF of $S\alpha S$ stable distribution in different alpha ( $\alpha$ )

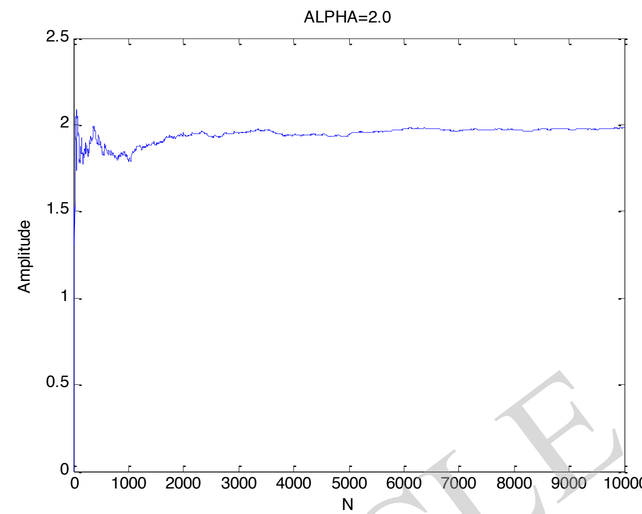



**Fig. (3).** Variance of $S\alpha S$ stable distribution with successively increase of sample numbers in different alpha( $\alpha$ ).

### B. Factional Lower Order Covariation

The covariance of $S\alpha S$ distribution is not exist because its variance is not limited. Hence, Covariation concept is put forward by Miller in 1978, it is similar to the covariance of Gaussian random process. Covariation of two $S\alpha S$ distribution random variables $X$ and $Y$ is defined as

$$[X,Y]_\alpha=\int_s xy^{<\alpha-1>}\mu(ds)\ ,\ 1<\alpha\le 2 \quad (2)$$

Where S denotes the unit circle, $<g>$ denotes the operation $z^{<\alpha>}=|z|^{\alpha}sign(z)$, the covariation coefficient of $X$ and $Y$ is defined as

$$\lambda_{XY}=\frac{[X,Y]_\alpha}{[Y,Y]_\alpha} \quad (3)$$

If the dispersion coefficient of $Y$ is $\gamma_y$, the covariation and covariation coefficient can be written as

$$[X,Y]_\alpha=\frac{E(XY^{<p-1>})}{E(|Y|^p)}\gamma_y\ ,\ 1\le p<\alpha\le 2 \quad (4)$$

$$\lambda_{XY}=\frac{E(XY^{<p-1>})}{E(|Y|^p)}\ ,\ 1\le p<\alpha\le 2 \quad (5)$$

According to the definition of covariation coefficient, the covariation coefficient of a real observation sequence $X(n)$ $(n=0,1,\ldots N)$ can be defined as:

$$\lambda(m)=\frac{E(X(n)X(n+m)^{<p-1>})}{E(|X(n+m)|^p)}\ ,\ 1\le p<\alpha\le 2 \quad (6)$$

$$\hat{\lambda}(m)=\frac{\sum_{m=1}^{N}X(n)\left|X(n+m)\right|^{p-1}sign[X(n+m)]}{\sum_{m=1}^{N}\left|X(n+m)\right|^{p}},1\le p<\alpha\le 2 \quad (7)$$

Where, $\hat{\lambda}(m)$ is the approximate estimation of $\lambda(m)$. Compared with (6), a more simplified fractional lower order moments method is used in array signal processing, it can be expressed as:

$$\lambda_{FLOM}(m)=E(X(n)X(n+m)^{<p-1>})\ ,\ 1\le p<\alpha\le 2 \quad (8)$$

$$\hat{\lambda}_{FLOM}(m)=\begin{cases}\frac{1}{L_2-L_1}\sum_{n=L_1+1}^{L_2}X(n)\left|X(n+m)\right|^{p-1}sign[X(n+m)] & if\ X(n)\ is\ real\\ \frac{1}{L_2-L_1}\sum_{n=L_1+1}^{L_2}X(n)\left|X(n+m)\right|^{p-2}X^{*}(n+m) & if\ X(n)\ is\ complex\end{cases},$$
$$1\le p<\alpha\le 2 \quad (9)$$

Where, $L_1=\max(0,-m)$, $L_2=\min(N-m,N)$.

