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RESEARCH ARTICLE

Cost Optimization Model of Second-hand Product for Extended Non-renewing Replacement-repair Warranty

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Abstract: This paper proposes maintenance policy and length of maintenance period for second-hand products from the user's perspective. When the second-hand product failure occurs during the basic warranty period, it is either replaced or minimally repaired. When the basic warranty period expires, the seller gives a discount to the user if he/she purchases the extended warranty for second-hand products. When the extended warranty period expires, the customer is in charge of the products' maintenance for a fixed length of time and replaces the products at the end of such a maintenance period. In time of maintenance period, the second-hand product is minimally repaired at each failure. Such a maintenance model can be viewed as a generalization of several existing maintenance models which can be obtained as special cases. Following this maintenance scheme, the expected total cost of the mathematical model is constructed, and then the length of maintenance period is obtained so that the expected total cost is minimized for the user. Finally, numerical examples are introduced to illustrate the proposed optimal replacement strategy.

Keywords: Maintenance period, Non-renewing warranty, Replacement warranty, Repair warranty, Replacement-repair warranty.

1. INTRODUCTION

The used/second-hand product market has been becoming increasingly important in total market since early twenty-first century. Recycling and reusing used/second-hand product has significant potential for reducing the environmental impacts, and protects the environment by preventing the second-hand second from becoming waste. Moreover, recycling of second-hand product is one of the important ways to achieve a low-carbon economy. In addition, from the perspective of the seller, he/she can sell the product to other consumers to obtain maximum profits through reutilizing the used product. But the consumer is most concerned about the functionality and reliability of the product, so a product should be sold with a basic warranty period.

During the basic warranty period, the seller provides either a free replacement (pro rata replacement) or minimal repair when the product fails. Following the expiration of the basic warranty period, the user is solely in charge of the failure of the product. Therefore, an extended warranty of the product will be becoming increasingly popular between the seller and the user which has been extensively discussed in the literature [1, 2]. Several scholars have carried out researches of replacement warranty and repair warranty among various warranty policies. Sahin and Polatoglu [3] investigated the replacement strategy when the renewing replacement warranty and non-renewing replacement warranty were expired. Subsequently, Jung and Park [4] extended their model by considering preventive maintenance activity. Jung *et al.* [5] presented an optimal replacement policy after the replacement warranty was expired by considering both expected cost and expected downtime. Yeh *et al.* [6] determined an optimal replacement strategy for the repairable system under free-repair warranty. Jung *et al.* [7] defined a new concept of the life cycle anew from the user's point of view and considered the optimal maintenance policy following the expiration of the renewing warranty. Recently, Jung

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et al. [8] presented an optimal maintenance strategy for repairable system after the non-renewing replacement-repair warranty was expired. What's more, Jung *et al*. [9] investigated an extend model by taking into account additional warranty period to the users after the original two-phase warranty was expired.

In contrast with various researches on new products, a literature review shows that only a few types of researches have been done on maintenance strategy for second-hand product to minimize the expected cost from the manufacturer's or the user's perspective [2]. In recent years, the concept of "warranty for second-hand products" has aroused the interest of several researchers, for example, Chattopadhyay and Murthy [10], Chukova *et al*. [11], and Shafiee *et al*. [12]. With a large number of second-hand products flooding into the market including automobiles, helicopters, TVs, furniture, *etc*, providing warranty services for second-hand products has been a relatively new marketing strategy employed by manufacturers or sellers. The second-hand warranty contract which is made by the seller and the user can assure the new users that the second-hand product will function well for a preset period of time or amount of usage.

In this paper, we will propose a decision model to determine the optimal length of maintenance period for second-hand products from the user's perspective. We will introduce two types of warranties which are the non-renewing free replacement-repair warranty (NFRRW) and the non-renewing pro rata replacement-repair warranty (NPRRW), and we also will compare the optimal results of these two warranty strategies *via* numerical examples.

The remainder of this article will be organized as follows. In section 2, we will derive the expected cost rate from the user's point of view by formulating the expected warranty cost and the length of the life cycle of the system. In section 3, we will discuss the optimal maintenance strategy and prove the uniqueness of the solution for the decision variable. Numerical results will be provided in section 4. Finally, concluding remarks will begiven in section 5.

