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RESEARCH ARTICLE

Medical Decision Making *via* the Arithmetic of Generalized Triangular Fuzzy Numbers

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Abstract:

Background:

When patient(s) approach to a medical expert to explain their problems, they often explain their conditions through vague linguistic expression [1]. Medical expert needs to prepare a list of potential symptoms for the particular diseases of the patients based on their vague linguistic statements. Together with the vagueness in medical documents and imprecise information gathered for decision making makes the medical experts' job more complex. Due to the occurrence of uncertainty in medical decision making exploitation of the Fuzzy Set (FST) is required. Generally in literature, type-I fuzzy set, Intuitionistic Fuzzy Sets (IFSs), Interval Valued Fuzzy Sets (IVFSs), and Picture Fuzzy Sets (PFSs) are extensively applied in medical decision making.

Objective:

Although different approaches have been used in medical decision making, no single evidence has been observed in use of Generalized Fuzzy Numbers (GFNs) in medical decision making. GFN has the ability to deal with vague/imprecise information in a supple way. Basically, the parameter height of GFN characterizes the grade of buoyancy of judgments of decision takers in a very specific comportment. Therefore, a maiden effort has been made to study medical diagnosis using arithmetic of GFNs, and finally to exhibit the techniques a case study has been carried out under this setting.

Method:

To achieve the proposed goal an algorithm is being formulated and to obtain patients-diseases relationship the arithmetic of GFNs is used as the composition of fuzzy relations.

Results:

In this study, two scenarios are taken into considerations. In scenario-I, TFNs are used while in scenario-II, GFNs are taken to characterize uncertainty and medical decision making has been carried out. The advantages of GFNs over the TFNs are observed through the comparison of both the approaches in medical decision making. Major advantage GFNs here is that it makes it possible to compare various diseases against each other's in a more acceptable manner and accordingly diseases of the patients can be detected directly.

Conclusion:

The advantage of the GFN approach has been observed from the case study where it is found that existing TFN approach provides illogical results while proposed one gives a rational result. Also, it has been established that proposed approach is efficient, simple, logical, technically sound and general enough for implementation.

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1. INTRODUCTION

It has been observed that more often many real world representation models are soiled with vagueness/ ambiguity or uncertainty due to deficiency of precision, deficiency in data, diminutive sample sizes or data acquired from specialist opinion, artificial/manmade error, etc. To tackle such kind of uncertainty Zadeh [2] devised a hypothesis called as fuzzy set theory (FST). Afterward, plenty of supplementary direct or indirect extensions have been made by numerous canvassers. Chen [3] further extended the conventional FST and developed generalized fuzzy numbers (GFN) where all the four arithmetic operations on GFNs were discussed using function principal. GFN can deal with uncertain data/information in a more bendy way in comparison to FST, as the height (*w)* that characterize the grade of buoyancy of judgments of decision takers in a very specific comportment.

Authors in [4] acknowledged that Chen's arithmetic operations are altering the type of membership functions (MFs) together with diminishing the deficiency of arithmetic operations on GFNs. Nevertheless, it has been observed that Chen initially decreased the height of the GFN of bigger one to the height of the smaller one and then converted it into a new GFN and carried out all the four basic arithmetic operations; and as a result, it lost data/information and did not provide accurate output.

The key disadvantage of Chen's method is to facilitate arithmetic operations between a fixed triangular GFN with dissimilar triangular GFNs with identical support but unusual heights where height of permanent GFN is smaller than the other GFNs then it is observed that every moment output triangular GFN is stay behind invariant which is really irrational. Dutta [5] supplementary studied arithmetic operations between triangular GFNs and it is observed that this approach produces different trapezoidal type GFNs in the arithmetic of triangular GFNs and which is found to be more realistic.

1.1. Related Works

Zadeh [6] first applied FST in the field of medicine and Sanchez [7, 8] fully formulated the finding models relating to fuzzy matrices in lieu of the medical understanding between symptoms and diseases. Nowadays, medical diagnosis using fuzzy variables is much more popular among researchers. Decisions making for medical diagnosis based on fuzzy numbers along with compositional rule of inference were studied [9]. Fuzzy set, rough set as well as soft set based medical analysis were discussed [10]. A procedure for medical diagnosis using fuzzy decision making was presented [11]. A system for medical diagnosis was proposed that was generated using fuzzy logic toolbox in MATLAB [12]. Specifically, it focused on medical diagnosis. A fuzzy inference system created to support medical diagnoses in real time was presented [13]; the concept of weighted hesitant fuzzy set (WHFS) was redefined and subsequently applied in medical diagnosis [14].

Interval valued fuzzy soft set was also studied to exhibited the technique with hypothetical case study [15]. A fuzzy diagnosis method based on the interval valued interview chart and the interval valued Intuitionistic fuzzy weighted arithmetic average operator was presented and studied the occurrence information symptoms as the weights [16]. Sanchez's approach was extended for medical diagnosis using interval valued fuzzy matrix [17], a fuzzy diagnosis method based on interval valued intuitonistic fuzzy sets (IVIFSs) was proposed in [18]. Medical diagnosis based on IVFN matrices was also discussed [19], also interval valued fuzzy sets was applied in medical diagnosis [1].

De and coworkers [20] first deliberated medical diagnosis with the notion of IFS, medical diagnosis using distance measures was also studied [21]. Advantages of type-2 fuzzy and switching relation between type-2 fuzzy sets were studied applied in medical diagnosis [22]. Some other studies were made in medical diagnosis for IFSs [23 - 25]. Works of [20] was responded in [26]. An approach on divergence measures for IFSs was propsed and applied in medical diagnosis [27]. Confirmation of the demand of IFSs in medical was studied [28]. Decision support system in ICU *via* IFSs was also discussed [29], modal operator based on IFSs and its application in medical decision making was also done [30].

Fuzzy soft set theory in medical diagnosis using fuzzy arithmetic operations was further studied [31]. The concept of bell-shaped fuzzy soft sets was first introduced and successfully applied in medical decision making [32]. Medical decision making using picture fuzzy set was also discussed [33]. An improved hybrid approach for training the adaptive network based fuzzy inference system with Modified Levenberg-Marquardt algorithm using analytical derivation scheme for computation of Jacobian matrix was introduced [34].

1.2. Purpose of the Research

It is palpable that when individual(s) suffer from any types of diseases he/she/they often move toward to a medical expert while medical expert queries a patient about his/her state, then patients use the linguistic variable/expression to elucidate their states which are generally vague [1]. Medical expert wants to assess a catalog of potential symptoms for the particular diseases of the patients based on their vague linguistic statements. Nevertheless, the associations between symptoms and diseases are not often one-to-one. The exposition of the identical disease may not be equal with dissimilar patients and even at dissimilar disease stages. Furthermore, it should be noted that a meticulous symptom can point out diverse diseases and in a number of circumstances in a meticulous patient may dismay the presumed formation of symptoms. On the other hand, knowledge base correlating the symptom-disease relationship comprises of ambiguity, imprecision, vagueness and uncertainty in medical decision making process. Most frequently, type-I fuzzy set, IFS, IVFSs, PFSs are employed for medical decision making purpose, but some situations it leads to counter intuitive results. Hence, GFNs can be employed as the representation of the state and symptoms of the patient that can be only known by medical experts with a very partial grade of precision. Therefore, triangular GFNs have been taken in this study as the GFN has the skill to deal with vague data/information in a more stretchy way because of the parameter height that characterize the degree of confidence of opinions of decision maker's as the height of GFN characterize the grade of buoyancy of judgments of decision takers in a very specific comportment. Thus, it can be opined that GFN has more importance in medical decision making process and can effectively be used.

