Send Orders for Reprints to reprints@benthamscience.ae

Open Access

Multi-period Mean-dynamic VaR Optimal Portfolio Selection: Model and Algorithm

Xing Yu*

Department of Mathematics & Applied Mathematics, Hunan University of Humanities, Science and Technology, Loudi, 417000, P.R. China

Abstract: This paper proposes the mean-dynamic VaR multi-period portfolio selection model with the transaction costs and the constraints on trade volumes. The Bat algorithm is applied to solve the multi-period mean-dynamic VaR model. Numerical results show that the Bat algorithm is effective and feasible to solve multi-period portfolio selection problems.

Keywords: Bat algorithm, efficient frontiers, mean-dynamic VaR, multi-period selection.

1. INTRODUCTION

The effective way to allocate capital among various assets is portfolio selection. In 1952, Markowitz proposed the famous mean-variance model for single-period portfolio selection [1], which set up a fundamental basis for modern finance. However, Markowitz's model confronts some challenges of assuming single period and complete market with no transaction costs. As we known, the actual investment process is a through multiple stage, and at every stage, investors may adjust the strategy, according to practical conditions, such as his budget or market conditions. Furthermore, the actual financial market always contains factors of transaction cost and transaction amount limit. It is natural to extend Markowitz's single period model to multi-period portfolio selection. Gulpinar [2] extended the multi-period mean–variance optimization framework to worst-case design with multiple rival return and risk scenarios. Giuseppe [3] was concerned with multi-period sequential decision problems for financial asset allocation. A model was proposed in which periodic optimal portfolio adjustments were determined with the objective of minimizing a cumulative risk measure over the investment horizon, while satisfying portfolio diversity constraints at each period and achieving or exceeding a desired terminal expected wealth target. Wei [4] solved the multi-period mean-dynamic semi variance portfolio model with genetic algorithm and formic group of algorithm. Cui [5] considered the mean-variance formulation in multi-period portfolio selection under no-shorting constraint, and derived the semi-analytical expression of the piecewise quadratic value function. Yan [6] proposed a class of multi-period semi-variance model of multi-period portfolio selection and used hybrid genetic algorithm to solve the model. The reult shows that the hybrid genetic algorithm with Particle Swarm Optimization (PSO) is effective and feasible.

*Address correspondence to this author at the Department of Mathematics & Applied Mathematics, Hunan University of humanities, science and technology, Loudi, 417000, P.R. China; Tel: +8613723804684;
E-mail: hnyuxing@163.com

Multi-period portfolio optimization is typically with complex con-straints, and the solution of such complex problems requires efficient optimization algorithms.To cope with this issue, studies employ neural networks and genetic algorithms. In the recent years, several novel metaheuristic algorithms have been proposed for global search. Such algorithms can increase the computational efficiency, solve larger problems, and implement robust optimization codes. Xin-She Yang [7] proposed a bat-inspired algorithm for solving nonlinear, global optimization problems. Preliminary studies suggest that the bat algorithm can have superior performance over genetic algorithms and particle swarm optimization. As we know, it is difficult to solve the optimal problem because it contains very complex programing such as nonlinear programming. Bat algorithm optimization of the process is a process of dynamic evolvement, unordered random population to update the current optimal solution continuously search process from beginning (local optimal solution) and the gradual and orderly in order to find the global optimal solution of dynamic process, the solution accuracy and the efficiency are very high. In this paper, we adopt bat algorithm to solve multi-period mean-dynamic variance (VaR) portfolio selection model.

The central problem considered in this paper is to determine multi-period discrete-time optimal portfolio strategies over a given finite investment horizon with bat algorithm. The organization of the rest of the paper is as follows. In Section 2 we describe the multi-period mean-dynamic VaR portfolio model. Section 3 is devoted to introduce the bat algorithm. In Section 4 a numerical example is presented to test the efficacy and feasibility of bat algorithm in solving multi-period portfolio selection problems. Finally, Section 5 concludes the paper.