2. MAINTENANCE MODEL FOLLOWING EXTENDED NON-RENEWING REPLACEMENT-REPAIR WARRANTY

We will investigate the maintenance model following the extended non-renewing replacement-repair warranty (NRRW) and its optimization of second-hand product in this section. The extended maintenance model is the combination of basic warranty, extended warranty and maintenance period. The basic warranty interval will be divided into two non-overlapping subintervals, non-renewing replace warranty(NRW) period and non-renewing minimal repair warranty(NMW) period. If the second-hand product fails during the NRW period of the basic warranty, the seller will replace it by another one either under NFRRW or under the NPRRW. Under the NFRRW, the seller will be fully responsible for the replacement cost. However, under the NPRRW, a pro-rated portion of replacement cost will be charged to the user proportionally to the usage age of the system when it fails. If the system fails during the NMW phase of the basic warranty period, the user is not responsible for any of the minimal repair cost under NFRRW, but should be responsible for the minimal repair cost under NPRRW. During the extended warranty period, the seller will be in charge of the minimal repair cost under both NFRRW and NPRRW. The minimal repair action restores the system to functioning state but will not change its hazard rate. Following the expiration of the basic warranty period, the user purchases an extended warranty from the seller which is of a fixed length α. When the user purchases extended warranty as $n(n\geq 1)$ times, the seller will give a special discount ϕ of purchasing extended warranty expense c_e. Hence, the user will not be in charge of the minimal repair expense during the extended warranty period under both NFRRW and NPRRW. Following the expiration of the extended warranty period, only the minimal repair action will be conducted and the user will just be responsible for the expense over the maintenance period, and the system will be replaced with another one with a fixed length τ at the end of such a maintenance period, regardless of the age of the system. Fig. **(1)** illustrates the maintenance model after the extended NRRW in this article.

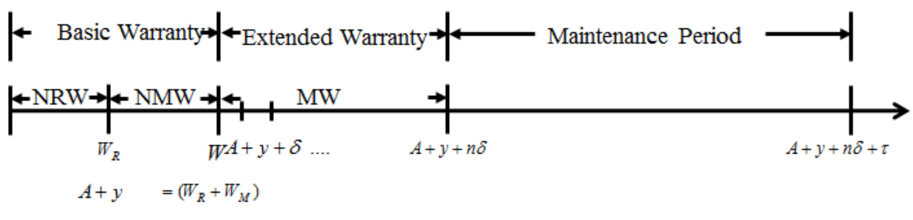

Fig. (1). Maintenance model with the NRRW following extended warranty.

3. FORMULATION OF EXPECTED COST RATE

Suppose that during the NRW phase of the basic warranty period, the system is replaced for k times, and y is the age of the system in use. We also assumed that k and y are known, and the k^{th} replacement of the second-hand product's age is A. Similar to the definition in Jung *et al*. [7], we define that the life cycle of a system begins with the installation of a second-hand product and ends when the system is replaced by another second-hand one at the end of maintenance period. Thus, under the non-renewing replacement-repair warranty strategy, the life cycle starts with the installation of a second-hand product and ends with the replacement of another second-hand one after the expiration of the maintenance period regardless of the number of replacements during the NRW phase. Therefore, the expected life cycle length of the proposed model is $w+n\delta+\tau$.

We use the cost-related random variables $C_{BW}, C_{EW}, C_M , C_r$ denoting cost of the basic warranty period, the cost of extended warranty period, the cost of maintenance period and the cost of the replacement at the end of the life cycle, respectively. Then, we will obtain the expected total cost ETC(τ) as follows.

$$ETC(\tau) = E(C_{BW}) + E(C_{EW}) + E(C_M) + E(C_r) \quad \textbf{(1)}$$

Each of the expected cost, which is given in Eq. (1), can be evaluated as below:

$$E(C_{BW}) = \begin{cases} c_{f,w}(k + \int_{A+y}^{A+y+w_M} h(t)\,\mathrm{d}t), & \text{NFRRW} \\ c_{r,w}\dfrac{(w_R - y)}{w_R} + c_{f,w}k + (c_{f,w} + c_{m,w})\int_{y}^{y+w_M} h(t)\,\mathrm{d}t, & \text{NPRRW} \end{cases} \quad \textbf{(2)}$$

$$E(C_{EW}) = \frac{1-\phi^k}{1-\phi}c_e + c_{few}\int_{A+y+w_M}^{A+y+w_M+n\delta} h(t)\,\mathrm{d}t \quad \textbf{(3)}$$

$$E(C_M) = (c_{m,m} + c_{f,m})\int_{A+y+w_M+n\delta}^{A+y+w_M+n\delta+\tau} h(t)\,\mathrm{d}t \quad \textbf{(4)}$$

and $E(C_r)=C_r$. Hence, the expected total cost can be represented as:

