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Exploring Stable Population Concepts from the Perspective of Cohort Change Ratios

David A. Swanson[1,*] and Lucky M. Tedrow[2]

[1]*Department of Sociology, University of California Riverside, Riverside, CA 92521 USA*

[2]*Department of Sociology, Western Washington University, Bellingham, WA 98225 USA*

Abstract. Cohort Change Ratios (CCRs) have a long history of use in demography. In spite of their history of use, they appear, however, to have been overlooked in regard to the major canon of formal demography, stable population theory. In this paper, CCRs are explored as a tool for examining the idea of a stable population. In comparing the approach using CCRs to the traditional analytical approach, benefits and drawbacks are noted. The paper also introduces an Index of Stability, which is used in a regression model to estimate the number of years before the population in question becomes (approximately) stable. The regression model works reasonably well and, as such, provides something not available in the traditional analytical approach, which is an estimate of the time to (approximate) stability for a given population.

Keywords: Stable Population Index, Hamilton-Perry Method, Numerical Solution, Demographic Theory.

1. INTRODUCTION

What is a stable population? It is a population with an invariable relative age structure and a constant rate of growth. That is, the proportion of people in each age group remains constant over time (Swanson and Stephan, 2004: 775). When the absolute number of people in each group is also constant over time, a stationary population exists, which is a special case of a stable population in which the growth rate is zero [1].

An important feature of the stable population model is that over time a population "forgets" its past age distribution when it is subject to constant rates regarding the components of change [2, 3]. This property is known as ergodicity. It implies that if one applies a constant set of fertility, mortality, and migration rates to two arbitrarily chosen age distribution, no matter how different, the two age distributions will ultimately converge to the same age distribution.

Alfred J. Lotka is generally credited with formulating the idea of a stable population and exploring many of its important features, including the finding that in the absence of migration, a population subject to constant fertility and mortality rates would eventually have a constant rate of natural increase [4, 5]. Continuing the analytical tradition established by Lotka, many researchers have examined the idea of a stable population and refined its underlying theory and extended its applications [2, 3, 6-14]. Most of this research has, however, been confined to examining a population not affected by migration. However, his is an un-necessarily restrictive assumption [3]. Nonetheless, other than the simple migration rates employed by Rogers [13, 15] and subsequent investigations of more refined model migration schedules [16], this restriction appears to remain a governing force in the examination of stable population ideas.

Another "unnecessarily restrictive" assumption that has governed much of the work on stable populations is defined by the so-called "two-sex" problem [12, 17, 18]. In this problem (which evidently stems from Lotka's 1907 formulation of a stable population), only one sex (virtually always women) was examined in the context of a stable population because of problems reconciling the numbers of births resulting from including both sexes. However, as Preston *et al.* show a "female-dominant" approach to fertility offers a convenient way around this problem [12].

The un-necessarily restrictive assumptions regarding migration and the inclusion of both sexes serve as primary motivations for the current paper. A secondary motivation is incorporating migration into examinations of the idea of a stable population in an easy-to-follow manner. To this end, the concept of a stable population is examined from the perspective of "Cohort Change Ratios."

2. COHORT CHANGE RATIOS

What are Cohort Change Ratios (CCRs)? They have a long history of use in demography. Under the rubric of "Census Survival Ratios," they have been used to estimate adult mortality [19, 20] and under the rubric of the "Hamilton-Perry" method, they are used to make population projections [21-23]. However, they appear to have been overlooked in regard to examining the concept of a stable population.

In this paper, CCRs are used to project a population to stability. Thus, the general ideas associated with CCRs are described in conjunction with the Hamilton-Perry method. The Hamilton-Perry Method is a variant of the cohort-component method that has far less intensive input data re-

*Address correspondence to this author at the Department of Sociology, University of California Riverside, Riverside, CA 92521 USA;
Tel: + 1 951 827 4373; Fax: +1 951 827 3330; E-mail: dswanson@ucr.edu