2. MULTI-PERIOD MEAN-DYNAMIC VAR PORTFOLIO MODEL

In this section, a general multi-period mean-dynamic VaR portfolio model is formulated. There are n risk assets traded in finance market. The aim of an investor is to allocate wealth in every beginning of T periods, respectively.

Suppose that $r_{it},\left(i=1,2\cdots n,t=1,2\cdots T\right)$ is the expected return of asset i at stage t. In this paper, it is obtained from history data. x_{it} is the amount of asset i at the beginning of stage t. At the end of stage t, buying amount is represented as b_{it} and selling amount as, s_{it} respectively, because there are transaction cost when buying or selling asset, and it is not wise to sell and buy asset at the same time. Therefore we should constraint $b_{it}\cdot s_{it}=0$. From the symbol assumption, $x_{i,t+1}=x_{it}\left(1+r_{it}\right)\left(1+b_{it}-s_{it}\right)-c_{it}$, where c_{it} is the transaction cost. For simply, let $x_t=\left(x_{1t},x_{2t}\cdots x_{nt}\right)'$, $\Sigma_t=\left(\sigma_{ij}^{(t)}\right)_{n\times n},\sigma_{ij}^{(t)}=\mathrm{cov}\left(r_{it},r_{jt}\right)$. VaR for the portfolio at stage t is f_t. Refer to normal suppose,

$$f_t=\Phi^{-1}\left(\alpha\right)\sqrt{x_t'\Sigma_t x_t}-r_t'x_t$$

where $\Phi\left(\cdot\right)$ is the distribution function of standard normal distribution. $\Phi^{-1}\left(\alpha\right)$ is its quantile under the confidence degree α. I_{pt} is the net portfolio return, whose expression is $I_{pt}=\sum_{i=1}^{n}x_{it}$.

The multi-period mean-dynamic VaR portfolio is as follows:

$$\min\sum_{t=1}^{T}f_t$$

$$s.t\begin{cases}f_t=\Phi^{-1}\left(c\right)\sqrt{x_t'\Sigma_t x_t}-r_t'x_t\\ x_{i,t+1}=x_{it}\left(1+r_{it}\right)\left(1+b_{it}-s_{it}\right)-c_{it}\\ S_T=\sum_{i=1}^{n}x_{iT}=\underline{S}\\ S_0=1\\ b_{it}\cdot s_{it}=0\\ b_{it},s_{it}\geq 0\end{cases}$$

where $\underline{S}$ is the minimal expectation. In actual case, we can suppose that $x_{i0}=\frac{1}{n}$ and $c_{i,t+1}=x_{it}\left(1+r_{it}\right)\left(b_{it}+s_{it}\right)\times 8\%,c_{i0}=0$

This is a multi-period decision of nonlinear optimization problems. It is time wasting if we use the general algorithm. Hence, the bat algorithm is adopted, which has higher computational efficiency in this paper.

3. BAT ALGORITHM

Bat Algorithm (BA) is a heuristic intelligent optimization algorithm proposed by Yang ias a pioneer. The principle of this algorithm is to solve swarm optimization problem by imitating bat echolocation, with which the advantages of simple model, quick convergence speeds and parallel processing.

In order to adopt the bat algorithm for our optimization problems, let us briefly review the basics of the bat algorithm. In the basic bat algorithm developed by Xin-She Yang (2011) [8], the following approximate or idealized rules are used:

1. All bats use echolocation to sense distance, and they also 'know' the difference between food/prey and background barriers in some magical way; and

2. Bats fly randomly with velocity V_i at position X_i with a frequency $f_{\min}$ (or λ), varying wavelength λ (or f) and loudness A_0 to search for prey. They can automatically adjust the wavelength (or frequency) of their emitted pulses and adjust the rate of pulse emission $r\in\left(0,1\right)$, depending on the proximity of their target.

