Send Orders for Reprints to reprints@benthamscience.ae

The Open Automation and Control Systems Journal, **2015**, *7*, **747-756** **747**

Open Access

Improved Nonlinear Robust Controller for STATCOM Based on System Immersion and Manifold Invariant Methodology

Chao Zhang[a], Aimin Zhang[b,*], Hang Zhang[a], Yunfei Bai[a] and Yingsan Geng[a]

[a]State Key Laboratory of Electrical Insulation and Power Equipment, Xi'an Jiaotong University, Xi'an, People's Republic of China; [b]School of Electronic and Information Engineering, Xi 'an Jiaotong University, Xi 'an, People's Republic of China

Abstract: The problem of designing an improved nonlinear robust controller is addressed adopting three perspectives for static synchronous compensator (STATCOM) system in the presence of time-delay, uncertainty parameter and external disturbances. At first, in order to reduce high degree coupling between system state variables and estimation errors, uncertain parameter is estimated by using the system immersion and manifold invariant (I&I) control which can ensure that the estimation errors converge to zero without using certainty equivalence principle. Secondly, since STATCOM system in practice almost contains time-delay nonlinearity and external disturbances, backstepping sliding mode control are adopted to design the control law recursively. At last, to solve the problem of robustness cannot be guaranteed before the motion orbit reaches sliding mode surface. The dissipation inequalities with external disturbances are constructed in each subsystem, which can ensure that nonlinear robust controller has a good stability and strong robustness in real time. Compared with adaptive backstepping sliding mode and adaptive backstepping, the contrastive simulations result show that the time of reaching steady state is shortened by at least 31%, and the oscillation amplitudes of transient responses are reduced by nearly half.

Keywords: Backstepping, I&I, sliding mode, STATCOM, time-delay.

1. INTRODUCTION

As one of flexible alternate current transmission system (FACTS), static synchronous compensator (STATCOM) are being widely used by several utilities to compensate their systems, such as providing voltage support; damping the power oscillation; reducing net loss and scheduling power flow [1-3]. But, in the practical systems, time-delay nonlinearity, parameter uncertainty and external disturbances can deteriorate the control performance of STATCOM system, which should not be neglected [4]. To solve these problems of nonlinearities, parameters and non-parameters uncertainties, the conventional nonlinear proportion integral differential (PID) technologies have been developed for nonlinear STATCOM system on the basis of feedback linearization. PID technology has been used in STATCOM controller design for projects where nonlinear characteristic influences were not seen as important [5]. However, it is inaccessible to acquire accurate system models which are the indispensable premise in PID controller design. Moreover, some useful nonlinearity can be canceled by feedback linearization.

The adaptive backstepping control has been applied to nonlinear controller by considering some of the state variables as "virtual control" and designing them for intermediate control laws [6-8]. The STATCOM controller has been designed effectively by using adaptive backstepping control, when parametric and non- parametric uncertainties are taken into consideration [9]. However, there are still some problems worth noting for estimating uncertain parameters. The main problem, there is no involved, is only the boundedness of estimation errors can be ensured, but it is not clearly explained whether the dynamics of transient response are unacceptable or not [10]. That is, the dynamics of estimation errors are a key factor which influence control system performance largely. If estimators are fixed, the estimation errors and coupling errors will be accumulated in constructing a (Control-Lyapunov function) CLF. As a consequence, stability and robustness cannot be guaranteed.

A new method named system immersion and manifold invariant (I&I) adaptive control has been presented to address the problems of the coupling between state variables and estimation errors [11, 12]. Based on the notions of system immersion and manifold invariance, I&I adaptive does not require the knowledge of constructing CLF. Moreover, a designed smooth function is introduced by this method, which can offset estimation errors as much as possible. Thus, the estimation errors would not be accumulated and transient stability can be guaranteed even if estimators reach a certain limit [13, 14]. It has been proved that I&I adaptive is used to estimate adaptive law of uncertain parameter effectively, and then the transient stability in closed-loop system is improved.

*Address correspondence to this author at the No.28, Xianning West Road, School of Electronic and Information Engineering, Xi 'an Jiaotong University, Xi'an, China; Tel: +86-29- 82668665-165, Fax: +86-29- 82668665-181; E-mail: zhangam@mail.xjtu.edu.cn

1874-4443/15 **2015 Bentham Open**

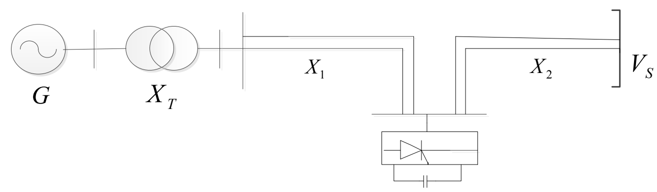

Fig. (1). STATCOM single machine infinite system model.

For STATCOM controller design with time-delay nonlinearity and external disturbances, backstepping control can together with sliding mode, control to design control law. The sliding mode is insensitive to parameter perturbation and external disturbances with matching condition [15]. In the process of the sliding mode motion, the robustness of control system can be guaranteed, when the orbit being in sliding mode is achieved. However, in the process of motion orbit reaching sliding mode surface, the robustness of control system cannot be guaranteed in real time [16]. To solve the problem, robust sliding control method was developed by [17-19], combining the advantages of both robust control and sliding control. Based on dissipation inequality, robust sliding control method can guarantee the robustness of STATCOM controller throughout the design process. A nonlinear STATCOM controller has been designed by robust sliding mode control, and a better performance of transient and steady state has been guaranteed [20].

In this paper, an improved nonlinear robust controller of STATCOM is designed to address the problems of the time-delay nonlinearity, uncertain parameter and external disturbances. Improvements are achieved in three aspects. First, the adaptive law of uncertain parameter is estimated by I&I adaptive control. The estimation errors converge to zero in finite time, which can solve the problem of the coupling between estimation errors and state variables. Second, for solving the problem of time-delay nonlinearity and external disturbances in STATCOM, backstepping sliding mode control is adopted to design control law recursively. The designed control law can improve the performance of transient and steady state for STATCOM. Third, in order to guarantee the robustness of nonlinear robust controller in real time, dissipation inequalities are constructed in each step. The simulations prove that all the state variables are globally bounded and converge to new stable equilibriums. Moreover, contrastive simulations show that our proposed controller gives better performance than other traditional controller under the same initial condition.