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quirements. Instead of mortality, fertility, migration, and total population data, which are required by the full-blown cohort-component method, the Hamilton-Perry method requires data only from the two most recent censuses [21-23]. The Hamilton-Perry method moves a population by age (and sex) from time t to time t+k using CCRs computed from data in the two most recent censuses. It consists of two steps. The first uses existing data to develop CCRs and the second applies the CCRs to the cohorts of the launch year population to move them into the future. The second step can be repeated infinitely, with the projected population serving as the launch population for the next projection cycle. The formula for the first step, the development of a CCR is:

$_{n}CCR_{x,i} = {}_{n}P_{x,i,t} / {}_{n}P_{x-k,i,t-k}$

where

$_{n}P_{x,i,t}$ is the population aged x to x+n in area i at the most recent census (t),

$_{n}P_{x-k,i,t-k}$ is the population aged x-k to x-k+n in area i at the 2nd most recent

census (t-k),

k is the number of years between the most recent census at time t

for area i and the one preceding it for area i at time t-k.

The basic formula for the second step, moving the cohorts of a population into the future is:

$_{n}P_{x+k,i,t+k} = ({}_{n}CCR_{x,i})*({}_{n}P_{x,i,t})$

where

$_{n}P_{x+k,i,t+k}$ is the population aged x+k to x+k+n in area i at time t+k

$_{n}CCR_{x,i} = {}_{n}P_{x,i,t} / {}_{n}P_{x-k,i,t-k}$

$_{n}P_{x,i,t}$ is the population aged x to x+n in area i at the most recent census (t),

k is the number of years between the most recent census at time t

for area i and the one preceding it for area i at time t-k.

Given the nature of the CCRs, 10-14 is the youngest age group for which projections can be made if there are 10 years between censuses. To project the population aged 0-4 and 5-9, one can use the Child Woman Ratio (CWR), or more generally a "Child Adult Ratio" (CAR). It does not require any data beyond what is available in the decennial census. For projecting the population aged 0-4, CAR is defined as the population aged 0-4 divided by the population aged 15-44. For projecting the population aged 5-9, CAR is defined as the population aged 5-9 divided by the population aged 20-49. Here are the CAR equations for projecting the population aged 0-4 and 5-9, respectively.

Population 0-4: $_{5}P_{0,t+k} = ({}_{5}P_{0,t} / {}_{30}P_{15,t}) * ({}_{30}P_{15,t+k})$

Population 5-9: $_{5}P_{5,t+k} = ({}_{5}P_{5,t} / {}_{30}P_{20,t}) * ({}_{30}P_{20,t+k})$

where

P = population,

t is the year of the most recent census

and t+k is the estimation year

There are other "adult" age groups that could be used to define CAR. The definitions shown in the two preceding equations are designed for a population in which fertility is at or below replacement, (i.e., the TFR is less than 2.1 or so), which correlates with the fact that first births tend to be postponed.

Projections of the oldest open-ended age group differ slightly from the CCR projections for the age groups beyond age 10 up to the oldest open-ended age group. If, for example, the final closed age group is 80-84, with 85+ as the terminal open-ended age group, then calculations for the $_{\infty}CCR_{85,i,t}$ require the summation of the three oldest age groups to get the population age 75+ at time t-k:

$_{\infty}CCR_{75,i,t} = {}_{\infty}P_{85,i,t} / {}_{\infty}P_{75,i,t-k}$

The formula for projecting the population 85+ of area i for the year t+k is:

$_{\infty}P_{85,imt+k} = ({}_{\infty}CCR_{75,i,t})* ({}_{\infty}P_{75,i,t})$

Table **1** provides an example of the Hamilton-Perry Method for the state of Alaska. It uses the country's 2000 census data and 2010 estimates by age to generate a 2020 population projection of the population by age. Since the population data are ten years apart for Alaska with a final open-ended age group of 85+, the conventions described above are used in terms of the CCRs, CAR, and the projection of age group 85+. Important to the subsequent discussion are the CCRs developed for the 2000-2010 period.

Table **1** shows that launching from a population of 710,231 in 2010, the Hamilton-Perry Method generates a 2020 population of 807,401. This projection corresponds to the increase in population between 2000 (626,932) and 2010 (710,231). This increase largely reflects Alaska's net in-migration and relatively young population.