1.3. Contributions

The applicability and legalization of the study has been shown by solving medical decision making problems *via* two ways. The current study made it feasible to instigate weights of all signs of diseases correctly and in view of that patients can be diagnosed straightforwardly. It is monitored that it confers efficient, flexible, simple, logical, technically sound solution for medical decision making problem. It is also attained that the GFNs have paramount significant role in medical decision making problem. The advantages of this study can be pointed out as follows: it offers an effort to carry out medical diagnosis by considering medical expert's medical knowledge in terms of GFNs; it has more usefulness because of its capability to address imprecision/vagueness/uncertainty in a proper manner; it also offers assistance to medical experts to carry out medical diagnosis providing better efficiency to the output and people living in the rural areas will be mostly benefited.

1.4. Objectives

In this paper, a maiden effort has been made to bring out medical diagnosis using GTFN *via* the approach [5].

2. PRELIMINARIES

Let X be a universe of discourse. Then the fuzzy subset A of X is defined by its membership function

$$\mu_A : X \to [0,1]$$

Which allocate a real number $\mu_A(x)$ in the interval [0, 1], to each element $x \in A$, wherever the value of $\mu_A(x)$ show the grade of membership of x in A.

2.1. Generalized Fuzzy Numbers (GFN)

The membership function of GFN A= $[a,b,c,d;w]$ where $a \le b \le c \le d$, $0 < w \le 1$ is defined as

$$\mu_A(x) = \begin{cases} 0, x < a \\ w\dfrac{x-a}{b-a}, a \le x \le b \\ w, b \le x \le c \\ w\dfrac{d-x}{d-c}, c \le x \le d \\ 0, x > d \end{cases}$$

If $w<1$, then the GFN *A* is called a trapezoidal GFN. If w_1 and w_2 , then GFN *A* is a normal trapezoidal fuzzy number [a, b, c, d]. If *a* = *b* and *c* = *d*, then *A* is a crisp interval .If *b* = *c* then *A* is a triangular GFN. If *a* = *b* = *c* and *w* = *1* then *A* is a real number.

2.2. α-cut of the Triangular GFN

Let *A* be a triangular GFN with heights *w* with MF

$$\mu_A(x)=\begin{cases} w\,\dfrac{x-a}{b-a}, & x\in[a,b] \\ w\,\dfrac{c-x}{c-b}, & x\in[b,c] \end{cases}$$

The αα-cut of the GTFNs *A* is

$$^{\alpha}A=\left[\frac{\alpha}{w}(b-a)+a,\,c-\frac{\alpha}{w}(c-b)\right],\alpha\in[0,w]$$

3. ARITHMETIC OPERATIONS ON GTFNs

Let *A* and *B* be two triangular GFNs with heights w_1 and w_2 respectively whose MFs are given

as $\mu_A(x)=\begin{cases} w_1\dfrac{x-a_1}{b_1-a_1}, & x\in[a_1,b_1] \\ w_1\dfrac{c_1-x}{c_1-b_1}, & x\in[b_1,c_1] \end{cases}$ and $\mu_B(x)=\begin{cases} w_2\dfrac{x-a_2}{b_2-a_2}, & x\in[a_2,b_2] \\ w_2\dfrac{c_2-x}{c_2-b_2}, & x\in[b_2,c_2] \end{cases}$.

The α α-cuts of the triangular GFNs *A* and *B* are

$$^{\alpha}A=\left[\frac{\alpha}{w_1}(b_1-a_1)+a_1,c_1-\frac{\alpha}{w_1}(c_1-b_1)\right],\alpha\in[0,w_1] \quad\text{and}\quad ^{\alpha}B=\left[\frac{\alpha}{w_2}(b_2-a_2)+a_2,c_2-\frac{\alpha}{w_2}(c_2-b_2)\right],\alpha\in[0,w_2]$$

respectively.

3.1. Theorem: *Addition of* Triangular *GFNs Produces a* Trapezoidal *GFN*

Proof: To evaluate addition of triangular GFNs *A* and *B,* it is needed to sum the α-cut of the triangular GFNs *A* and *B* using the method of arithmetic of intervals.

$$^{\alpha}A+{}^{\alpha}B=\left[\frac{\alpha}{w_1}(b_1-a_1)+a_1+\frac{\alpha}{w_2}(b_2-a_2)+a_2,\,c_1-\frac{\alpha}{w_1}(c_1-b_1)+c_2-\frac{\alpha}{w_2}(c_2-b_2)\right]\text{where } w=\min(w_1,w_2)$$

and $\alpha\in[0,w]$.

$$=\left[(a_1+a_2)+\alpha\left\{\frac{b_1-a_1}{w_1}+\frac{b_2-a_2}{w_2}\right\},(c_1+c_2)-\alpha\left\{\frac{c_1-b_1}{w_1}+\frac{c_2-b_2}{w_2}\right\}\right]$$

$$=\left[(a_1+a_2)+\alpha\left\{\frac{b_1-a_1}{w_1}+\frac{b_2-a_2}{w_2}\right\},(c_1+c_2)-\alpha\left\{\frac{c_1-b_1}{w_1}+\frac{c_2-b_2}{w_2}\right\}\right] \quad \textbf{(1)}$$

To gauge the output MF $\mu_{A+B}(x)$ we need to equate both the first and second component of (1) to x that provides

$$x=(a_1+a_2)+\alpha\left\{\frac{b_1-a_1}{w_1}+\frac{b_2-a_2}{w_2}\right\} \text{ and } x=(c_1+c_2)-\alpha\left\{\frac{c_1-b_1}{w_1}+\frac{c_2-b_2}{w_2}\right\}$$