$$ETC(\tau) = \begin{cases} c_{f,w}(k + \int_{A+y}^{A+y+w_M} h(t)\,\mathrm{d}t) + \dfrac{1-\phi^n}{1-\phi}c_e + c_{few}\int_{A+y+w_M}^{A+y+w_M+n\delta} h(t)\,\mathrm{d}t \\ +(c_{m,m} + c_{f,m})\int_{A+y+w_M+n\delta}^{A+y+w_M+n\delta+\tau} h(t)\,\mathrm{d}t + c_r, & \text{NFRRW} \\ c_{r,w}\dfrac{(w_R - y)}{w_R} + c_{f,w}k + (c_{f,w} + c_{m,w})\int_{A+y}^{A+y+w_M} h(t)\,\mathrm{d}t + \dfrac{1-\phi^n}{1-\phi}c_e \\ +c_{few}\int_{A+y+w_M}^{A+y+w_M+n\delta} h(t)\,\mathrm{d}t + (c_{m,m} + c_{f,m})\int_{A+y+w_M+n\delta}^{A+y+w_M+n\delta+\tau} h(t)\,\mathrm{d}t + c_r, & \text{NPRRW} \end{cases} \quad \textbf{(5)}$$

Subsequently, the expected cost rate per unit time during the life cycle of the system can be derived as:

$$C(\tau) = \frac{ETC(\tau)}{ECL(\tau)} = \frac{1}{w+n\delta+\tau}\left\{c_0 + (c_{m,m} + c_{f,m})\int_{A+y+w_M+n\delta}^{A+y+w_M+n\delta+\tau} h(t)\,\mathrm{d}t\right\} \quad \textbf{(6)}$$

where,

$$c_0=\begin{cases} c_{f,w}(k+\int_{A+y}^{A+y+w_M}h(t)\,\mathrm{d}t)+\dfrac{1-\phi^n}{1-\phi}c_e+c_{few}\int_{A+y+w_M}^{A+y+w_M+n\delta}h(t)\,\mathrm{d}t+c_r, & \text{NFRRW} \\ c_{r,w}\dfrac{(w_R-y)}{w_R}+c_{f,w}k+(c_{f,w}+c_{m,w})\int_{A+y}^{A+y+w_M}h(t)\,\mathrm{d}t+\dfrac{1-\phi^k}{1-\phi}c_e & \\ +c_{few}\int_{A+y+w_M}^{A+y+w_M+n\delta}h(t)\,\mathrm{d}t+c_r, & \text{NPRRW} \end{cases} \tag{7}$$

If the product of replacement is new, thus, the age A of the product is 0, and the extended warranty period $\delta = 0$, then Eq. (7) is degenerated to the following form, which is equivalent to the result of Jung *et al.* [13]:

$$C(\tau)=\frac{ETC(\tau)}{ECL(\tau)}=\frac{1}{w+n\delta+\tau}\left\{c_0+(C_{f,m}+C_{m,m})\int_{y+w_M}^{y+w_M+\tau}h(t)\,\mathrm{d}t\right\} \tag{8}$$

where,

$$c_0=\begin{cases} c_{f,w}(k+\int_{y}^{y+w_M}h(t)\,\mathrm{d}t)+c_r, & \text{NFRRW} \\ c_{r,w}\dfrac{(w_R-y)}{w_R}+c_{f,w}k+(c_{f,w}+c_{m,w})\int_{y}^{y+w_M}h(t)\,\mathrm{d}t+c_r, & \text{NPRRW} \end{cases} \tag{9}$$

If the age A of the product is 0, and do not consider the non-renewing repair warranty phase of the basic warranty period, which means that the value of $w_M$is 0, then Eq. (7) is reduced to the following form, which is same with Sahin and Polatoglu's [3]:

$$C(\tau)=\begin{cases} \dfrac{1}{w+\tau}\left\{c_{f,w}k+(c_{f,m}+c_{m,m})\int_{y}^{y+\tau}h(t)\,\mathrm{d}t+c_r\right\}, & \text{NFRW} \\ \dfrac{1}{w+\tau}\left\{c_{r,w}\dfrac{(w-y)}{w}c_{f,w}k+(c_{f,m}+c_{m,m})\int_{y}^{y+\tau}h(t)\,\mathrm{d}t+c_r\right\}, & \text{NPRW} \end{cases} \tag{10}$$