2. SYSTEM MODEL AND CONTROL OBJECTIVE

By generating or absorbing reactive power continuously, the STATCOM has been applied to compensate the reactive power in a power system, which can reduce the equivalent electric distances, improve the power system capacity, and enhance the transient stability with long transmission lines. The system structure drawing is shown in Fig. (**1**).

In Fig. (**1**), dynamic model of mathematical equivalent system can be constructed in the form of third-order differential equations.

$$\begin{cases} \dot{\delta} = \omega - \omega_0 \\ \dot{\omega} = \dfrac{\omega_0}{H}\Bigg[P_m - \dfrac{E_q{}' V_s \sin\delta}{X_1 + X_2} \\ \left(1 + \dfrac{X_1 X_2 I_q}{\sqrt{(X_2 E_q{}')^2 + (X_1 V_s)^2 + 2X_1 X_2 E_q{}' V_S \cos\delta}} \right) \\ -\dfrac{D}{H}(\omega - \omega_0) \Bigg] + \varepsilon_1 \\ \dot{I}_q = \dfrac{1}{T_q}(-I_q + I_{q0} + u_B) + \varepsilon_2 \end{cases} \tag{1}$$

where generator rotor angle δ, generator rotor angular speed ω and reactive current I_q are three state variables. $[\delta_0,\ \omega_0,\ I_{q0}]^{\mathrm{T}}$ are the initial operating points.

The parameters in model (1) are explained as follows. H is inertia constant; P_m is mechanical power on the generator shaft; D is unit damping coefficient and it is an uncertain parameter; u_B is equivalence input of the STATCOM regulator; $E_q{}'$ is transient electromotive force of the generator on the quadrature axis; T_q is inertial time constant of STATCOM; X_1 is total impedance from the generator to the injection of the STATCOM device. X_2 is total impedance from the injection of the STATCOM device to the infinite bus. ε_1 and ε_2 stands for the uncertain functions which model the uncertain disturbances imposed on the rotor and system susceptance. Damping coefficient D is viewed as the uncertain parameter, so $\theta = -D/H$ is also an uncertain parameter.

In order to simplify model (1), we can redefine $x_1 = \delta - \delta_0$, $x_2 = \omega - \omega_0$, $x_3 = I_q - I_{q0}$, and let

$$f(x_1) = \frac{X_1 X_2}{\sqrt{(X_2 E_q{}')^2 + (X_1 V_S)^2 + 2X_1 X_2 E_q{}' V_S \cos(x_1 + \delta_0)}},$$

$$k_1 = \frac{\omega_0}{H},\quad k_2 = \frac{\omega_0 E_q{}' V_S}{H(X_1 + X_2)},\quad k_3 = \frac{1}{T_q}$$

The simplified model can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \theta x_2 + k_1 P_m - k_2 \sin(\delta_0 + x_1) \\ \quad (1 + f(x_1)(x_3 + I_{q0})) + \varepsilon_1 \\ \dot{x}_3 = k_3(-x_3(t-d) + u_B) + \varepsilon_2 \end{cases}, \tag{2}$$

where d is delay time.

The regulated output can be expressed as $y = \begin{bmatrix} q_1 x_1 & q_2 x_2 \end{bmatrix}^{\mathrm{T}}$, where q_1 and q_2 are non-negative weighted coefficients whose proportional values are designed by the system controller designers.

The purpose of designing nonlinear robust controller is to guarantee that all the state variables are globally bounded and converge to new stable equilibriums. In addition, the improvement of robustness and enhancement of transient stability are achieved, when compared with other traditional controller.

3. DESIGN OF STATCOM NONLINEAR ROBUST CONTROLLER

The nonlinear robust controller contains an adaptive law ($\dot{\hat{\theta}}$) and a control law (u_B).

3.1. Adaptive Law Design

Based on the notions of system immersion and manifold invariance, a new method named I&I adaptive control can be adopted for designing the adaptive law of uncertain parameter [12].

Define estimation errors

$$e_\theta = \hat{\theta} - \theta + \beta(x_1, x_2), \tag{3}$$

A smooth function $\beta(x_1, x_2)$ is introduced to minimize the residual estimation errors. $\hat{\theta}$ is the estimation of θ. The derivative of e_θ along (2) is:

$$\begin{aligned} \dot{e}_\theta &= \dot{\hat{\theta}} + \sum_{k=1}^{2} \frac{\partial \beta}{\partial x_k} \times \dot{x}_k = \dot{\hat{\theta}} + \frac{\partial \beta}{\partial x_1} x_2 + \frac{\partial \beta}{\partial x_2}(\theta x_2 + k_1 P_m \\ &- k_2 \sin(\delta_0 + x_1)(1 + f(x_1)(x_3 + I_{q0})) + \varepsilon_1) \end{aligned}. \tag{4}$$

For offsetting the parameter-independent terms, $\hat{\theta}$ is selected as:

$$\begin{aligned} \dot{\hat{\theta}} &= -\frac{\partial \beta}{\partial x_1} x_2 - \frac{\partial \beta}{\partial x_2}((\hat{\theta} + \beta) x_2 + k_1 P_m \\ &- k_2 \sin(\delta_0 + x_1)(1 + f(x_1)(x_3 + I_{q0})) \end{aligned}. \tag{5}$$

By substituting (5) into (4), (4) is rewritten as

$$\dot{e}_\theta = -\frac{\partial \beta}{\partial x_2}(e_\theta x_2 - \varepsilon_1). \tag{6}$$

In order to guarantee the dynamics of e_θ, a CLF can be designed as:

$$V(e_\theta) = \frac{1}{2} e_\theta^{\,2}. \tag{7}$$

Theorem 1: Suppose that derivative of (7) is negative semi-definite, and then the stability and convergence of (3) is achieved on the basis of Lyapunov's theorem of stability.

Proof: Theoretically, there are a number of $\beta(x_1, x_2)$ that can be selected, which can ensure derivative of (7) is less than or equal to zero. Let $\beta(x_1, x_2) = \frac{1}{2} \rho x_2^{\,2}$ and $\rho \le -\frac{\kappa}{l_1^2}$, where $\kappa > 0$, $l_1 > \varepsilon_1$. This can be the most simplified form which can meet the requirement.