Then, express α in terms of x and putting $\alpha \geq 0$ and $\alpha \leq w$ in (1) we obtain α together with the domain of x,

$$\text{i.e, } \alpha=\frac{x-(a_1+a_2)}{\left\{\frac{b_1-a_1}{w_1}+\frac{b_2-a_2}{w_2}\right\}},x\in\left[(a_1+a_2),(a_1+a_2)+w\left\{\frac{b_1-a_1}{w_1}+\frac{b_2-a_2}{w_2}\right\}\right] \text{, and}$$

$$\alpha=\frac{(c_1+c_2)-x}{\left\{\frac{c_1-b_1}{w_1}+\frac{c_2-b_2}{w_2}\right\}},x\in\left[(c_1+c_2)-w\left\{\frac{c_1-b_1}{w_1}+\frac{c_2-b_2}{w_2}\right\},(c_1+c_2)\right]$$

that produces

$$\mu_{A+B}(x)=\begin{cases}\dfrac{x-(a_1+a_2)}{\left\{\frac{b_1-a_1}{w_1}+\frac{b_2-a_2}{w_2}\right\}}, & x\in\left[(a_1+a_2),w\left\{\frac{b_1-a_1}{w_1}+\frac{b_2-a_2}{w_2}\right\}+(a_1+a_2)\right]\\ w, & x\in\left[w\left\{\frac{b_1-a_1}{w_1}+\frac{b_2-a_2}{w_2}\right\}+(a_1+a_2),(c_1+c_2)-w\left\{\frac{c_1-b_1}{w_1}+\frac{c_2-b_2}{w_2}\right\}\right]\\ \dfrac{(c_1+c_2)-x}{\left\{\frac{c_1-b_1}{w_1}+\frac{c_2-b_2}{w_2}\right\}}, & x\in\left[(c_1+c_2)-w\left\{\frac{c_1-b_1}{w_1}+\frac{c_2-b_2}{w_2}\right\},(c_1+c_2)\right]\end{cases}$$

This is obviously a trapezoidal GFN with height w at

$$\left[w\left\{\frac{b_1-a_1}{w_1}+\frac{b_2-a_2}{w_2}\right\}+(a_1+a_2),(c_1+c_2)-w\left\{\frac{c_1-b_1}{w_1}+\frac{c_2-b_2}{w_2}\right\}\right] \text{ and range is } [(a_1+a_2),(c_1+c_2)].$$

Hence proved the theorem

3.2. Theorem: *Subtraction of Triangular GFNs Produces a Trapezoidal GFN*

Proof: Here also, to evaluate subtraction of triangular GFNs A and B, it is needed to subtract the α-cuts of A and B

using the method of interval arithmetic.

$$\alpha_{A}-\alpha_{B}=\left[\frac{\alpha}{w_1}(b_1-a_1)+a_1-\left\{c_2-\frac{\alpha}{w_2}(c_2-b_2)\right\},c_1-\frac{\alpha}{w_1}(c_1-b_1)-\left\{\frac{\alpha}{w_2}(b_2-a_2)+a_2\right\}\right] \tag{2}$$

where $w=\min(w_1,w_2)$ and $\alpha\in[0,w]$.

To evaluate the output MF$\mu_{A-B}(x)$ we need to equate both the first and second component of (2) to *x* which gives

$$x=\frac{\alpha}{w_1}(b_1-a_1)+a_1-\left\{c_2-\frac{\alpha}{w_2}(c_2-b_2)\right\}\text{and } x=c_1-\frac{\alpha}{w_1}(c_1-b_1)-\left\{\frac{\alpha}{w_2}(b_2-a_2)+a_2\right\}$$

Then, express α in terms of *x* and putting α ≥ 0 and α ≤ *w* in (2) we have α together with the domain of *x*

$$\alpha=\frac{x-(a_1-c_2)}{\left\{\frac{(b_1-a_1)}{w_1}+\frac{c_2-b_2}{w_2}\right\}},\ x\in\left[(a_1-c_2),(a_1-c_2)+w\left\{\frac{(b_1-a_1)}{w_1}+\frac{c_2-b_2}{w_2}\right\}\right]\text{and}$$

$$\alpha=\frac{(c_1-a_2)-x}{\left\{\frac{(c_1-b_1)}{w_1}+\frac{b_2-a_2}{w_2}\right\}},\ x\in\left[(c_1-a_2)-w\left\{\frac{(c_1-b_1)}{w_1}+\frac{b_2-a_2}{w_2}\right\},(c_1-a_2)\right]$$

The MF of the resulting GFN after subtraction of *A* and *B* is obtained as

$$\mu_{A-B}(x)=\begin{cases}\dfrac{x-(a_1-c_2)}{\left\{\frac{(b_1-a_1)}{w_1}+\frac{c_2-b_2}{w_2}\right\}}, & x\in\left[(a_1-c_2),(a_1-c_2)+w\left\{\frac{(b_1-a_1)}{w_1}+\frac{c_2-b_2}{w_2}\right\}\right]\\ w, & x\in\left[(a_1-c_2)+w\left\{\frac{(b_1-a_1)}{w_1}+\frac{c_2-b_2}{w_2}\right\},(c_1-a_2)-w\left\{\frac{(c_1-b_1)}{w_1}+\frac{b_2-a_2}{w_2}\right\}\right]\\ \dfrac{(c_1-a_2)-x}{\left\{\frac{(c_1-b_1)}{w_1}+\frac{b_2-a_2}{w_2}\right\}}, & x\in\left[(c_1-a_2)-w\left\{\frac{(c_1-b_1)}{w_1}+\frac{b_2-a_2}{w_2}\right\},(c_1-a_2)\right]\end{cases}$$

where $w=\min(w_1,w_2)$ and $\alpha\in[0,w]$.

Similarly, this is also a MF of the output trapezoidal GFN with height *w* at

$$\left[(a_1-c_2)+w\left\{\frac{(b_1-a_1)}{w_1}+\frac{c_2-b_2}{w_2}\right\},(c_1-a_2)-w\left\{\frac{(c_1-b_1)}{w_1}+\frac{b_2-a_2}{w_2}\right\}\right]\text{and range is}$$

$[(a_1-c_2),(c_1-a_2)]$.

Hence proved the theorem

3.3. Theorem: *Product of Triangular GFNs Produces a Trapezoidal Type GFN*

Proof: To evaluate product of triangular GFNs *A* and *B,* we need to initially product the αα-cuts of *A* and *B* using the approach of interval arithmetic.

$$^{\alpha}A.^{\alpha}B=\left[\left\{\frac{\alpha}{w_1}(b_1-a_1)+a_1\right\}.\left\{\frac{\alpha}{w_2}(b_2-a_2)+a_2\right\},\left\{c_1-\frac{\alpha}{w_1}(c_1-b_1)\right\}.\left\{c_2-\frac{\alpha}{w_2}(c_2-b_2)\right\}\right] \quad \textbf{(3)}$$

where $w=\min(w_1,w_2)$ and $\alpha\in[0,w]$.