Also, that If the product of replacement is new, and do not consider the non-renewing replacement warranty phase of the basic warranty period, which means that the value of w_R is 0, then Eq. (7) is degenerated to the following expected cost rate per unit time, which is same as the result of Yeh *et al.* [6]:

$$C(\tau)=\begin{cases} \dfrac{1}{w+\tau}\left\{c_{f,w}\int_{0}^{w}h(t)\,\mathrm{d}t+(c_{f,m}+c_{m,m})\int_{w}^{w+\tau}h(t)\,\mathrm{d}t+c_r\right\}, & \text{NFMW} \\ \dfrac{1}{w+\tau}\left\{(c_{r,w}+c_{m,w})\int_{0}^{w}h(t)\,\mathrm{d}t+(c_{r,m}+c_{m,m})\int_{w}^{w+\tau}h(t)\,\mathrm{d}t+c_r\right\}, & \text{NPMW} \end{cases} \tag{11}$$

4. OPTIMIZATION

In this section, we will derive the optimal length of maintenance period after the extended warranty by determining the value of τ, which minimizes $C(\tau)$. Hence, we consider the optimization problem as acquiring the value of τ*, which satisfies $C(\tau^*)=\min_{0\le\tau\le\infty}C(\tau)$. In order to get the value of τ* defined forward, we make the derivative of $C(\tau)$ with respect to $\boldsymbol{\tau}$ has a value of zero. Thus, we get the following expression:

$$h(A+y+w_M+n\delta+\tau)(w+n\delta+\tau)-\int_{A+y+w_M+n\delta}^{A+y+w_M+n\delta+\tau}h(t)dt=\frac{c_0}{c_{m,m}+c_{f,m}}, \tag{12}$$

We prove the uniqueness of the solution for Eq. (12) by using the concept of pseudo-convexity of a function. If the cost function given in Eq. (6) is pseudo-convexity, then it has only one local minimum, and thus there exists a unique global minimum [3].

4.1. Lemma

If F is an IFR distribution with strictly increasing failure rate function, then $C(\tau)$, given in Eq. (6), is pseudo-convex in $\tau \geq 0$ for a given n.

Proof. Let $G(\tau)$ represent the expression in the numerator of Eq. (6). Then we obtain the following relations.

$$C(0) = c_0/(w+n\delta) > 0,$$
$$C(\infty) = \infty,$$
$$\frac{\partial G(\tau)}{\partial \tau} = (c_{m,m} + c_{f,m})h(A + y + w_M + n\delta + \tau) > 0,$$
$$\frac{\partial^2 G(\tau)}{\partial \tau^2} = (c_{m,m} + c_{f,m})h'(A + y + w_M + n\delta + \tau) > 0.$$

Hence, $G(\tau)$ is positive and convex. Since $ETC(\tau)$ is linear in τ and positive, $C(\tau)$ is pseudo-convex in $G(\tau)$.

4.2. Theorem

If F is an IFR distribution with strictly increasing failure rate function and n ≥ 1 is given, then τ^* if and only if $(w+n\delta)h(A+y+w_M+n\delta)$, $\geq c_0/(c_{m,m}+c_{f,m})$ and $0 < \tau^* < \infty$ is the unique solution of Eq. (6) if and only if

$$(w+n\delta)h(A+y+w_M+n\delta) < c_0/(c_{m,m}+c_{f,m}).$$

Proof. With the use of the property of pseudo-convexity of $C(\tau)$ and Eq. (12), it is straightforward to prove Theorem 4.2. Thus, we obtain the optimal length of maintenance period of proposed model that follows the extended warranty from Theorem 4.2.

5. NUMERICAL EXAMPLES

This section will present numerical examples to illustrate the proposed methods discussed above under both NFRRW and NPRRW. The failure times of a system are assumed to follow a Weibull distribution with the failure rate of $h(t)=\beta\lambda^{\beta}t^{\beta-1}$ for $t \geq 0$, where $\lambda > 0$ and $\beta > 1$ ($h(t)$ is an increasing function), and the value of parameters can be obtained by Monte Carlo methods [14, 15]. The parameters $c_{m,m}$ and can c_r be obtained by Black-Scholes equations [16]. Suppose that the parameter values are given in Table **1** and $c_{r,w}=c_r$ for computational use. Using these parameter values, we derive the optimal length of the maintenance period, denoted by τ^*, and the corresponding expected cost rate $C(\tau^*)$.

Table 1. Parameter values assumed for our numerical example.