As defined above, we obtain the dynamics of e_θ that

$$\begin{aligned} V(e_\theta) &= e_\theta \dot{e}_\theta = -\frac{\partial \beta}{\partial x_2}(e_\theta^{\,2} x_2 - e_\theta \varepsilon_1) \\ &= -\rho(e_\theta x_2 - \frac{\varepsilon_1}{2})^2 + \frac{\rho \varepsilon_1^{\,2}}{4} \le -\rho(e_\theta x_2 - \frac{l_1}{2})^2 - \frac{\kappa}{4} \le 0 \end{aligned}, \tag{8}$$

which implies $\lim_{t \to \infty} e_\theta(t) = 0$ and Theorem 1 hold.

Remark 1: The proposed control law $\dot{\hat{\theta}}$ can offset the residual estimation errors $\hat{\theta} - \theta$ effectively. Based on system immersion and manifold invariant theory, a suitable smooth function $\beta(x_1, x_2)$ can not only ensure that the parametric form manifold $I_e = \left\{ (x, \hat{\theta}) \in R^3 \times R^1 \middle| \hat{\theta} - \theta + \varphi(x_1, x_2) = 0 \right\}$ is invariant and attractive, but also guarantee Z_e converges to zero. As a result, the closed-loop system consisting of (2) and (6) has a globally stable equilibrium when $\lim_{t \to \infty} e_\theta(t) = 0$. Moreover, the system (12) can be globally stable for arbitrary u_B, x and ε_1.

3.2. Control Law Design

In this section, three steps can be taken to design the control law for STATCOM with time-delay and external disturbances.

Step 1: Define error state variables z_i ($i = 1,2,3$) as:

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= x_2 - x_2^{\,*} \\ z_3 &= x_3 - x_3^{\,*} \end{aligned}, \tag{9}$$

where x_2 and x_3 are virtual control variables, x_2^* and x_3^* are virtual control input. The derivative of error variables z_i ($i=1,2,3$) along (2) is:

$$\begin{aligned}\dot{z}_1 &= x_2 \\ \dot{z}_2 &= \dot{x}_2 - \dot{x}_2^* = \theta x_2 + k_1 P_m - k_2 \sin(\delta_0 + x_1) \\ &\qquad (1 + f(x_1)(x_3 + I_{q0})) + \varepsilon_1 - \dot{x}_2^* \\ \dot{z}_3 &= \dot{x}_3 - \dot{x}_3^* = k_3(-x_3(t-d) + u_B) + \varepsilon_2 - \dot{x}_3^*\end{aligned} \tag{10}$$

Design the first CLF

$$V_1 = \frac{1}{2} z_1^2 . \tag{11}$$

The derivative of V_1 along (10) is:

$$\dot{V}_1 = z_1 z_2 + z_1 x_2^* . \tag{12}$$

The virtual control input x_2^* is:

$$x_2^* = -c_1 x_1 , \tag{13}$$

where c_1 is a positive constant. We can get $\dot{V}_1 \le 0$ if $z_2 = 0$.

Step 2: Design the second CLF

$$V_2 = \frac{1}{2}\sigma z_1^2 + \frac{1}{2} z_2^2 , \tag{14}$$

where σ is a constant. The derivative of V_2 is:

$$\dot{V}_2 = \sigma z_1 \dot{z}_1 + z_2 \dot{z}_2 . \tag{15}$$

The relationship between energy storage and energy supply is constructed on the basis of dissipation theory in [14].

$$M_1 = (\frac{1}{2} z_1^2 + \frac{1}{2} z_2^2)' + (\|y\|^2 - \gamma^2 \|\varepsilon_1\|^2)/2 . \tag{16}$$

Remark 2: The dissipation inequality with external disturbances is introduced to guarantee the robustness of the nonlinear STATCOM. The dissipation inequality with external disturbances can be expressed as:

$$V(x(T)) - V(x(0)) \le \int_0^T s(\Delta)\, dt , \tag{17}$$

where γ is disturbance attenuation constant, V(x) is an energy storage function, and $\mathrm{s}(\Delta) = \gamma\|\Delta\|^2 - \|y\|^2$ is an energy supply function, and $\Delta = (\varepsilon_1, \varepsilon_2)^{\mathrm{T}}$.

This implies that if the L_2 gain between the output and the input is equal to or less than a certain constant (disturbance attenuation constant γ), the energy is dissipated and the robustness of control system with regard to the input signal can be guaranteed.

Substituting the regulated output $y = \begin{bmatrix} q_1 x_1 & q_2 x_2 \end{bmatrix}^{\mathrm{T}}$ and (12) into (16), (16) can be rewritten as:

$$\begin{aligned} M_1 &= \dot{V}_1 + z_2 \dot{z}_2 + \frac{1}{2}(\|y\| - \gamma^2 \|\varepsilon_1\|^2) \\ &= -\alpha z_1^2 - \frac{1}{2}(\gamma\varepsilon_1 - \frac{z_2}{\gamma})^2 - \frac{1}{4}\gamma^2 \varepsilon_1^2 + z_2(\eta_1 x_1 + \eta_2 x_2 \\ &+ \theta x_2 + k_1 P_m - k_2 \sin(\delta_0 + x_1)(1 + f(x_1)(x_3 + I_{q0}))) \end{aligned} \tag{18}$$

where $\alpha = \sigma c_1 - \frac{1}{2} q_1^2 - \frac{1}{2} q_2^2 c_1^2$, $\eta_1 = \sigma - \frac{1}{2} c_1 q_2^2 + \frac{1}{2\gamma^2} c_1$, $\eta_2 = \frac{1}{2\gamma^2} + \frac{1}{2} q_2^2 + c_1$

In order to guarantee $M_1 \le 0$, can be designed as:

$$x_3^* = \left(\left(\frac{\eta_1 x_1 + \eta_2 x_2 + (\hat{\theta} + \beta) x_2 + k_1 P_m - c_2 z_2}{k_2 \sin(\delta_0 + x_1)} \right) - 1 \right) \frac{1}{f(x_1)} - I_{q0} \tag{19}$$

where c_2 is a positive constant. By selecting σ, we can guarantee $\alpha > 0$. And then, substituting (19) to (18), we get

$$M_1 = -\alpha z_1^2 - \frac{1}{2}(\gamma\varepsilon_1 - \frac{z_2}{\gamma})^2 - \frac{1}{4}\gamma^2 \varepsilon_1^2 - c_2 z_2^2 - z_2 e_\theta x_2 \tag{20}$$

By *Remark* 1, we know $\lim_{t\to\infty} e_\theta = 0$, and then $M_1 \le 0$.