### C. Fractioanl Lower Order Covariace

Because the fractional lower order covariation and fractional lower order moments provide $\alpha$ for $1<\alpha\le 2$ and the range from 0 to 1 is not defined, hence, the fractional lower order covariace (FLOC) is given it can provide $\alpha$ for $0<\alpha\le 2$. The fractional lower order auto-covariace(FLOAC) of N pairs of observations $X(n)$ $(n=0,1,\mathrm{K}\ N)$ based on the definition of FLOC can be defined as:

$$R_d(m)=E\left\{X(n)^{<a>}X(n+m)^{<b>}\right\},\qquad \text{for}\quad 0\le a<\alpha/2,\ 0\le b<\alpha/2 \quad (10)$$

Where $0<\alpha\le 2$, if $X(n)$ is real, the FLOAC is estimated by the sample FLOAC $\hat{R}_d(m)$.

$$\hat{R}_d(m)=\frac{1}{L_2-L_1}\sum_{n=L_1+1}^{L_2}\left|X(n)\right|^{a}\left|X(n+m)\right|^{b}sign[X(n)X(n+m)] \quad (11)$$

And if $X(n)$ is complex, the FLOAC is estimated by the sample FLOAC $\hat{R}_d(m)$

$$\hat{R}_d(m)=\frac{1}{L_2-L_1}\sum_{n=L_1+1}^{L_2}\left|X(n)\right|^{a-1}\left|X(n+m)\right|^{b-1}X^{*}(n)X^{*}(n+m) \quad (12)$$

Where, $L_1=\max(0,-m)$, $L_2=\min(N-m,N)$, $*$ denotes the conjugate operation.

## 3. THE FLO-TFAR TIME-FREQUENCY ESTIMATION METHOD

### A. The Non-stationary $S\alpha S$ Process TFAR Model

A stationary AR process x[n] can be defined by

$$x[n]=-\sum_{i=1}^{M}a_i x[n-i]+e[n]=-\sum_{i=1}^{M}a_i(\Psi^i x)[n]+e(n) \quad (13)$$

Where $a_i$ is the AR model parameters, M is the AR model order, $e(n)$ is stationary independent identically distributed(I.I.D)Gaussian process $(\Psi^i x)[n]=x[n-i]$, for example $(\Psi^1 x)[n]=x[n-1]$. Because a lot of signals are non-stationary in real-world signal processing, the type (13) is improved and a non-stationary AR process x[n] is defined by

$$x[n]=-\sum_{i=1}^{M}a_i[n]x[n-i]+e[n]=-\sum_{i=1}^{M}a_i[n](\Psi^i x)[n]+e(n) \quad (14)$$

A new non-stationary TFAR process x[n] is defined by [4, 5]

$$x[n]=-\sum_{i=1}^{M}\sum_{l=-L}^{L}a_{i,l}e^{j\frac{2\pi}{N}nl}x[n-i]+e[n]=-\sum_{i=1}^{M}\sum_{l=-L}^{L}a_{i,l}(\mathrm{Z}^l\Psi^i x)[n]+e(n) \quad (15)$$

Where $a_i[n]=\sum_{l=-L}^{L}a_{i,l}e^{j\frac{2\pi}{N}nl}$, $(\mathrm{Z}^l x)[n]=e^{j\frac{2\pi}{N}nl}x[n]$, $a_{i,l}$ is the TFAR model extension parameters, $M$ and $L$ is the model order, among $M$ is the order in time domain and $L$ is the order in frequency domain. When $L=0$, the TFAR model will degenerate into the AR model. Because $e(n)$ is a non-stationary process and it's variance is time-varying, it can be defined as

$$\sigma^2[n]=\sum_{l=-2L}^{2L}\sigma_l e^{j\frac{2\pi}{N}nl} \quad (16)$$

According to the type (14),(15) method, we also defines a time-frequency non-stationary $S\alpha S$ process TFAR model as

$$X[n]=-\sum_{i=1}^{M}\sum_{l=-L}^{L}a_{i,l}e^{j\frac{2\pi}{N}nl}X[n-i]+U[n]=-\sum_{i=1}^{M}\sum_{l=-L}^{L}a_{i,l}(\mathrm{Z}^l\Psi^i x)[n]+U(n) \quad (17)$$