λ	β	k	w	ϕ	n	α	$c_{m,m}$	$c_{m,w}$	$c_{f,m}$	$c_{f,w}$	c_{few}	c_e	c_r
1	2	1	0.5	0.95	2	0.5	2	1	1	1	1	1	30

In Fig. **1**, when w_r =0.1, y=0.25 w_r , A=0.1, and the expense of replacement c_r=30, we consider various choices of β under NFRRW. For example, when β=2, the optimal maintenance period τ^*=1.5662, and $C(\tau^*)$=18.5473. From Fig. **(1)**, we can also obtain the conclusion that when β increases, τ^* decreases, and $C(\tau^*)$ increases. The triangular sign symbols the optimal maintenance period τ^* and $C(\tau^*)$ in Fig. **(1)**. In Fig. **(2)**, the parameter value β=2, and the other parameter is the same with Fig. **(1)**. we consider various choices of parameter A under NFRRW. For example, when A=0.25, the optimal maintenance period τ^*=1.5086, and $C(\tau^*)$=19.1016. When A increases, τ^* decreases, and $C(\tau^*)$ increases. Figs.**(3** and **4)** consider the situation under NPRRW, and they have the similar results with 1 and 2.

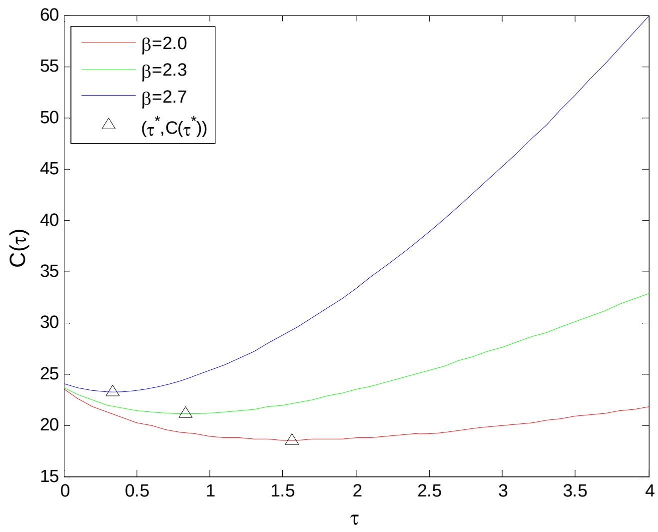

Fig. (2). Optimal maintenance period under NFRRW when β takes different values.

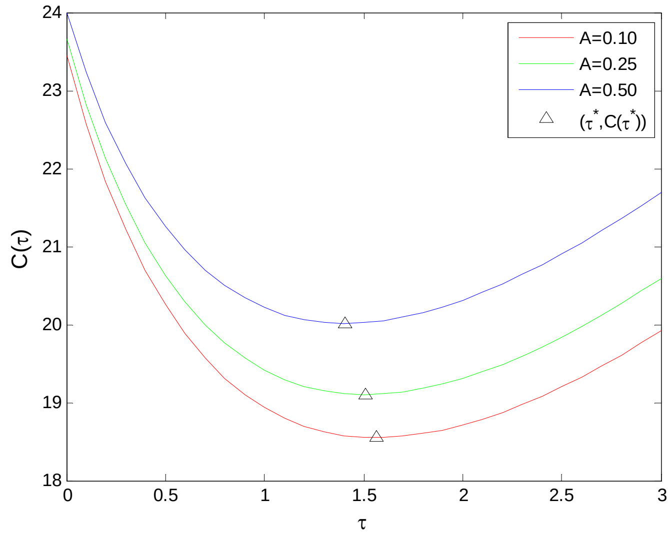

Fig. (3). Optimal maintenance period under NPRRW when initial age A takes different values.

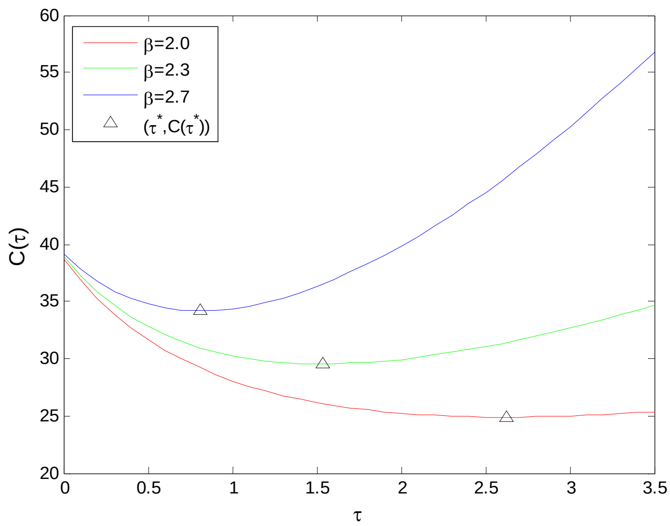

Fig. (4). Optimal maintenance period under NFRRW when β takes different values.