To obtain the required MF $\mu_{AB}(x)$ we need to equate both the first and second component of (3) to *x* which gives:

$$x=\left\{\frac{\alpha}{w_1}(b_1-a_1)+a_1\right\}.\left\{\frac{\alpha}{w_2}(b_2-a_2)+a_2\right\}$$

$$\Rightarrow x=\frac{\alpha^2}{w_1w_2}(b_1-a_1)(b_2-a_2)+\frac{\alpha}{w_1}a_2(b_1-a_1)+\frac{\alpha}{w_2}a_1(b_2-a_2)+a_1a_2$$

$$\Rightarrow \alpha^2\frac{(b_1-a_1)(b_2-a_2)}{w_1w_2}+\alpha\left\{\frac{a_2(b_1-a_1)}{w_1}+\frac{a_1(b_2-a_2)}{w_2}\right\}+(a_1a_2-x)=0$$

this is a quadratic equation and solving we obtain

$$\alpha=\frac{-\{a_2(b_1-a_1)w_2+a_1(b_2-a_2)w_1\}+\sqrt{\{a_2(b_1-a_1)w_2+a_1(b_2-a_2)w_1\}^2-4w_1w_2(b_1-a_1)(b_2-a_2)(a_1a_2-x)}}{2(b_1-a_1)(b_2-a_2)}$$

Similarly

$$\text{Similarly}\quad x=\left\{c_1-\frac{\alpha}{w_1}(c_1-b_1)\right\}.\left\{c_2-\frac{\alpha}{w_2}(c_2-b_2)\right\}\text{gives}$$

gives

$$\alpha=\frac{c_1(c_2-b_2)w_1+c_2(c_1-b_1)w_2-\sqrt{\left\{c_1(c_2-b_2)w_1+c_2(c_1-b_1)w_2\right\}^2-4w_1w_2(c_1-b_1)(c_2-b_2)(c_1c_2-x)}}{2(c_1-b_1)(c_2-b_2)}$$

Now, setting α ≥ 0 and α ≤ *w* in (3) we get the required MF of the resulting GFN after multiplication of *A* and *B* simultaneously with the domain of *x*

$$\mu_{AB}(x)=\begin{cases}\dfrac{-\{a_2(b_1-a_1)w_2+a_1(b_2-a_2)w_1\}+\sqrt{\{a_2(b_1-a_1)w_2+a_1(b_2-a_2)w_1\}^2-4w_1w_2(b_1-a_1)(b_2-a_2)(a_1a_2-x)}}{2(b_1-a_1)(b_2-a_2)}\\ \text{if } x\in[a_1a_2,a_1a_2+\dfrac{w^2}{w_1w_2}(b_1-a_1)(b_2-a_2)+\dfrac{w}{w_1}a_2(b_1-a_1)+\dfrac{w}{w_2}a_1(b_2-a_2)]\\ w,\ x\in[a_1a_2+\dfrac{w^2}{w_1w_2}(b_1-a_1)(b_2-a_2)+\dfrac{w}{w_1}a_2(b_1-a_1)+\dfrac{w}{w_2}a_1(b_2-a_2),\\ \qquad c_1c_2+\dfrac{w^2}{w_1w_2}(c_1-b_1)(c_2-b_2)-\dfrac{w}{w_1}c_2(c_1-b_1)-\dfrac{w}{w_2}c_1(c_2-b_2)]\\ \dfrac{c_1(c_2-b_2)w_1+c_2(c_1-b_1)w_2-\sqrt{\{c_1(c_2-b_2)w_1+c_2(c_1-b_1)w_2\}^2-4w_1w_2(c_1-b_1)(c_2-b_2)(c_1c_2-x)}}{2(c_1-b_1)(c_2-b_2)}\\ \text{if } x\in[c_1c_2+\dfrac{w^2}{w_1w_2}(c_1-b_1)(c_2-b_2)-\dfrac{w}{w_1}c_2(c_1-b_1)-\dfrac{w}{w_2}c_1(c_2-b_2),c_1c_2]\end{cases}$$

This is obviously a MF of trapezoidal type GFN with height *w* at

$$\left[a_1a_2+\frac{w^2}{w_1w_2}(b_1-a_1)(b_2-a_2)+\frac{w}{w_1}a_2(b_1-a_1)+\frac{w}{w_2}a_1(b_2-a_2),c_1c_2+\frac{w^2}{w_1w_2}(c_1-b_1)(c_2-b_2)-\frac{w}{w_1}c_2(c_1-b_1)-\frac{w}{w_2}c_1(c_2-b_2)\right]$$ and range

is $[a_1a_2,c_1c_2]$.

Hence proved the theorem

3.4. Theorem: *Division of Triangular GFNs Produces a Trapezoidal Type GFN*

Proof: To divide triangular GFNs *A* and *B,* it is needed to divide the α-cut of *A* and *B* using the method of interval arithmetic.

$$\alpha_{A}/\alpha_{B}=\left[\frac{\frac{\alpha}{w_1}(b_1-a_1)+a_1}{\left\{c_2-\frac{\alpha}{w_2}(c_2-b_2)\right\}},\frac{c_1-\frac{\alpha}{w_1}(c_1-b_1)}{\left\{\frac{\alpha}{w_2}(b_2-a_2)+a_2\right\}}\right] \quad \textbf{(4)}$$

where $w=\min(w_1,w_2)$ and $\alpha\in[0,w]$.

To evaluate the MF $\mu_{A/B}(x)$ we need to equate both the first and second component of (4) to *x,* which gives

$$x=\frac{\frac{\alpha}{w_1}(b_1-a_1)+a_1}{\left\{c_2-\frac{\alpha}{w_2}(c_2-b_2)\right\}} \text{ and } x=\frac{c_1-\frac{\alpha}{w_2}(c_1-b_1)}{\left\{\frac{\alpha}{w_2}(b_2-a_2)+a_2\right\}}$$

Then, expressing α in terms of x and then setting $\alpha \geq 0$ and $\alpha \leq w$ in (4) we obtain α together with the domain of x

i.e., $$\alpha = \frac{c_2 x - a_1}{\left\{\frac{(b_1 - a_1)}{w_1} + \frac{(c_2 - b_2)x}{w_2}\right\}},\ x \in \left[a_1/c_2, \frac{\frac{w}{w_1}(b_1 - a_1) + a_1}{c_2 - \frac{w}{w_2}(c_2 - b_2)}\right]$$ and

$$\alpha = \frac{c_1 - a_2 x}{\left\{\frac{(c_1 - b_1)}{w_1} + \frac{(b_2 - a_2)x}{w_2}\right\}},\ x \in \left[\frac{c_1 - \frac{w}{w_1}(c_1 - b_1)}{a_2 + \frac{w}{w_2}(b_2 - a_2)}, c_1/a_2\right]$$

The MF of the output trapezoidal GFN is obtained as

$$\mu_{A/B}(x) = \begin{cases} \dfrac{c_2 x - a_1}{\left\{\frac{(b_1 - a_1)}{w_1} + \frac{(c_2 - b_2)x}{w_2}\right\}}, & x \in \left[a_1/c_2, \dfrac{\frac{w}{w_1}(b_1 - a_1) + a_1}{c_2 - \frac{w}{w_2}(c_2 - b_2)}\right] \\ w, & x \in \left[\dfrac{\frac{w}{w_1}(b_1 - a_1) + a_1}{c_2 - \frac{w}{w_2}(c_2 - b_2)}, \dfrac{c_1 - \frac{w}{w_1}(c_1 - b_1)}{a_2 + \frac{w}{w_2}(b_2 - a_2)}\right] \\ \dfrac{c_1 - a_2 x}{\left\{\frac{(c_1 - b_1)}{w_1} + \frac{(b_2 - a_2)x}{w_2}\right\}}, & x \in \left[\dfrac{c_1 - \frac{w}{w_1}(c_1 - b_1)}{a_2 + \frac{w}{w_2}(b_2 - a_2)}, c_1/a_2\right] \end{cases}$$

Obviously this is a MF of a trapezoidal type GFN with height w at

$$\left[\frac{\frac{w}{w_1}(b_1 - a_1) + a_1}{c_2 - \frac{w}{w_2}(c_2 - b_2)}, \frac{c_1 - \frac{w}{w_1}(c_1 - b_1)}{a_2 + \frac{w}{w_2}(b_2 - a_2)}\right]$$ and range is $\left[a_1/c_2, c_1/a_2\right]$.