Step 3: Define sliding mode surface $s = \lambda_1 z_1 + \lambda_2 z_2 + z_3 = 0$, where λ_1 and λ_2 are positive parameters. The third CLF can be defined as:

$$V_3 = V_2 + \frac{1}{2} s^2 + \int_{t-d}^{t} q(x(\tau))\, d\tau , \tag{21}$$

where $q(x(\tau))$ is pending non-negative function. The derivative of V_3 along (2) is:

$$\dot{V}_3 = \dot{V}_2 + s\dot{s} + q(x(t)) - q(x(t-d)) . \tag{22}$$

Defined the relationship between energy storage and energy supply as:

$$M_2 = \dot{V}_3 + \frac{1}{2}(\|y\|^2 - \gamma^2\|\varepsilon_1\|^2 - \gamma^2\|\varepsilon_2\|^2) . \tag{23}$$

Substituting the regulated output and (22) into (23), the equation (23) can be rewritten as:

$$\begin{aligned} M_2 &= \dot{V}_2 + \frac{1}{2}(\|y\|^2 - \gamma^2\|\varepsilon_1\|^2 - \gamma^2\|\varepsilon_2\|^2) + s\dot{s} + q(x(t)) - q(x(t-d)) \\ &= -\alpha z_1^2 - \frac{1}{2}(\gamma\varepsilon_1 - \frac{z_2}{\gamma})^2 - \frac{1}{4}\gamma^2\varepsilon_1^2 - z_2x_2e_\theta - c_2z_2^2 - \frac{1}{2}\gamma^2\varepsilon_2^2 \\ &+ s\left(\lambda_1x_2 + \lambda_2\dot{x}_2 + \lambda_2c_1x_1 - k_3x_3(t-d) + k_3u_B + \varepsilon_2\right) \\ &+ s\left\{\frac{1}{\xi_1 f(x_1)}\Big[\eta_1x_1 + (\eta_2 + \hat{\theta} + \beta + c_2)((\hat{\theta}+\beta)x_2 + k_1P_m\right. \\ &\left. - \xi_1(1 + f(x_1)(x_3 + I_{q0})) + (\dot{\hat{\theta}} + \dot{\beta})x_2 + \frac{z_2}{\gamma^2}\right] \\ &- \left(\frac{\xi_2x_2}{\xi_1^2 f(x_1)} + \chi f(x_1)\right)(\eta_1x_1 + \eta_2x_2 + (\hat{\theta}+\beta)x_2 + k_1P_m \\ &+ c_2z_2) + \chi\xi_1 f(x_1)\Big\} + q(x(t)) - q(x(t-d)) \end{aligned} \tag{24}$$

where $\chi = \frac{X_1 + X_2}{k_1X_1X_2}$, $\xi_1 = k_2\sin(x_1 + \delta_0)$ and $\xi_2 = k_2\cos(x_1 + \delta_0)$

Let $R = \frac{\xi_2x_2}{\xi_1^2 f(x_1)} + \chi f(x_1)$, $K = \eta_2 + \hat{\theta} + \beta + c_2$ and $T = (\hat{\theta}+\beta)x_2 + k_1P_m - \xi_1(1 + f(x_1)(x_3 + I_{q0})$, (24) can be re-written as:

$$\begin{aligned} M_2 &= \dot{V}_2 + z_3\dot{z}_3 + \frac{1}{2}(\|y\|^2 - \gamma^2\|\varepsilon_1\|^2 - \gamma^2\|\varepsilon_2\|^2) + s\dot{s} + q(\tau) \\ &= -\alpha z_1^2 - \frac{1}{2}(\gamma\varepsilon_1 - \frac{z_2}{\gamma})^2 - \frac{1}{4}\gamma^2\varepsilon_1^2 - z_2x_2e_\theta - c_2z_2^2 - \frac{1}{2}\gamma^2\varepsilon_2^2 \\ &+ s\left(\lambda_1x_2 + \lambda_2\dot{x}_2 + \lambda_2c_1x_1 - k_3x_3(t-d) + k_3u_B + \varepsilon_2\right) + \mathrm{s} \\ &\left\{\frac{1}{\xi_1 f(x_1)}\left[\eta_1x_1 + KT + (\dot{\hat{\theta}} + \dot{\beta})x_2 + \frac{z_2}{\gamma^2}\right] - R(\eta_1x_1 + \eta_2x_2 + (\hat{\theta}+\beta)x_2\right. \\ &\left. + k_1P_m + c_2z_2) + \chi\xi_1 f(x_1)\right\} + q(x(t)) - q(x(t-d)) \end{aligned} \tag{25}$$

where $q(\tau) = q(x(t)) - q(x(t-d))$

Define $h(x_3(t-d)) = |k_3x_3(t-d)|$, where $h(x_3(t-d))$ is introduced to compensate time-delay term of STATCOM. Furthermore, to ensure dissipation inequality, we can define the nonnegative function $q(x(t)) = |sk_3x_3(t)|$, and get $q(x(t-d)) = |sk_3x_3(t-d)|$, which is a very simple form, but not the only form.