In Table **2**, the optimal maintenance period denoted by τ^*, is obtained by minimizing the expected cost rate function $C(\tau)$ of Eq. (6). We list the value of τ^* and corresponding expected cost rate $C(\tau^*)$ when the parameters β and A take different values. Table **3** provides similar patterns with Table **2** except that the parameter w_r takes a different value.

Table 2. Optimal maintenance policies with NRRW when w_r=0.1.

β	A	y	Optimal policy	NFRRW			NPRRW		
				c_r			c_r		
				30	50	70	30	50	70
2	0.1	$0.25w_r$	τ^*	1.5662	2.5085	3.2681	2.6249	3.8555	4.8520
			$C(\tau^*)$	18.5473	24.2012	28.7587	24.8995	32.2832	38.2622
		$0.50w_r$	τ^*	1.5578	2.5021	3.2627	2.3040	3.4467	4.3711
			$C(\tau^*)$	18.6467	24.3125	28.8762	23.1237	29.9803	35.5267
	0.25	$0.25w_r$	τ^*	1.5086	2.4646	3.2313	2.5921	3.8302	4.8307
			$C(\tau^*)$	19.1016	24.8378	29.4377	25.6024	33.0316	39.0345
		$0.50w_r$	τ^*	1.5000	2.4581	3.2258	2.2683	3.4194	4.3481
			$C(\tau^*)$	19.2000	24.9487	29.5549	23.8097	30.7161	36.2885
	0.5	$0.25w_r$	τ^*	1.4100	2.3904	3.1692	2.5367	3.7879	4.7951
			$C(\tau^*)$	20.0102	25.8922	30.5654	26.7702	34.2773	40.3206
		$0.50w_r$	τ^*	1.4012	2.3837	3.1637	2.2081	3.3734	4.3095
			$C(\tau^*)$	20.1069	26.0024	30.6821	24.9486	31.9404	37.5569
2.3	0.1	$0.25w_r$	τ^*	0.8352	1.4615	1.9505	1.5361	2.3203	2.9366
			$C(\tau^*)$	21.0713	28.6127	34.8474	29.5457	39.7420	48.2151
		$0.50w_r$	τ^*	0.8221	1.4503	1.9403	1.3194	2.0563	2.6345
			$C(\tau^*)$	21.2090	28.7850	35.0406	27.1635	36.5618	44.3590
	0.25	$0.25w_r$	τ^*	0.7514	1.3912	1.8869	1.4742	2.2656	2.8856
			$C(\tau^*)$	21.8429	29.6097	35.9773	30.6557	41.0281	49.6110
		$0.50w_r$	τ^*	0.7380	1.3800	1.8767	1.2549	1.9998	2.5821
			$C(\tau^*)$	21.9784	29.7825	36.1719	28.2198	37.7997	45.7088
	0.5	$0.25w_r$	τ^*	0.6072	1.2728	1.7806	1.3721	2.1762	2.8025
			$C(\tau^*)$	23.0889	31.2725	37.8762	32.5401	43.2155	51.9840
		$0.50w_r$	τ^*	0.5932	1.2614	1.7704	1.1479	1.9073	2.4965
			$C(\tau^*)$	23.2189	31.4453	38.0727	30.0084	39.9036	48.0031
2.7	0.1	$0.25w_r$	τ^*	0.3340	0.7593	1.0801	0.8079	1.3168	1.7046
			$C(\tau^*)$	23.2421	32.9897	41.2467	34.1915	47.8196	59.4336
		$0.50w_r$	τ^*	0.3166	0.7441	1.0659	0.6556	1.1402	1.5083
			$C(\tau^*)$	23.4039	33.2295	41.5371	31.0799	43.5618	54.1739
	0.25	$0.25w_r$	τ^*	0.2255	0.6656	0.9932	0.7205	1.2372	1.6287
			$C(\tau^*)$	24.1302	34.3821	42.9589	35.7644	49.8505	61.7720
		$0.50w_r$	τ^*	0.2074	0.6501	0.9789	0.5651	1.0587	1.4310
			$C(\tau^*)$	24.2800	34.6205	43.2524	32.5190	45.4638	56.3826
	0.5	$0.25w_r$	τ^*	0.0333	0.5055	0.8469	0.5749	1.1065	1.5048
			$C(\tau^*)$	25.3918	36.6573	45.8366	38.4560	53.3707	65.8328
		$0.50w_r$	τ^*	0.0136	0.4894	0.8324	0.4134	0.9242	1.3044
			$C(\tau^*)$	25.5076	36.8873	46.1314	34.9544	48.7498	60.2129