Hence proved the theorem

4. A NORMALIZED APPROACH

To conquer the difficulty of computation (arithmetic operation), Dutta [5] adopted traditional computational loom (α-cut technique). For which first normalized the specified GFNs and carry out (computations) arithmetic operations. Then the consequential fuzzy number obtained is a normal fuzzy number. To get resulting GFN, truncated the resulting normal fuzzy number at the least height of the given GFNs [5].

For example, suppose A=[a_1,b_1,c_1;w_1] and B=[a_2,b_2,c_2;w_2] are triangular GFNs. To carry out arithmetic between *A* and *B*, first simplified both the GFNs and then execute arithmetic operations between the normalized fuzzy numbers [5]. P=[p,q,r], is the output fuzzy number. To obtain exact output trapezoidal GFN [5], truncated the normal fuzzy number *P at w=min (w1,w2)*. After that the MFs of the output trapezoidal GFNs are obtained as follows:

$$\mu_{A+B}(x)=\begin{cases}\dfrac{x-(a_1+a_2)}{(b_1+b_2)-(a_1+a_2)}, x\in[(a_1+a_2), w\{(b_1+b_2)-(a_1+a_2)\}+(a_1+a_2)]\\ w, \quad x\in[w\{(b_1+b_2)-(a_1+a_2)\}+(a_1+a_2), w\{(c_1+c_2)-(b_1+b_2)\}+(c_1+c_2)]\\ \dfrac{(c_1+c_2)-x}{(c_1+c_2)-(b_1+b_2)}, x\in[(c_1+c_2)-w\{(c_1+c_2)-(b_1+b_2)\}, (c_1+c_2)]\end{cases}$$

$$\mu_{A-B}(x)=\begin{cases}\dfrac{x-(a_1-c_2)}{(b_1-b_2)-(a_1-c_2)}, x\in[(a_1-c_2), w\{(b_1-b_2)-(a_1-c_2)\}+(a_1-c_2)]\\ w, \quad x\in[w\{(b_1-b_2)-(a_1-c_2)\}+(a_1-c_2),(c_1-a_2)-w\{(c_1-a_2)-(b_1-b_2)\}]\\ \dfrac{(c_1-a_2)-x}{(c_1-a_2)-(b_1-b_2)}, x\in[(c_1-a_2)-w\{(c_1-a_2)-(b_1-b_2)\},(c_1-a_2)]\end{cases}$$

$$\mu_{AB}(x)=\begin{cases}\dfrac{-\{a_2(b_1-a_1)+a_1(b_2-a_2)\}+\sqrt{\{a_2(b_1-a_1)+a_1(b_2-a_2)\}^2-4(b_1-a_1)(b_2-a_2)(a_1a_2-x)}}{2(b_1-a_1)(b_2-a_2)},\\ \text{if } x\in[a_1a_2,(b_1-a_1)(b_2-a_2)w^2+\{a_2(b_1-a_1)+a_1(b_2-a_2)\}w+a_1a_2]\\ w, x\in[(b_1-a_1)(b_2-a_2)w^2+\{a_2(b_1-a_1)+a_1(b_2-a_2)\}w+a_1a_2, c_1c_2,(c_1-b_1)(c_2-b_2)w^2-\{c_1(c_2-b_2)+c_2(c_1-b_1)\}w+c_1c_2]\\ \dfrac{c_1(c_2-b_2)+c_2(c_1-b_1)-\sqrt{\{c_1(c_2-b_2)+c_2(c_1-b_1)\}^2-4(c_1-b_1)(c_2-b_2)(c_1c_2-x)}}{2(c_1-b_1)(c_2-b_2)},\\ \text{if } x\in[(c_1-b_1)(c_2-b_2)w^2-\{c_1(c_2-b_2)+c_2(c_1-b_1)\}w+c_1c_2, c_1c_2]\end{cases}$$

$$\mu_{A/B}(x)=\begin{cases}\dfrac{c_2x-a_1}{x(c_2-b_2)+(b_1-a_1)}, x\in\left[a_1/c_2, \dfrac{(b_1-a_1)w+a_1}{c_2-(c_2-b_2)w}\right]\\ w, \quad x\in\left[\dfrac{(b_1-a_1)w+a_1}{c_2-(c_2-b_2)w}, \dfrac{c_1-(c_1-b_1)w}{(b_2-a_2)w+a_2}\right]\\ \dfrac{c_1-a_2x}{(b_2-a_2)x+(c_1-b_1)}, x\in\left[\dfrac{c_1-(c_1-b_1)w}{(b_2-a_2)w+a_2}, c_1/a_2\right]\end{cases}$$

5. NUMERICAL EXAMPLES

Let *A* and *B* be triangular GFNs whose MF are given as

$$\mu_A(x)=\begin{cases}0.8\dfrac{x-20}{5}, x\in[20,25]\\ 0.8\dfrac{30-x}{5}, x\in[25,30]\end{cases} \quad \text{and} \quad \mu_B(x)=\begin{cases}0.7\dfrac{x-4}{2}, \quad x\in[4,6]\\ 0.7\dfrac{8-x}{2}, \quad x\in[6,8]\end{cases}.$$

To evaluate the MFs of the trapezoidal GFNs *A+B* using approach [5], we first sum up α-cut of the triangular GFNs *A* and *B*.

α-cuts of A and B are ${}^{\alpha}A=\left[20+\frac{5}{0.8}\alpha,30-\frac{5}{0.8}\alpha\right]$ and ${}^{\alpha}B=\left[4+\frac{2}{0.7}\alpha,8-\frac{2}{0.7}\alpha\right]$ respectively.