Where $U(n)$ is stationary $S\alpha S$ distribution process, according to the type (16), we define the dispersion coefficient of stationary $S\alpha S$ process as

$$\gamma[n]=\sum_{l=-2L}^{2L}\gamma_l e^{j\frac{2\pi}{N}nl} \quad (18)$$

The generalized TF-Yule-Walker equations and parameter estimation

When the traditional Yule - Walker equations is solved, the TFAR model coefficient is got, we can also compute the generalized TF-Yule-Walker equations to get TFAR model parameters $a_{i,l}$ for non-stationary $S\alpha S$ process, We multiply (17) by $X^{<P-1>}[n-i']$ and take expectation, This yields

$$C_X[n,i]=-\sum_{i'=1}^{M}\sum_{l'=-L}^{L}a_{i',l'}e^{j\frac{2\pi}{N}nl'}C_X[n-i',i-i']+C_{U,X}[n,i] \quad (19)$$

Where $E\{\eta_1\eta_2^{<p-1>}\} = <\eta_1,\eta_2>_p$ $(1 < p < \alpha \le 2)$ is $p$ order covariance of $\eta_1,\eta_2$ [6], $\eta^{<P>} = |\eta|^{p-1}\cdot\eta^*$, $C_X[n,i] \overset{\Delta}{=} E\{X[n]X^{<P-1>}[n-i]\} \overset{\Delta}{=} E\{X[n]|X[n-i]|^{P-2}X^*[n-i]\}$ is the auto-covariance function of the $X[n]$, $C_{U,X}[n,i] \overset{\Delta}{=} E\{U[n]X^{<P-1>}[n-i]\} \overset{\Delta}{=} E\{U[n]|X[n-i]|^{P-2}\cdot X^*[n-i]\}$ is the cross- covariance function of the $X[n]$ and $U[n]$. $*$ indicate conjugate, $\eta^{-<P>} = (\eta^*)^{<P>} = (\eta^{<P>})^*$,

Because the $X[n]$ and $U[n]$ is independent each other, type (19) can be simplified as

$$C_X[n,i] = -\sum_{i'=1}^{M}\sum_{l'=-L}^{L} a_{i',l'}e^{j\frac{2\pi}{N}nl'}C_X[n-i',i-i'] \quad , \quad i > 0 \tag{20}$$

We assume the observed value $X[n]$ is $[0,N-1]$, both sides of type (20) yields is taken the length-N discrete Fourier transform(DFT)

$$\bar{\lambda}_X[i,l] = -\sum_{i'=1}^{M}\sum_{l'=-L}^{L} a_{i',l'}\bar{\lambda}_X[i-i',l-l']e^{-j\frac{2\pi}{N}i'(l-l')} \tag{21}$$

$$\bar{\lambda}_X[i,l] \overset{\Delta}{=} \sum_{n=0}^{N-1} C_X[n,i]e^{-j\frac{2\pi}{N}nl} \tag{22}$$

$$\bar{\lambda}_X[i,l] \overset{\Delta}{=} \sum_{n=0}^{N-1} E\{X[n]X^{<P-1>}[n-i]\}e^{-j\frac{2\pi}{N}nl} \tag{23}$$

We call (21) equations for the generalized TF - Yule - Walker equations, when $p = 2$, it degenerate into the TF - Yule - Walker equations of the non-stationary Gaussian process. (21) include $(2L+1)M$ separated equations and the solved parameters $a_{i',l'}$ are $(2L+1)M$, so, the model coefficient $a_{i',l'}$ can be got by solving the equations (21).

$\lambda_X[i,l]$ is similar to the Cohen- class ambiguity function (AF) of the second order correlation function in the time-frequency distribution in type (22-23), where the auto-correlation is replaced by the auto-covariance, we can name it for discrete fractional lower order ambiguity function(FLO-AF), and $\bar{\lambda}_X[i,l]$ is called fractional lower order expected ambiguity function(FLO - EAF), it indicate the statistical auto-covariance of the time shift $i$ and frequency shift $l$ in the time-frequency domain. In order to solve the model coefficients $a_{i',l'}$ of (21), we define according to the frequency shift $\Lambda_l = diag\left\{e^{-j\frac{2\pi}{N}l}, e^{-j\frac{2\pi}{N}l}, \cdots, e^{-j\frac{2\pi}{N}l},\right\}$, $M \times M$ Toeplitz matrix $\tilde{\lambda}_l$ is defined as

$$\tilde{\lambda}_l = \lambda_l\Lambda_l \tag{24}$$

$$\lambda_l = \begin{bmatrix} \bar{\lambda}_X[0,l] & \bar{\lambda}_X[-1,l] & \cdots & \bar{\lambda}_X[-M+1,l] \\ \bar{\lambda}_X[1,l] & \bar{\lambda}_X[0,l] & \cdots & \bar{\lambda}_X[-M,l] \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\lambda}_X[M-1,l] & \bar{\lambda}_X[M,l] & \cdots & \bar{\lambda}_X[0,l] \end{bmatrix} \tag{25}$$