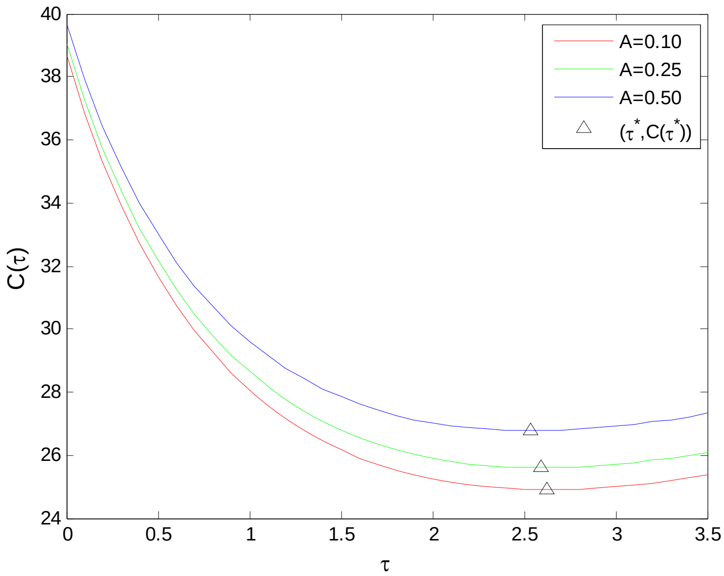

Fig. (5). Optimal maintenance period under NPRRW when initial age *A* takes different values.

Table 3. Optimal maintenance policies with NRRW when w_r=0.2.

β	A	y	Optimal policy	NFRRW			NPRRW		
				c_r			c_r		
				30	50	70	30	50	70
2	0.1	$0.25w_r$	τ*	1.5914	2.5278	3.2843	2.6396	3.8669	4.8616
			C(τ*)	18.2483	23.8669	28.4061	24.5379	31.9012	37.8696
		$0.50w_r$	τ*	1.5741	2.5146	3.2732	2.3131	3.4538	4.3771
			C(τ*)	18.4445	24.0873	28.6391	22.8788	29.7227	35.2624
	0.25	$0.25w_r$	τ*	1.5343	2.4841	3.2476	2.6045	3.8398	4.8388
			C(τ*)	18.8055	24.5048	29.0858	25.2268	32.6387	38.6326
		$0.50w_r$	τ*	1.5166	2.4707	3.2364	2.2749	3.4244	4.3523
			C(τ*)	18.9997	24.7244	29.3183	23.5495	30.4466	36.0141
	0.5	$0.25w_r$	τ*	1.4366	2.4102	3.1858	2.5452	3.7944	4.8005
			C(τ*)	19.7193	25.5615	30.2148	26.3710	33.8660	39.9032
		$0.50w_r$	τ*	1.4183	2.3966	3.1744	2.2104	3.3751	4.3109
			C(τ*)	19.9100	25.7795	30.4464	24.6621	31.6506	37.2655
2.3	0.1	$0.25w_r$	τ*	0.8747	1.4951	1.9812	1.5668	2.3475	2.9621
			C(τ*)	20.6594	28.0983	34.2712	28.9908	39.1016	47.5210
		$0.50w_r$	τ*	0.8482	1.4726	1.9606	1.3400	2.0745	2.6515
			C(τ*)	20.9317	28.4391	34.6533	26.8014	36.1430	43.9047
	0.25	$0.25w_r$	τ*	0.7916	1.4250	1.9175	1.5031	2.2913	2.9097
			C(τ*)	21.4365	29.0934	35.3963	30.0731	40.3622	48.8930
		$0.50w_r$	τ*	0.7647	1.4023	1.8970	1.2736	2.0165	2.5977
			C(τ*)	21.7051	29.4354	35.7816	27.8312	37.3574	45.2330
	0.5	$0.25w_r$	τ*	0.6492	1.3071	1.8114	1.3976	2.1992	2.8242
			C(τ*)	22.6977	30.7558	37.2894	31.9065	42.5036	51.2232
		$0.50w_r$	τ*	0.6210	1.2841	1.7907	1.1630	1.9211	2.5097
			C(τ*)	22.9563	31.0981	37.6786	29.5691	39.4174	47.4874
2.7	0.1	$0.25w_r$	τ*	0.3859	0.8051	1.1229	0.8517	1.3570	1.7429
			C(τ*)	22.7527	32.2743	40.3832	33.4172	46.8262	58.2910
		$0.50w_r$	τ*	0.3513	0.7745	1.0943	0.6854	1.1672	1.5341
			C(τ*)	23.0785	32.7496	40.9568	30.5971	42.9332	53.4469

(Table 3) contd.....