Now,

$$ {}^{\alpha}A+{}^{\alpha}B=\left[20+\frac{5}{0.8}\alpha,30-\frac{5}{0.8}\alpha\right]+\left[4+\frac{2}{0.7}\alpha,8-\frac{2}{0.7}\alpha\right] \quad \textbf{(5)} $$

$$ =\left[20+\frac{5}{0.8}\alpha+4+\frac{2}{0.7}\alpha,30-\frac{5}{0.8}\alpha+8-\frac{2}{0.7}\alpha\right] $$

$$ =\left[24+\left(\frac{5}{0.8}+\frac{2}{0.7}\right)\alpha,38-\left(\frac{5}{0.8}+\frac{2}{0.7}\right)\alpha\right] $$

Equating both the first and second component of (5) to x and expressing α in terms of x, we have

$$ \alpha=0.7\frac{x-24}{6.375}\text{ and }\alpha=0.7\frac{38-x}{6.375}. $$

Putting $\alpha \geq 0$ and $\alpha \leq 0.7$ in (5) we obtain α together with the domain of x,

$x\in[24,30.375]$ for $\alpha=0.7\frac{x-24}{6.375}$ and $x\in[31.625,38]$ for $\alpha=0.7\frac{38-x}{6.375}$ while

$x\in[30.375,31.625]$ for $\alpha=0.7$.

Thus the resultant MF of the trapezoidal GFN is

$$ \mu_{A+B}(x)=\begin{cases} 0.7\dfrac{x-24}{6.375}, x\in[24,30.375] \\ \quad 0.7,\ x\in[30.375,31.625] \\ 0.7\dfrac{38-x}{6.375}, x\in[31.625,38] \end{cases} $$

On the other hand, to employ normalized approach, first we normalize the triangular GFN A and B by dividing the MFs by 0.8 and 0.7 respectively and which are

$$ \mu_A(x)=\begin{cases} \dfrac{x-20}{5}, x\in[20,25] \\ \dfrac{30-x}{5}, x\in[25,30] \end{cases} \quad \text{and} \quad \mu_B(x)=\begin{cases} \dfrac{x-4}{2}, & x\in[4,6] \\ \dfrac{8-x}{2}, & x\in[6,8] \end{cases}. $$

Using approach [35] of arithmetic of normal fuzzy numbers, we have the following resultant normal fuzzy number

$$\mu_{A+B}(x) = \begin{cases} \dfrac{x-31}{7}, & x \in [24,31] \\ \dfrac{38-x}{7}, & x \in [31,38] \end{cases}$$

Discretizing the fuzzy number at minimum height 0.7, we have

$$\mu_{A+B}(x) = \begin{cases} \dfrac{x-24}{7}, & x \in [24,28.9] \\ 0.7, & x \in [28.9,33.1] \\ \dfrac{38-x}{7}, & x \in [33.1,38] \end{cases}$$

Similarly, the other consequential trapezoidal GFNs can be evaluated using approach [5] and normalized approach which are respectively given in the following Table (**1**).

Table 1. Resulting GFNs obtained using approach [5] and Normalized Approach.

Arithmetic Operations	Membership Functions	
	Approach [5]	Normalized Approach [5]
Subtraction	$\mu_{A-B}(x) = \begin{cases} 0.7\dfrac{x-12}{6.375}, x \in [12, 18.375] \\ 0.7, \ x \in [18.375, 19.625] \\ 0.7\dfrac{26-x}{6.375}, x \in [19.625, 26] \end{cases}$	$\mu_{A-B}(x) = \begin{cases} \dfrac{x-12}{7}, x \in [12, 16.9] \\ 0.7, \ x \in [16.9, 21.1] \\ \dfrac{26-x}{7}, x \in [21.1, 26] \end{cases}$
Multiplication	$\mu_{A\times B}(x) = \begin{cases} 0.7\dfrac{-46+\sqrt{324+22.4x}}{14}, x \in [80, 146.25] \\ 0.7, \ x \in [146.25, 153.75] \\ 0.7\dfrac{76-\sqrt{400+22.4x}}{14}, \ x \in [153.75, 240] \end{cases}$	$\mu_{A\times B}(x) = \begin{cases} \dfrac{-60+\sqrt{400+40x}}{20}, x \in [80, 89.1] \\ 0.7, \ x \in [89.1, 174.9] \\ \dfrac{100-\sqrt{400+40x}}{20}, \ x \in [174.9, 240] \end{cases}$
Division	$\mu_{A/B}(x) = \begin{cases} 0.7\dfrac{8x-20}{2x+4.375}, x \in \left[\dfrac{20}{8}, \dfrac{24.725}{6}\right] \\ 0.7, \ x \in \left[\dfrac{24.725}{6}, \dfrac{25.625}{6}\right] \\ 0.7\dfrac{30-4x}{2x+4.375}, x \in \left[\dfrac{25.625}{6}, \dfrac{30}{4}\right] \end{cases}$	$\mu_{A/B}(x) = \begin{cases} \dfrac{8x-20}{4x+5}, x \in \left[\dfrac{20}{8}, \dfrac{23.5}{6.6}\right] \\ 0.7, \ x \in \left[\dfrac{23.5}{6.6}, \dfrac{26.5}{5.4}\right] \\ \dfrac{30-4x}{2x+5}, x \in \left[\dfrac{26.5}{5.4}, \dfrac{30}{4}\right] \end{cases}$

6. RANKING OF TRAPEZOIDAL GFNs

Let A = [a, b, c, d; w] be a trapezoidal GFN and $[L_{\overline{x_\alpha}}, R_{\overline{x_\alpha}}]$ be the α α-cut of *A*. After that the value of *A* can be defined as:

$$V_\mu(A) = \int_0^w \frac{L_{\overline{x_\alpha}} + R_{\overline{x_\alpha}}}{2} f(\alpha) d\alpha \text{ and}$$

Where $f(\alpha)$ is a non-negative and non-decreasing function on $[0, w]$ with $f(0) = 0$ and $\int_0^w f(\alpha)d\alpha = w$.

.

Here, we also choose $f(\alpha) = \frac{2\alpha}{w}$, $\alpha \in [0, w]$.

Thus, the value of the trapezoidal GFN *A* is evaluated as:

$$V_{\mu}(A) = \frac{a + d + 2(b + c)}{6} w \quad \textbf{(6)}$$

7. METHODOLOGY

As medical documents, information even medical decision making process fouled with vagueness/imprecision/uncertainty. Again, knowledge base correlating the symptom-disease relationship comprises of ambiguity, imprecision, vagueness and uncertainty in medical decision making process. Consequently fuzzy set theory became popular aid in medical decision making process. Generally, type-I fuzzy set, IFSs, IVFSs and PFSs are used for medical decision making. However, these tools provide counterintuitive results. For example,

De and colleagues first deliberated medical diagnosis with the notion of IFS [20]. Then, a further study in medical diagnosis has been done using the same IFS medical data and criticized the earlier study having with different results [21]. Numerous studies have been made on medical diagnosis problem for the same IFSs data and obtained different results from the existing studies ([22], [36 - 39]). Later, a comparative study has been made *via* the existing distance measures for the same medical diagnosis problem and obtained different results except with the second study in this sequence [28]. It is seen that for the same diagnosis problem addressed by various researchers using different approaches lead to different chaotic results. Similar types of counter intuitive results also occur for other uncertainty modeling tools.