Based on Cauchy-Schwartz inequality theorem, a relationship can be obtained.

$$-sh(x_3(t-d)) \le |sk_3x_3(t-d)| \le |s||k_3x_3(t-d)| . \tag{26}$$

Substituting $q(x(t))$, $q(x(t-d))$ and (27) into (26), (26) can be rewritten as:

$$\begin{aligned} M_2 &\le -\alpha z_1^2 - \frac{1}{2}(\gamma\varepsilon_1 - \frac{z_2}{\gamma})^2 - \frac{1}{4}\gamma^2\varepsilon_1^2 - z_2x_2e_\theta - c_2z_2^2 - \frac{s^2}{2\gamma^2} \\ &- \frac{1}{2}(\gamma\varepsilon_2 - \frac{s}{\gamma})^2 + s\left(\lambda_1x_2 + \lambda_2\dot{x}_2 + \lambda_2c_1x_1 + k_3u_B\right) + \mathrm{s}\left\{\frac{1}{\xi_1 f(x_1)}\right. \\ &\left[\eta_1x_1 + KT + (\dot{\hat{\theta}} + \dot{\beta})x_2 + \frac{z_2}{\gamma^2}\right] - R(\eta_1x_1 + \eta_2x_2 + (\hat{\theta}+\beta)x_2 + k_1P_m \\ &\left. + q(x(t)) + c_2z_2) + \chi\xi_1 f(x_1)\right\} \end{aligned} \tag{27}$$

the control law is then designed as:

$$\begin{aligned} u_B = \ &\frac{1}{k_3}\Big\{-\lambda_1x_2 - \lambda_2\dot{x}_2 - \lambda_2c_1x_1 \\ &- \frac{1}{\xi_1 f(x_1)}\left[\eta_1x_1 + KT + (\dot{\hat{\theta}} + \dot{\beta})x_2 + \frac{z_2}{\gamma^2}\right] \\ &+ R(\eta_1x_1 + \eta_2x_2 + (\hat{\theta}+\beta)x_2 + k_1P_m + c_2z_2) \\ &- \chi\xi_1 f(x_1) - \mu|k_3x_3(t)| - \delta s\Big\} \end{aligned} \tag{28}$$

where δ is a non-negative sliding mode observer gain, and μ is a sign function, which is defined as $\mu = -1$ when $s > 0$ and $\mu = 1$ when $s < 0$.

Substituting the (28) into (27), (27) is rewritten as:

$$\begin{aligned} M_2 &\le -\alpha z_1^2 - \frac{1}{2}(\gamma\varepsilon_1 - \frac{z_2}{\gamma})^2 - \frac{1}{4}\gamma^2\varepsilon_1^2 - z_2x_2e_\theta - c_2z_2^2 \\ &- \frac{s^2}{2\gamma^2} - \frac{1}{2}(\gamma\varepsilon_2 - \frac{s}{\gamma})^2 - \delta s^2 \end{aligned} . \tag{29}$$

By Remark 1 and $\delta > 0$, we can obtain $M_2 \le 0$. The dissipation inequality is guaranteed.

3.3. Proof of System Stability

From (24) and (30), we can get

$$\begin{aligned} M_2 &= \dot{V}_3 + \frac{1}{2}(\|y\|^2 - \gamma^2\|\varepsilon_1\|^2 - \gamma^2\|\varepsilon_2\|^2) \\ &\le -\alpha z_1^2 - \frac{1}{2}(\gamma\varepsilon_1 - \frac{z_2}{\gamma})^2 - \frac{1}{4}\gamma^2\varepsilon_1^2 - z_2x_2e_\theta . \\ &- c_2z_2^2 - \frac{s^2}{2\gamma^2} - \frac{1}{2}(\gamma\varepsilon_2 - \frac{s}{\gamma})^2 - \delta s^2 \le 0 \end{aligned} \tag{30}$$

Define $V = 2V_3$, (30) is rewritten as:

$$\dot{V} = 2\dot{V}_3 \le \gamma^2\|\varepsilon_1\|^2 + \gamma^2\|\varepsilon_2\|^2 - \|y\|^2 . \tag{31}$$

Integrating both sides of (31), we obtain

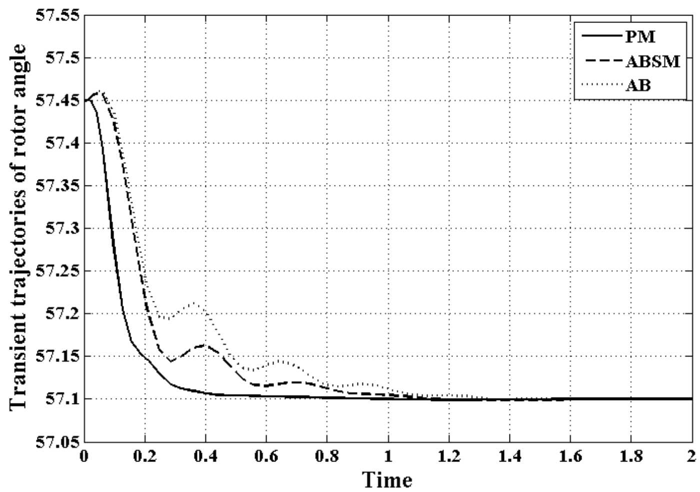

Fig. (2). Transient trajectories of rotor angle when $\gamma_1 = 1$ and $d = 0.02s$.

$$V(x(t)) - V(x(0)) \le \int_0^T (\gamma^2\|\varepsilon_1\|^2 + \gamma^2\|\varepsilon_2\|^2 - \|y\|^2)\, dt \qquad (32)$$

The dissipation inequality holds. This implies that the proposed nonlinear robust controller involving adaptive law $\dot{\hat{\theta}}$ and control law u_B can guarantee the stability and robustness of STATCOM control system.

Furthermore, we will prove that all the state variables are globally bounded and converge to new stable equilibriums.

By (28), we have:

$$\begin{aligned}&\dot{V}_3 + \frac{1}{2}\|y\|^2 = -\alpha z_1^{\ 2} - \frac{1}{2}(\gamma\varepsilon_1 - \frac{z_2}{\gamma})^2 - \frac{1}{4}\gamma^2\varepsilon_1^{\ 2} - z_2 x_2 e_\theta \\ &-c_2 z_2^{\ 2} - \frac{s^2}{2\gamma^2} - \frac{1}{2}(\gamma\varepsilon_2 - \frac{s}{\gamma})^2 - \delta s^2 \le 0\end{aligned} \quad . \qquad (33)$$

We can obtain $\dot{V}_3 \le 0$, so, $V_3(t) \le V_3(0)$, where $t \ge 0$. Obviously, the boundness of $V_3(0)$ is guaranteed, and then z_1 , z_2 , s , x_1 and x_2 are also bounded. Define $U = -\dot{V}_3$, and integrate both sides:

$$\int_0^t U(\tau)\, d\tau = V_3(0) - V_3(t) \qquad (34)$$

As $V_3(0)$ is bounded and $V_3(t)$ is non-increasing bounded, so U is bounded, we have $\lim_{t\to\infty}\int_0^t U(\tau) < 0$. Based on Barbalat's lemma, we can obtain $\lim_{t\to\infty} U(t) = 0$. Thus, if $t \to \infty$, we have $z_1 \to 0$, $z_2 \to 0$, $x_1 \to 0$, $x_2 \to 0$ and $s \to 0$. Substituting these states into $z_3 = s - \lambda_1 z_1 - \lambda_2 z_2$, we can prove that $z_3 \to 0$. This implies that all the states x_1 , x_2 and x_3 are strictly bounded and converge to x_1^* , x_2^* and x_3^* . All the states are globally bounded and asymptotically stable. The boundness of STATCOM holds.