Since this touches on the implicit recognition of the components of population change in the Hamilton-Perry projection for Alaska, it is worthwhile to note here the Hamilton Perry method can be described in terms of these components. That is, the Hamilton-Perry Method can be expressed in terms of the fundamental demographic equation. Since the fundamental equation is:

$P_{i,t+k} = P_{i,t} + B_i – D_i + I_i - O_i$

where

$P_{i,t}$ = Population of area i at time t (e.g., the launch date)

$P_{i,t+k}$ = Population of area i at time t+k (e.g., the projection target date)

B_i = Births in area i between time t and t+k

D_i = Deaths in area i between time t and t+k

I_i = In-migrants in area i between time t and t+k

O_i = Out-migrants in area i between time t and t+k

The first equation we showed can be expressed as

$_{n}CCR_{x,i} = {}_{n}P_{x,i,t} / {}_{n}P_{x-k,i,t-k}$

since

$_{n}CCR_{x,i} =({}_{n}P_{x-k,i,t-k} + B_i – D_i + I_i - O_i)/({}_{n}P_{x-k,i,t-k})$

Table 1. A Hamilton-Perry Population Projection for Alaska: Base Year Data (2000-2010), Launch Year(2010) and Target Year 2020)

	2000 Population	2000 Proportion by Age	2010 Population	2010 Proportion BY AGE	2000-2010 CCR	ABS Difference
Total Population: 0 to 4 years	47,591	0.0759	53,996	0.0760	0.34274	0.0001
Total Population: 5 to 9 years	53,771	0.0858	50,887	0.0716	0.37033	0.0141
Total Population: 10 to 14 years	56,661	0.0904	50,816	0.0715	1.06776	0.0188
Total Population: 15 to 19 years	50,094	0.0799	52,141	0.0734	0.96969	0.0065
Total Population: 20 to 24 years	39,892	0.0636	54,419	0.0766	0.96043	0.0130
Total Population: 25 to 29 years	42,987	0.0686	55,419	0.0780	1.10630	0.0095
Total Population: 30 to 34 years	46,486	0.0741	47,706	0.0672	1.19588	0.0070
Total Population: 35 to 39 years	55,723	0.0889	45,833	0.0645	1.06621	0.0243
Total Population: 40 to 44 years	58,326	0.0930	47,141	0.0664	1.01409	0.0267
Total Population: 45 to 49 years	53,515	0.0854	54,726	0.0771	0.98211	0.0083
Total Population: 50 to 54 years	41,437	0.0661	56,300	0.0793	0.96526	0.0132
Total Population: 55 to 59 years	27,423	0.0437	49,971	0.0704	0.93378	0.0266
Total Population: 60 to 64 years	17,327	0.0276	35,938	0.0506	0.86729	0.0230
Total Population: 65 to 69 years	12,626	0.0201	22,202	0.0313	0.80961	0.0111
Total Population: 70 to 74 years	9,881	0.0158	13,148	0.0185	0.75882	0.0028
Total Population: 75 to 79 years	6,863	0.0109	8,892	0.0125	0.70426	0.0016
Total Population: 80 to 84 years	3,695	0.0059	5,985	0.0084	0.60571	0.0025
Total Population: 85 years and over	2,634	0.0042	4,711	0.0066	0.68643	0.0024
Total Population	626,932	1.0000	710,231	1.0000		0.2115
S						0.10573

The second equation we show can be expressed as

$_nP_{x+k,i,t+k} = (_nCCR_{x,i})*(_nP_{x,i,t})$

since

$_nP_{x+k,i,t+k} = ((_nP_{x-k,i,t-k} + B_i - D_i + I_i - O_i)/(_nP_{x-k,i,t-k}))*(_nP_{x,i,t})$

Where x+k >= 10 then

$_nCCR_{x,i} = (_nP_{x-k,i,t-k} - D_i + I_i - O_i)/(_nP_{x-k,i,t-k})$

and since $N_i = I_i - O_i$

$_nCCR_{x,i} = (_nP_{x-k,i,t-k} - D_i + N_i)/(_nP_{x-k,i,t-k})$

where x+k >= 10

These equations clearly reveal that the Hamilton-Perry Method expresses the individual components of change (birth, deaths, and migration) in terms of Cohort Change Ratios and incorporates these components of change in the projections made from it.