Therefore, GFNs can be employed as the representation of the state and symptoms of the patient that can be only known by medical experts with a very partial grade of precision. Therefore, triangular GFNs have been taken in this study as the GFN has the skill to deal with vague data/information in a more stretchy way because of the parameter height that characterize the degree of confidence of opinions of decision maker's in a very specific deportment. Advantages of GFNs over the normal fuzzy numbers have been demonstrated in the case study section.

The methodology for medical decision making can be summarized as suppose *P*, *S* and *D* are the sets of patients, symptoms and diseases respectively. To identify the patients and their corresponding disease that they are suffering from, it is utmost important to determine the symptoms and accordingly need to formulate the medical knowledge *via* fuzzy relations say, symptoms-diseases (***Q***) and patient-symptoms fuzzy (***R***) with the help of doctors/experts/physicians.

To gauge the patients-diseases relation (***T***) the following composition can be adopted

$$\mu_T(p_i, d_k) = \sum_{s \in S} \left[\mu_Q(p_i, s) . \mu_R(s, d_k) \right] \quad \textbf{(7)}$$

The entries of the patients-diseases relation (***T)*** will be Trapezoidal GFNs. Then, ranking of the Trapezoidal GFNs can be computed using value. It should be noted that the maximum value in each row indicate that the patient is likely to have the disease. If doctor/expert/physician does not satisfy with the result, then modification will be done on symptoms-diseases (***Q***) and compute from the beginning. A flowchart has been depicted in Fig. (**1**) in appendix section for easy understanding of the readers.

7.1. Case Study

In this segment, a hypothetic case study for medical decision making has been carried out using arithmetic of triangular GFNs *via* value.

Suppose *Pranjal, Rupjit*,and *Soumendra* are admitted in Assam Medical College hospital at Dibruagrh, India. The symptoms of the patients are found to be *temperature, headache, cough* and *stomach pain*. The diseases for these symptoms found by doctor/expert/physician are *Viral Fever, Typhoid* and *Malaria.* Require to identify which patient is exactly suffering from which disease.

To perform medical diagnosis Mathematically, let *P* = {*Pranjal, Rupjit, Soumendra*}, be the set of patients; *S* = {*temperature, headache, cough*, *stomach pain*} be the set of symptoms and *D* = {*Viral Fever, Typhoid, Malaria*} be the set of diseases respectively.

7.1.1. Scenario-I

The medical knowledge *via* fuzzy relations say, symptoms-diseases (***Q***) and patient-symptoms fuzzy (***R***) have been compiled from Çelik and Yamakand (2013) and respectively depicted in Tables (**2 and 3**). Here, heights of all the fuzzy numbers are considered equal to unit (i.e., *w=1*).

Table 2. Patient-symptom relation.

R	*Temperature*	*Headache*	*Cough*	*Stomach Problem*
Pranjal	[6, 7, 8]	[2, 3, 4]	[4, 5, 6]	[1, 2, 3]
Rupjit	[5, 6, 7]	[1, 2, 3]	[2, 3, 4]	[4, 5, 6]
Soumendra	[2, 3, 4]	[4, 5, 6]	[2, 3, 4]	[5, 6, 7]

Table 3. Symptom-disease relation.

Q	*Viral Fever*	*Typhoid*	*Malaria*
Temperature	[8, 9, 10]	[4, 5, 6]	[0,1,2]
Headache	[2, 3, 4]	[4, 5, 6]	[4, 5, 6]
Cough	[4, 5, 6]	[1, 2, 3]	[4, 5, 6]
Stomach problem	[1, 2, 3]	[7, 8, 9]	[7, 8, 9]

Using the proposed composition of fuzzy relation (7), the following patients-diseases relation Table (**4**) is obtained.

Table 4. The resultant patients-diseases relation.

	Viral Fever	*Typhoid*	*Malaria*
Pranjal	[69,101,141]	[43, 76,117]	[31,63,103]
Rupjit	[54,85,124]	[54,86,126]	[40,71,110]
Soumendra	[37,69,109]	[61,94,135]	[59,91,131]

As all entries here are triangular fuzzy numbers with w=1

Thus, the value of the triangular fuzzy number *A = [a,b,c]* will be

$$V_{\mu}(A) = \frac{a+c+4b}{6} \tag{8}$$

Using equation (8) we have the following resultant value of each entries of patients-diseases relation Table (**5**).

Table 5. The value of resultant patients-diseases relation.

	Viral Fever	*Typhoid*	*Malaria*
Pranjal	**102.3333**	77.3333	64.3333
Rupjit	86.3333	**87.3333**	72.3333
Soumendra	70.3333	**95.3333**	92.3333

The maximum value in each row indicates that the patient is likely to have the disease. Thus, we can conclude that *Pranjal* is suffering from *viral fever* while *Rupjit* and *Soumendr* both are suffering from *Typhoid*. But, proper analysis of the medical data set it could be found that *Rupjit* suffers from *viral fever*. Since for the patient *Rupjit* the maximum value among the symptoms occurs for *temperature* while for the symptom *temperature* the maximum value among the diseases occurs for *viral fever* and hence it general intuition that *Rupjit* is suffering from *viral fever*. However, analytical data corroborates the same fact about patients *Pranjal* and *Soumendra*.

7.1.2. Scenario-2

It is observed that due to consideration of heights of fuzzy numbers lead to illogical results. Therefore, here details relationships between symptoms and diseases are studies and accordingly degree of relationships have been assigned in terms of Triangular GFNs.

A *viral fever* is characterized by a *very high fever* that is often remittent where the fever abates for a little while and then shoots up once again. Hence, *temperature* is much more than 104^{o}F. On the other hand, for *typhoid* temperature is more than 104^{o}F. Again, for *malariatemperature* is high i.e., it lies between 103^{o}F-104^{o}F. Thus, keeping all these in mind, the degree of relationships (*temperature, viral fever), (temperature, typhoid)* and *(temperature, malaria)* have been precisely assigned in a very specific comportment in terms of triangular GFNs. It is also well known that *headache* is a common symptom for the three diseases and in a similar fashion degree of relationships of *headache* with the diseases have been assigned. *Cough* is a major symptom of *viral fever* while it has less relationship with *typhoid* and *malaria*. The symptom *stomach pain* has less relationship with the diseases under study.

A detail the data set regarding symptoms-diseases (***Q***) and patient-symptoms fuzzy (***R***) have been depicted in Table (**6 and 7**).

Table 6. Patient-symptom relation.

R	*Temperature*	*Headache*	*Cough*	*Stomach Problem*
Pranjal	[6,7,8;0.8]	[2,3,4;0.4]	[4,5,6;0.6]	[1,2,3;0.2]
Rupjit	[5,6,7;0.7]	[1,2,3;0.3]	[2,3,4;0.6]	[4,5,6;0.3]
Soumendra	[2,3,4;0.4]	[4,5,6;0.7]	[2,3,4;0.2]	[5,6,7;0.8]

Table 7. Symptom-disease relation.