$a = \left[a_{-L},\cdots,a_L\right]^T$, $a_l = \left[\bar{\lambda}_X[1,l],\cdots,\bar{\lambda}_X[M,l]\right]^T$,

$\theta = \left[\theta_{-L},\cdots,\theta_L\right]^T$, $\theta_l = \left[a_{1,l},\cdots,a_{M,l}\right]^T$ and we get

$$\tilde{\lambda}\theta = -a \text{ or } \theta = -\tilde{\lambda}^{-1}a \tag{26}$$

Where $\tilde{\lambda}$ is $(2L+1)M \times (2L+1)M$ dimensional Toeplitz matrix, the length of $\theta$ and $a$ is $(2L+1)M$, the coefficients $a_{i,l}$ of the non-stationary $S\alpha S$ process $X[n]$ of FLO-TFAR model is got by solving (25) equations. Since $\gamma = 2\sigma^2$, when $i = 0$, let $C_{U,X}[n,0] = \gamma[n]$, (19) can be written as

$$\gamma[n] = C_X[n,0] + \sum_{i'=1}^{M}\sum_{l'=-L}^{L} a_{i',l'}e^{j\frac{2\pi}{N}nl'}C_X[n-i',0-i'] \tag{27}$$

The FLO-TFAR time-frequency spectrum estimation

The FLO-TFAR time-frequency spectrum estimation of a time-varying $S\alpha S$ process $X[n]$ is defined as

$$\hat{S}_\alpha(n,\theta) = \frac{\hat{\gamma}[n]}{\left|\sum_{i=1}^{\hat{M}}\hat{a}_i[n]e^{-j2\pi i\theta}\right|^\alpha} \tag{28}$$

Where if $X[n]$ and the pumping signal $U[n]$ are non-stationary Gaussian distribution, $\gamma = 2\sigma^2$, $\alpha = 2$, (28) will degenerate as

$$\hat{S}_\alpha(n,\theta) = \frac{2\hat{\sigma}^2[n]}{\left|\sum_{i=1}^{\hat{M}}\hat{a}_i[n]e^{-j2\pi i\theta}\right|^2} \tag{29}$$

This expression is the TFAR model time-frequency spectrum estimation [4], and (28) can be known as a generalized TFAR model time-frequency spectrum estimation. When $X[n]$ and $U[n]$ are stationary $S\alpha S$ process, (28) can be written as

$$\hat{S}_\alpha(e^{j2\pi f}) = \frac{\hat{\gamma}}{\left|1-\sum_{i=1}^{\hat{M}}\hat{a}_ie^{-j2\pi f}\right|^\alpha} \tag{30}$$

This expression is the AR $\alpha$ spectrum of the stationary $S\alpha S$ process in [11].

## 4. SIMULATION RESULTS

### A. Parameters Estimation Comparison

We defined a stationary whiten $S\alpha S$ process $U(n)$ of length $N = 256$, A FLO-TFAR model is thought to generate the non-stationary signal $x[n]$ by passing normalized stationary $U(n)$ through a time-varying IIR filter, we let $M=3$, $L=2$ ($i=1,2,3$, $l=2,1,0,-1,-2$), the filter parameters $A_{i,l}$ is defined as

$$A_{i,l}=\begin{bmatrix} a_{1,2} & a_{2,2} & a_{3,2} \\ a_{1,1} & a_{2,1} & a_{3,1} \\ a_{1,0} & a_{2,0} & a_{3,0} \\ a_{1,-1} & a_{2,-1} & a_{3,-1} \\ a_{1,-2} & a_{2,-2} & a_{3,-2} \end{bmatrix}^{\mathrm{T}} \tag{31}$$