It is assumed that the loudness varies from a large(positive) A_0 to a minimum constant value $A_{\min}$.

In general, the frequency f in a range $\left[f_{\min},f_{\max}\right]$corresponds to a range of wavelengths $\left[\lambda_{\min},\lambda_{\max}\right]$. For example, if the frequency range is $\left[20kHz,50kHz\right]$, then the wavelengths range is $\left[0.7mm,17mm\right]$. Obviously, we can choose the ranges freely to suit different applications. It can be assumed that $f\in\left[0,f_{\max}\right]$, the higher the frequency, the shorter the wavelength, the shorter the flight distance flight, which is usually in the range of a few meters. Pulse frequency usually ranges from 0 to 1, where 0 means no pulse and 1 means the maximum frequency.

Based on the above analysis, the main steps of the bat algorithm can be described as follows:

Step 1. Initialization of parameters, objective function $f\left(X\right)$, $X=\left(x_1,x_2 L\ x_d\right)'$. The initial bat populations $X_i\left(i=1,2L\ n\right)$ and V_i, pulse frequency is f_i at X_i. Initialize the pulse rate r_i and sound loudness A_i.

Step 2. To generate a new solution by adjusting the frequency and change the speed and position.

Step 3. If $rand>r_i$, to choose a solution from the best solutions, in the vicinity of which format a local solution.

Step 4. To produce a new solution through random flight.

Step 5. If $rand < A_i$ & $f\left(X_i\right) < f\left(X_i^*\right)$ then to accept the above solution, and increasing r_i reducing A_i.

Step 6. Arrange bats and find the best X^*.

Step 7. If it does not meet the end condition, return to step 2.

Step 8. Output the global optimal position.

For the bats in simulations, we have to define the rules how their positions X_i and velocities V_i in a $d-$ dimensional search space are updated. The new solutions X_i^t and velocities V_i^t at time step t are given as:

$$f_i = f_{\min} + \left(f_{\max} - f_{\min}\right)\beta, \beta \in \left[0,1\right]$$

$$V_i^{t+1} = V_i^t + \left(X_i^t - X^*\right) f_i$$

$$X_i^{t+1} = X_i^t + V_i^t$$

where X^* is the current global best solution which is located after comparing all the solutions among all the n bats at each iteration t. We can use f_i (or λ_i) to adjust the velocity change while fixing the other factor λ_i (or f_i), depending on the type of the problem of interest.

For the local search part, once a solution is selected among the current best solutions, a new solution for each bat is generated locally using random walk as fellows:

$$X_{new} = X_{old} + \varepsilon A^t, \varepsilon \in \left[-1,1\right]$$

where A^t is the average loudness of all the bats at this time step.

Emission of loudness A_i and pulse rate r_i should be updated with the iterative process. Once the bat finds prey, loudness will be gradually reduced, and the pulse rate will gradually improve. Loudness can choose any convenient value. For example, we can use $A_0 = 100, \mathrm{A}_{\min} = 1$ or $A_0 = 0, \mathrm{A}_{\min} = 100$. We assume that $A_{\min} = 0$, which means a bat just found prey, then temporarily stop the sound. Pulsed emission loudness A_i and rate r_i updating formula are as follows:

$$A_i^t = \alpha A_i^{t-1}, r_i^t = r_i^0 \left[1 - e^{-\gamma t}\right]$$

where α and γ are constants.For any $0 < \alpha < 1$ and $\gamma > 0$, we have

$$A_i^t \to 0, r_i^t \to r_i^0 \; as \; t \to \infty$$

Loudness emitted by each bat and pulse rate were randomly given at initialization. In general, we define the initial loudness A_i^0 is usually between $\left[1,2\right]$ the initial emission rate r_i^0 which is near 0. The loudness and emission rate will vary with the search process of constantly updated, and gradually to the optimal solution.