Note that the fundamental equation can be generalized to include age groups (as well as sex, race, and ethnicity).

3. A STABLE POPULATION: THE TRADITIONAL APPROACH

Although Preston *et al*. [3] point out that the assumption of no migration is un-necessarily restrictive, stable population theory has largely been examined using this restriction. It also has largely been examined in terms of a single sex due to the so-called "two-sex" problem, which Preston *et al*. [3] also argue is un-necessarily restrictive.

The Lotka Integral Equation as given by (Preston *et al*. [3] is

$$B(t) = \int_0^t N(a,t)m(a)da + G(t)$$

where

B(t) = number of births at time t

N(a,t) = number of persons aged a at time t

m(a) = rate of bearing female children for women aged a

G(t) = births to women alive at time 0

As Preston *et al*. [3] observe, the N(a,t) function for women born after time 0 can be expressed in terms of the number of births into their cohort and the probability of surviving to age a, p(a):

$N(a,t) = B(t-a)*p(a)$,

where t>0

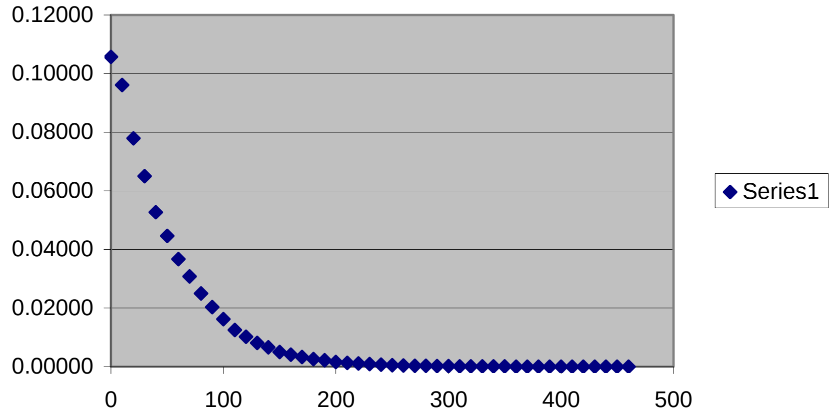

Fig. (1). Stability Index (***S***) over time (in Years) as the Alaskan Population moves to Stability.

Making this substitution into the preceding equation yields

$$B(t) = \int_0^t B(t-a)*p(a)*\ m(a)da + G(t)$$

And since the value of G(t) goes to zero over time (e.g., in about 50 years), the birth sequence can be expressed as

$$B(t) = \int_0^t B(t-a)*p(a)*\ m(a)da$$

where t > 50

The preceding Equation can be solved when an expression for B(t) is substituted into its left and right hand sides. Lotka showed that an exponential birth series would do this. Let $B(t) = B*e^{pt}$

Then

$$B*e^{pt} = \int_0^t B*e^{p(t-a)} *p(a) *m(a)da$$

where t > 50

and cancelling the common term, $B*e^{pt}$ from both sides yields

$$1 = \int_0^t B*e^{pa} *p(a)*m(a)da$$

4. A STABLE POPULATION: THE CCR APPROACH

The CCR approach simply takes the cohort change ratios found at a current point in time and holds them constant until the population reaches stability. To determine when a population has reached stability, the well-known "Index of Dissimilarity" is employed as an "Index of Stability" (***S***). The index is defined as:

$$S = 100*\{0.5* \sum \left| ({}_np_x/\textstyle\sum_n P_x)_{t+y} - ({}_np_x/\textstyle\sum_n P_x)_t \right| \}$$

where

y = number of years between census counts/projection cycles

x = age

n = width of the age group (in years)

t = year

S compares the relative age distribution at one point in time (t+y) with the relative age distribution at the preceding point in time (t) and measures the percentage that one distribution would have to be re-allocated to match the other. ***S*** ranges from 0 to 100; a score of zero means that there is no allocation error, and 100 means that the maximum allocation error exists. This can mean several things, but a common interpretation is that half of the numbers at one point in time would have to be re-allocated and half of the numbers at the preceding point in time would have to be re-allocated.