4. SIMULATIONS

In this section, simulation analysis and discussion of the nonlinear robust controller are given by considering a single-machine infinite bus system with STATCOM as an application example. The modeled STATCOM system involves time-delay nonlinearity, uncertain parameter and external disturbances. The following parameters in (1) are given:

$H = 8$, $E_q' = 1.108$, $P_m = 1.0$, $X_1 = 0.84$, $X_2 = 0.52$, $T_q = 0.03$, $K_c = 1$, $q_1 = 0.4$, $q_2 = 0.6$, $\lambda_1 = 1$, $\lambda_2 = 1$, $\sigma = 1$, $d = 0.02, 0.04$, $\gamma = 1, 5$. The tracked system state variables involve generator rotor angle δ , generator rotor angular speed ω and reactive current I_q . The initial points are given as: $\delta_0 = 57.1^\circ$, $\omega_0 = 314.159\ rad/s$, $I_0 = 0$. The external disturbances are designed as: $\varepsilon_1 = e^{-2t}\sin(5t)$, $\varepsilon_2 = e^{-2t}\cos(5t)$ respectively. Three different cases are discussed as follows.

4.1. Different Control Methods

The dynamic responses trajectories of the state variables are simulated by using the proposed method (PM), adaptive backstepping sliding mode (ABSM) and adaptive backstepping (AB) respectively.

Figs. (**2-4**) show the transient trajectories of the state variables involving rotor angle, rotor angular speed and reactive current respectively. The comparison between the proposed method and the two methods are investigated, when $\gamma = 1$

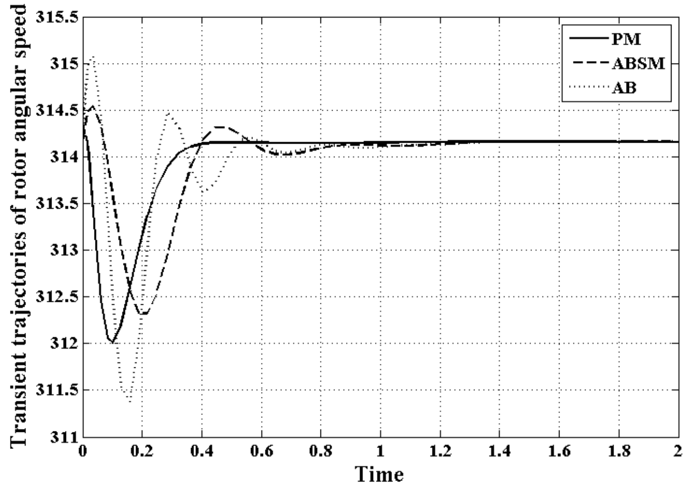

Fig. (3). Transient trajectories of rotor angular speed when $\gamma_1 = 1$ and $d = 0.02s$.

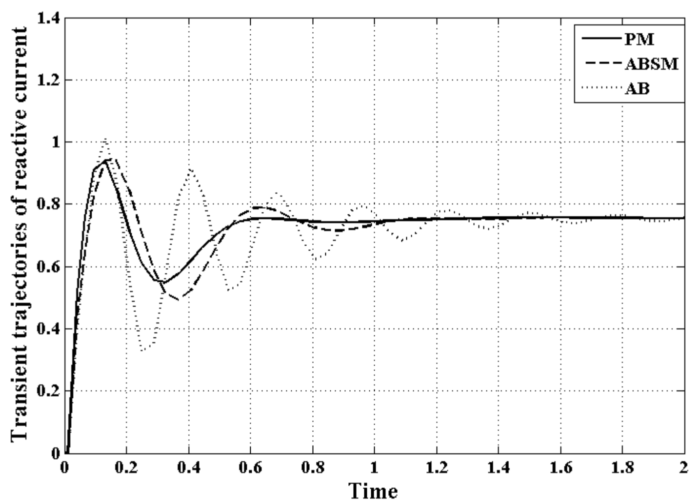

Fig. (4). Transient trajectories of reactive current when $\gamma_1 = 1$ and $d = 0.02s$.

and $d = 0.02s$. Taking Fig. (**4**) for example, the transient responses trajectories fluctuate fast and tend to be stable after 1.6s or more under ABSM and AB. Instead, by using the proposed method, transient responses trajectory fluctuate more smoothly and converge to stable state after 0.6s, suggesting that the proposed method results in better system performance.

4.2. Different Disturbance Attenuation Constants

Compared with Figs. (**3-5**), the simulations between the proposed method and the two methods ABSM and AB were investigated, when $\gamma = 5$ and $d = 0.02s$.

Figs. (**5-7**) show the transient trajectories of the three state variables when $\gamma = 5$ and $d = 0.02s$. The transient trajectories depart from the initial state and fluctuate strongly without an appropriate control. From Figs. (**2-4**) and Figs. (**5-7**), the contrastive simulations show that all transient trajectories fluctuate fast, and system reaches the stable state slowly, when $\gamma = 5$. Instead, it spends less time in converging to the stable state, when $\gamma = 1$. Especially, the transient trajectories fluctuate powerfully and cannot reach to steady state under AB. Consequently, the disturbance attenuation constant γ is a key factor impacting on the system performance. A smaller γ can result in better stability and convergence.

4.3. Different Time-delay Terms

To investigate the influence of time-delay term, the contrastive simulations under our proposed method are performed at $d_1 = 0.02$ and $d_1 = 0.04$ respectively, when $\gamma = 1$.