4. EMPIRICAL ANALYSIS

In this part, it focuses on an empirical research of an optimal portfolio model with five risk assets in Chinese market who show a good momentum in 2013. We choose the day data of five stocks NO 002583, 600694, 600089, 601166 and 600276 from 2013-01-04 09:01:00 to 2014-4-1 15:00:00, which is from CSMAR database in China. But for simply, we construct the optimal portfolio model only suppose a stage conclude 5 moths. That is, there is 5 stages. From solving the model, the dynamic portfolio is

At stage 1, $b_{11} = 0, b_{21} = 0.1183, b_{31} = 0.1581, b_{41} = 0, b_{51} = 0$ and $s_{11} = s_{21} = s_{31} = 0,\ s_{41} = s_{51} = 0$.

At stage 2, $b_{12} = 0, b_{22} = 0.577, b_{32} = 0.0856, b_{42} = 0, b_{52} = 0$ and $s_{21} = 0.3423, s_{22} = s_{32} = 0,\ s_{42} = 0, s_{52} = 0.451$.

At stage 3, $b_{31} = b_{32} = b_{33} = b_{34} = b_{35} = 0$ and $s_{31} = 0, s_{32} = 0.31, s_{33} = 0, s_{34} = s_{35} = 0$.

At stage 4 not do any tradings.

At stage 5, $b_{51} = 0, b_{52} = 0, b_{53} = 0.318, b_{54} = 0, b_{55} = 0.032$ and $s_{51} = 0, s_{52} = 0,\ s_{53} = 0, s_{54} = 0.1264, s_{55} = 0$.

CONCLUSION

Since practical portfolio selection problems involve intertemporal decisions, the single static portfolio model is necessary to be extended to multiperiod case. This paper constructs the dynamic portfolio model, in which it focus on minimax the cumulate risk under the constraints. It also considers the wealth budget and transaction costs, which is very important in dynamic investing. Because bats algorithm has advantages in solving optimization problems, with higher efficiency and precision, this paper adopts bats algorithm to solve the problem. At last, an empirical study choosing five stocks from Chinese market to test validity of the models, giving the strategies. In order to contrast study for proposed the Bat algorithmd, the results compared between the general stochastic dynamic programming approach and our approach. It shows that the results are robust under different structure parameters.

CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

ACKNOWLEDGEMENTS

This research is supported by scientific research fund of Hunan provincial education department (NO 2A077).

REFERENCES

[1] H. M. Markowitz, "Portfolio selection", *Journal of finance*, vol. 7, pp. 77-91, 1952.

[2] N. Gulpinar, and B. Rustem, "Worst-case robust decisions for multi- period mean–variance portfolio optimization", *European Journal of Operational Research*, vol. 183, pp. 981-1000, 2007.

[3] G. C. Calafiore, "Multi-period portfolio optimization with linear control policies", *Automatica*, vol. 44, pp. 2463-2473, 2008.

[4] W. Yan, R. Miao, and S. Li, "Multi-period semi-variance portfolio selection: Model and numerical solution", *Applied Mathematics and Computation*, vol. 194, no. 1, pp. 128-134, 2007.

[5] X.Y. Cui, and J. J. Gao, "Optimal muti-period mean-variance policy under no-shorting constraint", *European Journal of Operational Research*, vol. 234, pp. 459-468, 2014.

[6] W. Yan, and R. Miao, "Multi-period semi-variance portfolio selection: Model and numerical solution", *Applied Mathematics and Computation*, vol. 194, pp. 128-134, 2007.

[7] Y. Xinshe. *Nature-Inspired Metaheuristic Algorithms*, Frome, UK: Luniver Press, 2011.

[8] Y. Xinshe. "Bat algorithm for multi-objective optimization", *International Journal of Bio-Inspired Computation*, vol. 3, no. 5, pp. 267-274, 2011.

Received: December 18, 2014 Revised: March 17, 2015 Accepted: March 18, 2015