β	*A*	*y*	Optimal policy	NFRRW			NPRRW		
				c_r			c_r		
				30	50	70	30	50	70
	0.25	$0.25w_r$	τ*	0.2792	0.7119	1.0361	0.7632	1.2763	1.6660
			C(τ*)	23.6724	33.6686	42.0844	34.9501	48.8081	60.5768
		$0.50w_r$	τ*	0.2434	0.6810	1.0075	0.5939	1.0847	1.4558
			C(τ*)	23.9778	34.1430	42.6656	32.0029	44.7963	55.6147
	0.5	$0.25w_r$	τ*	0.0916	0.5533	0.8905	0.6156	1.1434	1.5400
			C(τ*)	25.0248	35.9644	44.9553	37.5633	52.2355	64.5396
		$0.50w_r$	τ*	0.0529	0.5214	0.8614	0.4399	0.9482	1.3273
			C(τ*)	25.2713	36.4257	45.5414	34.3655	48.0026	59.3638

From Figs. **(2 - 5)**, Tables **2** and **3**, we summarize the effect of the relevant parameters on τ* and $C(\tau^*)$ as follows:

1. When the value of β increases, τ* decreases, and $C(\tau^*)$ increases under both NFRRW and NPRRW. Thus, we can conclude reasonably that when the system functions with a high failure rate, it is more beneficial for the user to utilize the system for short maintenance period.
2. When the initial age *A* of the second-hand product increases, τ* decreases, and $C(\tau^*)$ increases under both NFRRW and NPRRW. Therefore, we can also draw the conclusion that it is more beneficial for the user to use the system for short maintenance period when the initial age increases.
3. When the value of c_r increases, τ* becomes longer, and $C(\tau^*)$ increases. Thus, it is more beneficial for the user to utilize the system for long maintenance period when the expense of replacement increases.
4. The expected cost rate is always higher under the NPRRW than under the NFRRW.

CONCLUSION

This paper has investigated the optimal maintenance period following an extended replacement-repair warranty of the second-hand product. Due to the saturated market and the development of science and technology, increasingly complex and high priced products flow into the market. Also an extended warranty of the second-hand product when the basic replacement-repair warranty is expired attracts more attention of the user. Furthermore, the effective maintenance of the system following the extended warranty by the user makes sure the system works well and minimizes the operating cost. Therefore, we present a model of extended replacement-repair warranty .When the initial age is zero and there is no extended warranty period, our model will degenerate to Jung *et al.* [8]. When A=0, α=0, and w_m=0, our model is degenerated to the result of Sahin and Polatoglu [3]. Also note that if A=0, α=0, and w_R=0, it is same as the result of Yeh *et al.* [6]. In addition, we make a contrast between NFRRW and NPRRW. Subsequently, we analyze the effect of several parameters on the length of maintenance period and expected cost rate per unit time. The generalization of this extended model of the two-dimensional cases would be very interesting, so this will be our future research orientation.

LIST OF ABBREVIATIONS

ϕ	=	The discount rate of purchasing extended warranty expense
τ	=	Length of maintenance period after the NRRW is expired
ä	=	Length of each extended warranty
A	=	The age of the second-hand product
C(τ)	=	Expected cost rate per unit time
Ce	=	Unit cost of purchasing extended warranty
Cfew	=	Unit failure cost during the extended warranty period
Cfm	=	Unit failure cost during the maintenance period
Cf,w	=	Unit failure cost of minimal repair during the warranty period
Cmm	=	Unit cost of minimal repair during the maintenance period
Cm,w	=	Unit cost of minimal repair during the warranty period
Cr	=	Unit replacement cost at the end of the maintenance period
Crw	=	Unit replacement cost during the warranty period

ECL(τ)	=	Expected length of life cycle
ETC(τ)	=	Expected total cost
F(t), f(t)	=	Life distribution and probability density function of T
H(t)	=	Failure rate function
K	=	Number of replacements during the NRRW period
N	=	Number of extended warranty periods
NFMW	=	Non-renewing free minimal repair warranty
NFRRW	=	Non-renewing free replacement-repair warranty
NFRW	=	Non-renewing free replacement warranty
NMW	=	Non-renewing minimal repair warranty
NPMW	=	Non-renewing pro rata repair warranty
NPRRW	=	Non-renewing pro rata replacement-repair warranty
NPRW	=	Non-renewing pro rata replacement warranty
NRRW	=	Non-renewing replacement-repair warranty
NRW	=	Non-renewing replacement warranty
T	=	Time to failure of a system
W	=	Basic two-phase warranty period
WM	=	NMW period
Wr	=	NRW period
Y+A	=	Age of the system in use at the end of the NRRW period

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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