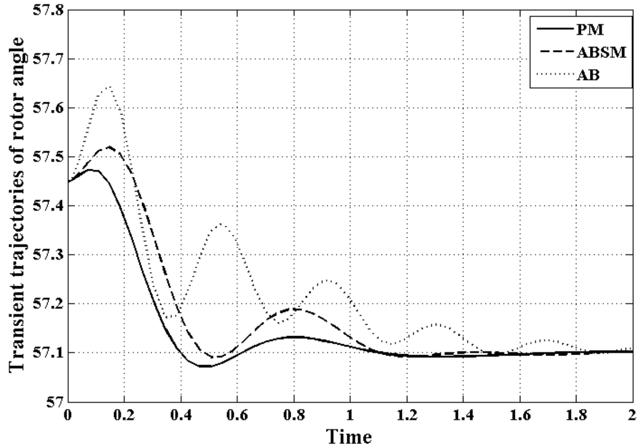

Fig. (5). Transient trajectories of rotor angle when $\gamma = 5$ and $d = 0.02s$.

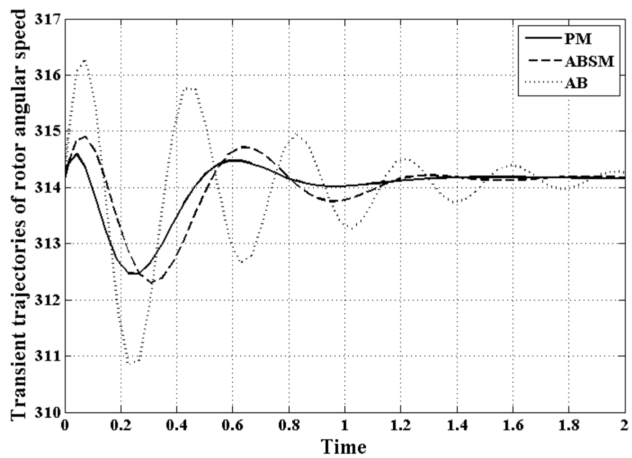

Fig. (6). Transient trajectories of rotor angular speed when $\gamma = 5$ and $d = 0.02s$.

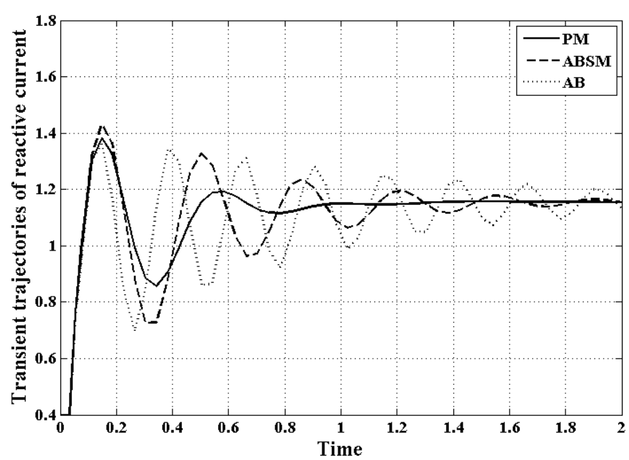

Fig. (7). Transient trajectories of reactive current when $\gamma = 5$ and $d = 0.02s$.

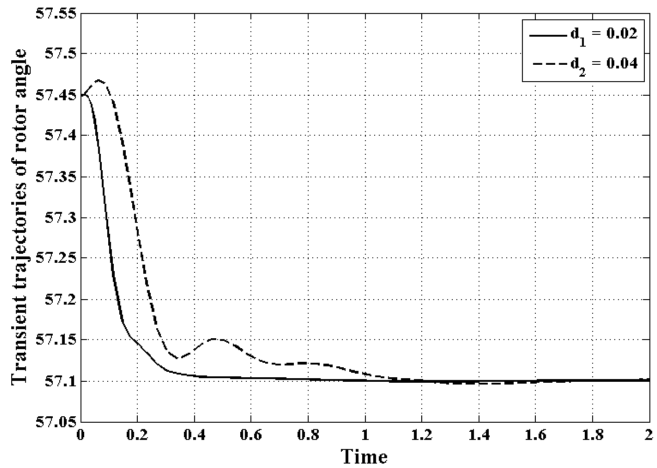

Fig. (8). Transient trajectories of rotor angle in different time-delay.

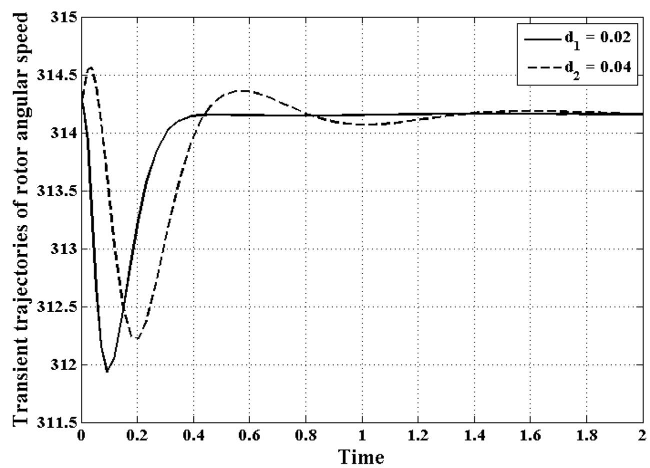

Fig. (9). Transient trajectories of rotor angular speed in different time-delay.

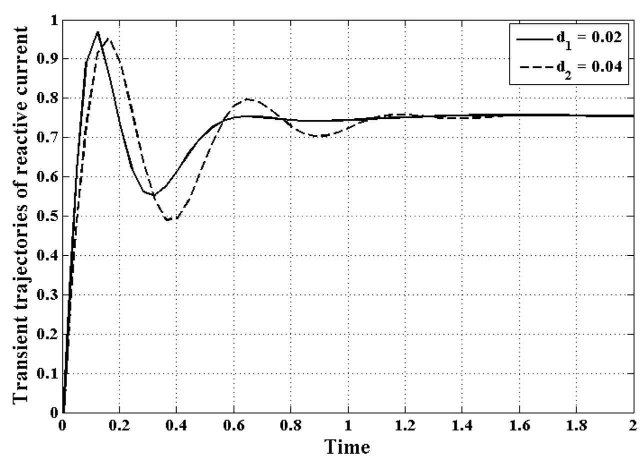

Fig. (10). Transient trajectories of reactive current in in different time-delay.