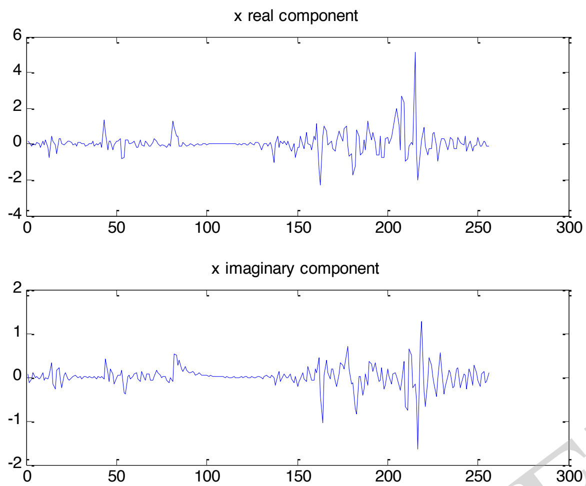



**Fig. (4).** The Observation signal $x[n]$ in time domain.

The $x[n]$ is a non-stationary process, it's time-domain waveform diagram of the Real part and imaginary part are shown in (Fig. **4**), it's evolutionary time-frequency spectrum which is computed from the given TF parameters $A_{i,l}$ is shown in (Fig. **5**).

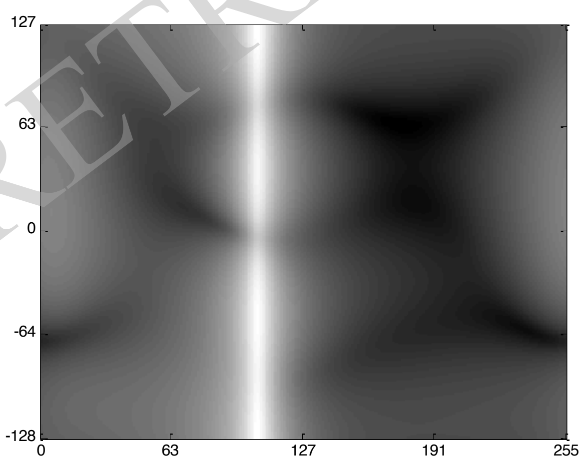



**Fig. (5).** The evolutionary time-frequency spectrum of $x[n]$.

We assume that parameters estimation of TFAR model algorithm is $\hat{a}_{i,l}$ and FLO - TFAR model parameter estimation is $\hat{a}'_{i,l}$. We have done the following comparison in order to compare parameter estimation of two methods. Let $\alpha=1.3$, $N=256$, we estimate the model parameter by performing the TFAR model algorithm and FLO-TFAR model algorithm, After independent 20 times estimates is runned , the averaged parameters are shown in (Table **1**).

**Table 1. The parameter estimation of TFAR model and FLO-TFAR model.**

| The Actual Parameters | | The Estimated Parameters Based on TFAR Model | The Estimated Parameters Based on FLO-TFAR Model |
|---|---|---|---|
| $a_{1,2}$ | 0.2774 + 0.1809i | 0.2399+ 0.1452i | 0.2699 + 0.1779i |
| $a_{1,1}$ | 0.1762 + 0.1503i | 0.1865+ 0.2122i | 0.1508 + 0.1713i |
| $a_{1,0}$ | -0.2140- 0.1005i | -0.2558 - 0.0701i | -0.2324 - 0.0839i |
| $a_{1,-1}$ | 0.0744 + 0.1909i | 0.0869 + 0.2195i | 0.0693 + 0.2206i |
| $a_{1,-2}$ | 0.1601 + 0.1545i | 0.2263 + 0.1847i | **0.1490 + 0.1438i** |
| $a_{2,2}$ | -0.0145+0.0404i | -0.0441+ 0.1057i | -0.0356 + 0.0534i |
| $a_{2,1}$ | 0.0565- 0.0074i | 0.1344 - 0.0532i | 0.0877 + 0.0149i |
| $a_{2,0}$ | 0.2221+ 0.1832i | 0.2821 + 0.2277i | 0.1868 + 0.2081i |
| $a_{2,-1}$ | 0.0529 + 0.1038i | 0.1480 + 0.1358i | 0.0881 + 0.1169i |
| $a_{2,-2}$ | -0.0889- 0.0562i | -0.1345 - 0.0560i | -0.1071 - 0.0427i |
| $a_{3,2}$ | 0.0541 + 0.0448i | 0.1069 + 0.0281i | 0.0461 - 0.0421i |
| $a_{3,1}$ | 0.0807 - 0.0163i | 0.0672 + 0.0224i | 0.0337 - 0.0592i |
| $a_{3,0}$ | -0.1783- 0.0609i | -0.2319 - 0.0145i | -0.2020 - 0.0526i |
| $a_{3,-1}$ | 0.1087 - 0.0698i | 0.1103 - 0.0191i | 0.0603 - 0.0707i |
| $a_{3,-2}$ | -0.0116+0.0304i | 0.0107 + 0.0506i | **-0.0105 + 0.0236i** |