Q	*Viral Fever*	*Typhoid*	*Malaria*
Temperature	[8,9,10;0.9]	[4,5,6;0.8]	[0,1,2;0.7]
Headache	[2,3,4;0.8]	[4,5,6;0.7]	[4,5,6;0.6]
Cough	[4,5,6;0.8]	[1,2,3;0.2]	[4,5,6;0.5]
Stomach problem	[1,2,3;0.3]	[7,8,9;0.4]	[7,8,9;0.3]

Using the composition of relations (7), patient-disease relation (***T***) is evaluated and depicted in Table (**8**).

Table 8. The resultant trapezoidal GFNs of the patient-disease relation.

T	*Viral Fever*	*Typhoid*	*Malaria*
Pranjal	[69, 78.7639,126.1528, 141;0.2]	[43, 60.9435, 92.8958,117;0.2]	[31, 47.0190, 81.5190, 103; 0.2]
Rupjit	[54, 70.9256 100.4613,124;0.3]	[54, 69.2738, 104.6071,126;0.2]	[40, 62.8265, 80.2265,110;0.3]
Soumendra	[37, 51.6984, 88.6508,109;0.2]	[61, 74.3673, 116.1531,135;0.2]	[59, 72.7357, 111.4310,131;0.2]

Then, each entry of the patient-disease relation (***T***) is defuzzified using value and is given in Table (**9**). It should be noted that maximum value in each row indicates that the patient is likely to have the disease.

Table 9. The value of the trapezoidal GFNs of the patient-disease relation.

T	*Viral Fever*	*Typhoid*	*Malaria*
Pranjal	**20.6611**	15.5893	13.0359
Rupjit	**26.0387**	17.5921	21.8053
Soumendra	14.2233	**19.2347**	18.6111

8. RESULTS AND DISCUSSIONS

Normally, a disease is pigeonholed by symptoms which encourage the patient to approach to a medical expert. A collection of experimental assessments are commenced to recognize the happening of a disease. In the area of medical decision making, plenty of variables are there which persuade the decision making process and accordingly, distinguish the opinions of the doctors/physicians. Due to so many factors to analyze for the finding of the disease of a patient makes the medical expert's job complicated.

As most of the medical decision making setback involves dealing with vagueness and uncertainties and it is needed to contain of all the information into investigation. In the case study, two scenarios are taken into considerations. In scenario-I, TFNs are used to characterize uncertainty and medical decision making has been carried out in this study. To organize the data base, for the patients *Pranjal Rupjit* and *Soumendra*; four symptoms *Temperature*, *Headache*, *Cough* and *Stomachpain* and three diseases, *Viral Fever*, *Typhoid* and *Malaria* are considered. Finally, patients-diseases relation has been computed. It is found that *Pranjal* suffers from *viral fever* while *Rupjit* and *Soumendra* both are suffering from *Typhoid*. But, which is irrational for the patient *Rupjit*. That is, it is clear from the data set that *Rupjit* is suffering from *viral fever* but the approach gives that it is typhoid. Therefore, in scenario-II, triangular GFNs are assigning instead of TFNs to characterize uncertainty and medical decision making has been performed here using proposed arithmetic on GFNs as composition of relation and finally values have been evaluated using equation (2). It is found that *Pranjal* and *Rupjit* are suffering from *viral fever* while *Soumendra* is suffers from typhoid which is more realistic.

CONCLUSION

GFN, a direct generalization of FST, has received more and more attention in the field of decision making, science and technology, risk assessment, etc. because of the height of GFN to characterize the grade of buoyancy of judgments of decision takers in a very specific comportment. In this editorial the concept of arithmetic on triangular GFN proposed by Dutta [5] has been reviewed which can successfully covenant with vagueness, imprecision, uncertainty with vagueness, imprecision, uncertainty and also applied in medical decision making for first time. For this purpose, this document presents an appropriate algorithm to perform medical decision making in which arithmetic on triangular GFNs have been used as composition of fuzzy relations. A case study has been performed *via* two scenarios. The advantage of the proposed approach has been observed from the cast study where it is found that existing approach provides illogical results while proposed one gives a rational result. Hence, it is established that proposed approach is efficient, simple, logical, technically sound and general enough for implementation not only in medical decision making but also in other allied fields.

In some situations, existence of different types of uncertainties leads to imprecise fuzzy set membership functions and as a consequence ordinary FST is inappropriate to model such types of uncertainties. IVFS and type-II fuzzy sets are capable of modeling uncertainties, therefore as an extension of this editorial generalized IVFSs and type-II fuzzy sets will be studied along with attempt will be made to employ in medical investigations.

CONSENT FOR PUBLICATION

Not applicable.

CONFLICT OF INTEREST

The authors declare no conflict of interest, financial or otherwise.

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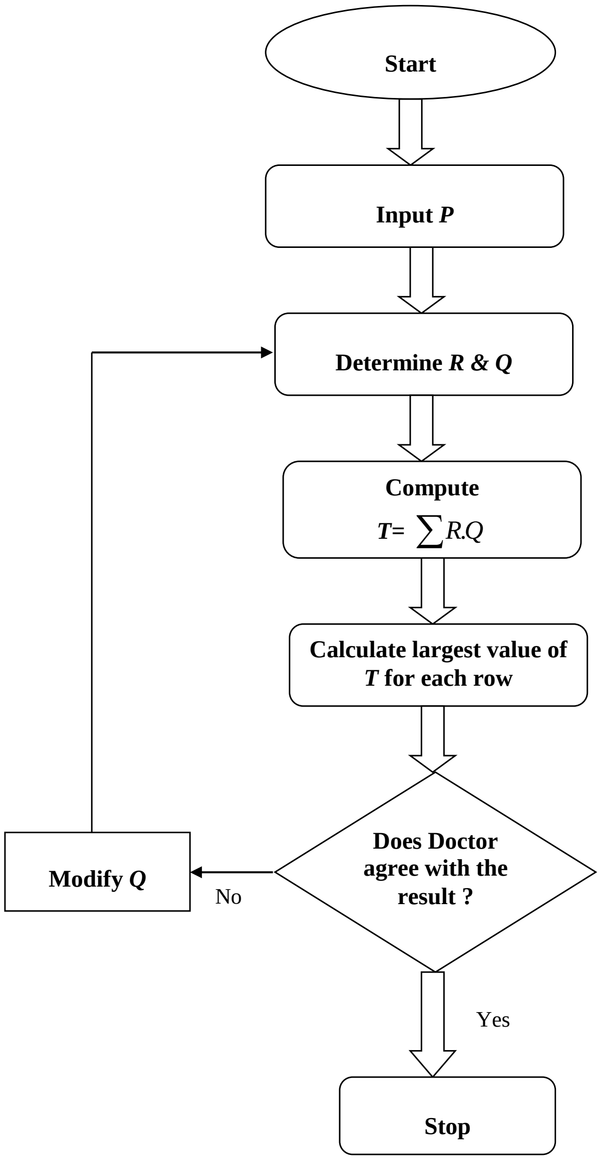
APPENDIX



Appendix (1). Flowchart for the medical diagnosis

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