In Figs. (**8-10**), two different time-delay terms are simulated to investigate its influences. The results of these Figs. reveal that the proposed method can ensure that the state variables of the nonlinear STATCOM are globally bounded and transient responses will eventually converge to a stable value regardless of what time-delay is considered. Moreover, it can be seen clearly from Fig. (**10**) that the transient trajectories attain stability and eventually converge to equilibrium points after a very short time (0.6s), when $d_1 = 0.02$. Nevertheless, the response time of reactive current is 1.6s or more, when $d_1 = 0.04$. Thus, the time-delay d is a crucial nonlinear factor deteriorating the transient and steady performance of the STATCOM system. This result is consistent with the theoretical analysis.

CONCLUSION

In this paper, we propose an improved nonlinear robust controller for STATCOM. The proposed control strategy gives some advantages such as: (a) the design of the adaptive law and the control law are taken apart, which can reduce algorithm complexity significantly; (b) The proposed controller avoids high degree coupling between system state variables and estimation errors by means of designing adaptive law of uncertain parameter; (c) For solving the problem of time-delay nonlinearity and external disturbances, the control law is designed by backstepping sliding mode control, guaranteeing transient stability and robustness of STATCOM control system; (d) In order to guarantee the robustness of control system throughout the controller design process, the dissipation inequalities are constructed in each subsystem. By comparing with the conventional control methods, the nonlinear robust controller has advantages in terms of the oscillation amplitude and convergence time regardless of what delay time is considered.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

ACKNOWLEDGEMENTS

This work was supported by National Natural Science Foundation of China (51177126, 61105126) and Major Technological Innovation Project Special Fund of Shaanxi Province (2008ZKC01-09).

REFERENCES

[1] P. Rao, M. Crow, and Z. Yang, "Statcom control for power system voltage control applications", *IEEE Transactions on Power Delivery,* vol. 15, no. 4, pp. 1311-1317, 2000.

[2] E. Acha, and B. Kazemtabrizi, "A new statcom model for power flows using the newton–raphson method", *IEEE Transactions on Power Systems,* vol. 28, no. 3, pp. 2455-2465, 2013.

[3] K. Josh, A. Behal, A. K. Jain, and N. Mohan, "A comparative study of control strategies for fast voltage regulation with statcoms", In: *30th Annual Conference of IEEE Industrial Electronics Society*, 2004.

[4] Y. Zhao, J. Yang, and G. Wen, "h ∞ control for uncertain switched nonlinear singular systems with time delay", *Nonlinear Dynamics*, vol. 74, no. 3, pp. 649-665, 2013.

[5] Y. Ni, L. Jiao, S. Chen, and B. Zhang, "Application of a nonlinear pid controller on statcom with a differential tracker", In: *Proceedings of Energy Management and Power Delivery,* Singapore, 1998.

[6] J. Zhou, C. Wen, and Y. Zhang, "Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis", *IEEE Transactions on Automatic Control*, vol. 49, no. 10, pp. 1751-1759, 2004.

[7] Y. Xu, S. Tong, and Y. Li, "Adaptive fuzzy fault-tolerant control of static var compensator based on dynamic surface control technique", *Nonlinear Dynamics*, vol. 73, no. 3, pp. 2013-2023, 2013.

[8] W. Bing, "Nonlinear robust control based on adaptive backstepping design for statcom", *Journal of Northeastern University,* vol. 24, no. 3, pp. 221-224, 2003.

[9] D. Bresch-Pietri, J. Chauvin, and N. Petit, Eds., "Adaptive backstepping controller for uncertain systems with unknown input time-delay, application to si engines," In: *49th IEEE Conference on Decision and Control*, USA, 2010.

[10] Z. -G. Hou, A. -M. Zou, F. -X. Wu, and L. Cheng, "Adaptive dynamic surface control of a class of uncertain nonlinear systems in pure-feedback form using fuzzy backstepping approach", In: *IEEE International Conference on Automation Science and Engineering,* USA, 2008.

[11] A. Astolfi, and R. Ortega, "Immersion and invariance: a new tool for stabilization and adaptive control of nonlinear systems", *IEEE Transactions on Automatic Control,* vol. 48, no. 4, pp. 590-606, 2003.

[12] A.R.L. Zachi, "Dynamic control of uncertain manipulators through immersion and invariance adaptive visual servoing", *The International Journal of Robotics Research*, vol. 25, pp. 1149-1159, 2006.

[13] Sellami, D. Arzelier, R. M'hiri, and J. Zrida, "A sliding mode control approach for systems subjected to a norm-bounded uncertainty", *International Journal of Robust and Nonlinear Control*, vol. 17, no. 4, pp. 327-346, 2007.

[14] L. Zhang, A. Zhang, Z. Ren, G. Li, C. Zhang, and J. Han, "Hybrid adaptive robust control of static var compensator in power systems", *International Journal of Robust and Nonlinear Control,* vol. 24, no. 12, pp. 1707-1723, 2014.

[15] S. Dadras, and H. Momeni, "Adaptive sliding mode control of chaotic dynamical systems with application to synchronization", *Mathematics and Computers in Simulation,* vol. 8, no. 12, pp. 2245-2257, 2010.

[16] D. Swaroop, J. K. Hedrick, P. P. Yip, and J. C. Gerdes, "Dynamic surface control for a class of nonlinear systems", *IEEE Transactions on Automatic Control*, vol. 45, no. 10, pp. 1893-1899, 2000.

[17] S. Wang, and J. Fei, "Robust adaptive sliding mode control of mems gyroscope using t–s fuzzy model", *Nonlinear Dynamics,* vol. 77, no. 1-2, pp. 361-371, 2014.

[18] E. P. Rya, "An integral invariance principle for differential inclusions with applications in adaptive control", *SIAM Journal on Control and Optimization*, vol. 3, pp. 960-980, 1998.

[19] D. Ye, and X. Zhao, "Robust adaptive synchronization for a class of chaotic systems with actuator failures and nonlinear uncertainty", *Nonlinear Dynamics,* vol. 76, no. 2, pp. 973-983, 2014.

[20] P. F. Puleston, S. A. González, and F. Valenciaga, "A statcom based variable structure control for power system oscillations damping", *International Journal of Electrical Power & Energy Systems*, vol. 29, pp. 241-250, 2007.

Received: September 16, 2014 Revised: December 23, 2014 Accepted: December 31, 2014