(Table **1**) shows that the parameter estimate of FLO - TFAR model algorithm is closer to the actual value, and TFAR model algorithm have greater deviation. In order to further compare the performance of the two algorithms under different characteristic index $\alpha$, we measured them by the mean square error estimation precision, the parameter estimation mean square error (MSE) of TFAR and FLO-TFAR model is defined respectively as $MSE$ and $MSE'$

$$MSE=\sqrt{\sum_{i=1}^{3}\sum_{l=-2}^{2}(\hat{a}_{i,l}-a_{i,l})^2} \tag{32}$$

$$MSE'=\sqrt{\sum_{i=1}^{3}\sum_{l=-2}^{2}(\hat{a}'_{i,l}-a_{i,l})^2} \tag{33}$$

The mean square error curve is shown in (Fig. **6**) when the characteristics of the process index $\alpha$ (Alpha) change from 1.0 to 2.0. The simulation show that the TFAR model parameters error change from 1 db - 11 db, and the FLO-TFAR model parameter estimation error maintain around -12 db.

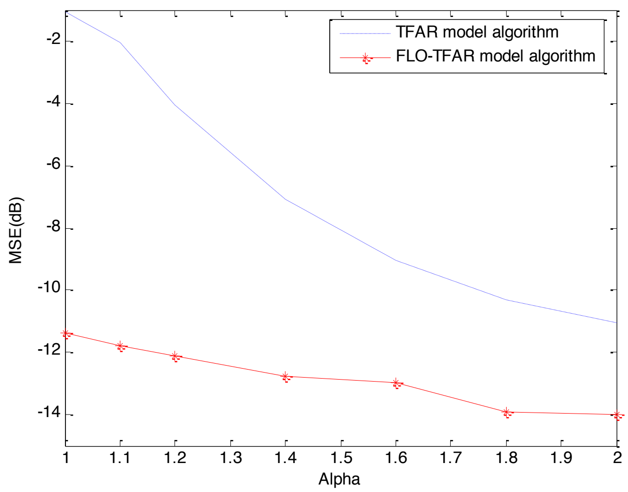



**Fig. (6).** The MSE comparison of the TFAR model and FLO-TFAR model.

### B. Time-frequency Spectrum Estimation Comparison

We respectively use the parameter estimation algorithm based on TFAR model and FLO - TFAR model parameter estimation algorithm to estimate the time-frequency spectrum, simulation spectrum diagram is shown in (Fig. **7**, Fig. **8**).

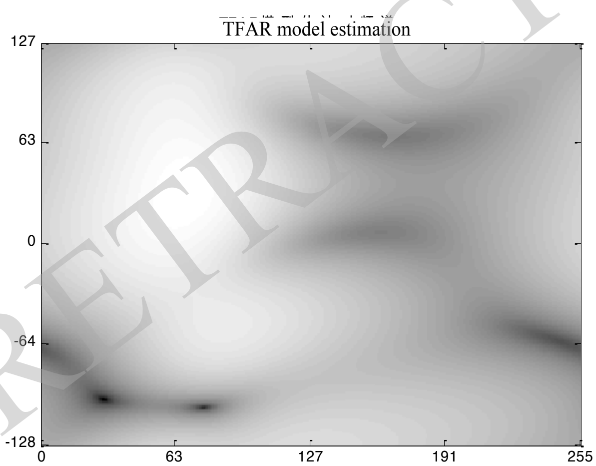



**Fig. (7).** The TFAR model time-frequency spectrum.

The result show that the estimated spectrum based on TFAR model (Fig. **7**) is different from the actual spectrum (Fig. **5**), and the proposed FLO - TFAR model spectrum estimation is very close to the actual time-frequency distribution (Fig. **5**).

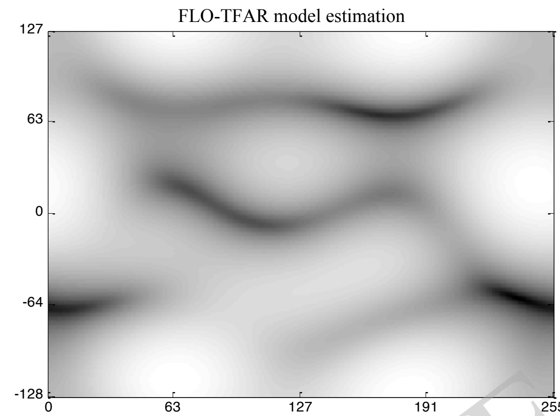



**Fig. (8).** The FLO-TFAR model time-frequency spectrum.

## CONCLUSION

We propose a FLO-TFAR model spectrum method which can work in $S\alpha S$ stable distribution environment by combining the stationary process $\alpha$ spectrum with the existing AR model time-frequency algorithm and using fractional low-order covariance instead of the second order autocorrelation matrix. The dimension of the method is extended to 3-D(time, frequency, amplitude). Simulations show that the proposed algorithm has good performance in parameter estimation and spectrum estimation, FLO - TFAR model algorithm has more advantages when $\alpha$ is smaller. Hence, the proposed FLO - TFAR model algorithm has better toughness in this paper and wider applicability. In order that the FLO - TFAR model can be applied to more field, the next step, we will extended it to the fractional lower order autoregressive moving average (FLO-TFARMA) model, and realize the parameter estimation and spectrum estimation.

## CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

## ACKNOWLEDGEMENTS

Declared none.

## REFERENCES

[1] Y. Grenier, "Time-dependent ARMA modeling of non-stationary signals," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 31, no. 8, pp. 899-911, 1983.

[2] M. Jachan, G. Matz, and F. H. lawatsch, "Tfarma. models: Order estimation and stabilization," *IEEE ICASSP-2005*, vol. 4, pp. 301-304, 2005.

[3] M. Jachan, G. Matz and F. Hlawatsch, "Time-frequency autoregressive random processes: Modeling and fast parameter estimation," *IEEE ICASSP*, 2003, pp.125-128.

[4] M. Jachan, G. Matz, and F. Hlawatsch, "Least-squares and maximum-likelihood TFAR parameter estimation for nonstarionary processes," *IEEE ICASSP*, vol.3, pp. 492-495, 2006.

[5] M. Jachan,G. Matz and F. Hlawatsch, "Vector time-frequency AR models for nonstationary multivariate random processes," *IEEE Transactions on Signal Processing*, vol. 57, no.12, pp. 4646-4658, 2009.

[6] X. Ma ,C. L. Nikias, "Parameter estimation and blind channel identification in impulsive signal environments," *IEEE Transactions on Signal Processing*, vol. 43, no.12, pp. 2884-2897, 1995.

[7] M. Shao, C. L. Nikias, "Singnal processing with fractional lower order moments instable processed and their applications," *Proceedings IEEE,* vol. 81, no. 7, pp. 986-1010, 1993.

[8] T.H Liu, and J. M. Mendel, "A subspace-based direction finding algorithm using fractional lower order statistics," *IEEE Transactions Signal Processing*, vol. 49, no. 8, pp. 1605-1613, 2001.

[9] E. N. Kuruoglu, *Signal Processing in* $\alpha$ *Stable Noise Environments: A Least Lp-NORM Approach*, Britain: Signal Processing and Communication Laboratory, University of Cambridge, 1998.

[10] S. Min, and C. L. Nikias, "Signal processing with fractional lower order moments: Stable processes and their applications," In: *Proceedings on IEEE*, vol. 81, no. 7, pp. 986-1010, 1993.

[11] S. Wang, X. Zhu, X. Li, Y. Wang, "Pectrum estimation for AR $S\alpha S$ process based on FLOC," *Acta Electronica Sinica*, vol. 35, no. 9, pp. 1637-1641, 2007.

[12] S. Wang, and X. Zhu "$\alpha$ spectrum estimation method for ARMA $S\alpha S$ process based on FLOC," *Journal on Communications*, vol. 28, no. 7, pp. 98-103, 2007.

[13] H. Wang, J. Long, and D. Zha, "Pseudo cohen time-frequency distributions in infinite variance noise environment," *Applied Mechanics and Materials,* vol. 475-476, no. 2014, pp. 253-258, 2013.

[14] J. Long, H. Wang, D. Zha, "Fractional low-order adaptive time-frequency distribution based on stable distribution noise," *Computer Engineering*, vol. 37, no. 18, pp. 81-83, 2011.

Received: June 16, 2015 Revised: August 10, 2015 Accepted: September 19, 2015