S exploits the idea that when a population is stable, the sum of the differences between the relative size of corresponding age groups at time t+y and time t is zero. Thus, at a point time when the sum of the differences across all of the corresponding age groups is zero at that point in time and the preceding point in time (or very nearly so), the population has reached stability. The advantage of using the Index of Dissimilarity as ***S*** is that it provides ***S*** with a bounded measure (between 0 and 1) and has a clear interpretation. This index could, of course, be used in conjunction with the traditional approach, but it does not appear in the literature in regard to measuring population stability. With ***S***, one has a potential tool for examining the length of time to stability for given population.

The examination of the CCR approach to the idea of a stable population starts by using the case of Alaska. The CCRs (from the 2000-2010 period) are held constant from the launch year (2010) to a year where ***S*** = 0 (relative to the preceding year in the projection cycle). This occurs at the year 2470. Table **2** displays this by showing the information at for the 2000-2010 launch period and the information at the period where stability is reached, 2370-2380. ***S*** = .10573 at the launch year of 2010; by 2380, ***S*** = .0.0000.

Fig. (**1**) provides the change in ***S*** from 2010 to 2470. As it shows, the path to stability is monotonic but not linear. It initially declines rapidly to the point where S is approximately equal to .01, but the change in ***S*** slows substantially around 2120. From there to 2470, ***S*** moves incrementally to zero.

Table 2. The Population of Alaska at Start (2000-10) and at Achieving Stability (2460-70)

	2000 Population	2000 Proportion by Age	2010 Population	2010 Proportion by Age	2000-2010 CCR	ABS Difference	Projected 2460	2460 Proportion by Age	Projected 2470	2470 Proportion by Age	ABS Difference
Total Population: 0 to 4 years	47,591	0.0759	53,996	0.0760	0.34274	0.0001	1,474,294	0.0698	1,588,233	0.0698	0.0000
Total Population: 5 to 9 years	53,771	0.0858	50,887	0.0716	0.37033	0.0141	1,679,618	0.0796	1,809,411	0.0796	0.0000
Total Population: 10 to 14 years	56,661	0.0904	50,816	0.0715	1.06776	0.0188	1,461,280	0.0692	1,574,200	0.0692	0.0000
Total Population: 15 to 19 years	50,094	0.0799	52,141	0.0734	0.96969	0.0065	1,511,861	0.0716	1,628,702	0.0716	0.0000
Total Population: 20 to 24 years	39,892	0.0636	54,419	0.0766	0.96043	0.0130	1,302,766	0.0617	1,403,460	0.0617	0.0000
Total Population: 25 to 29 years	42,987	0.0686	55,419	0.0780	1.10630	0.0095	1,552,579	0.0735	1,672,572	0.0735	0.0000
Total Population: 30 to 34 years	46,486	0.0741	47,706	0.0672	1.19588	0.0070	1,446,200	0.0685	1,557,951	0.0685	0.0000
Total Population: 35 to 39 years	55,723	0.0889	45,833	0.0645	1.06621	0.0243	1,536,636	0.0728	1,655,369	0.0728	0.0000
Total Population: 40 to 44 years	58,326	0.0930	47,141	0.0664	1.01409	0.0267	1,361,367	0.0645	1,466,578	0.0645	0.0000
Total Population: 45 to 49 years	53,515	0.0854	54,726	0.0771	0.98211	0.0083	1,400,858	0.0663	1,509,142	0.0664	0.0000
Total Population: 50 to 54 years	41,437	0.0661	56,300	0.0793	0.96526	0.0132	1,219,797	0.0578	1,314,079	0.0578	0.0000
Total Population: 55 to 59 years	27,423	0.0437	49,971	0.0704	0.93378	0.0266	1,214,262	0.0575	1,308,087	0.0575	0.0000
Total Population: 60 to 64 years	17,327	0.0276	35,938	0.0506	0.86729	0.0230	982,051	0.0465	1,057,921	0.0465	0.0000
Total Population: 65 to 69 years	12,626	0.0201	22,202	0.0313	0.80961	0.0111	912,560	0.0432	983,082	0.0432	0.0000
Total Population: 70 to 74 years	9,881	0.0158	13,148	0.0185	0.75882	0.0028	691,718	0.0328	745,196	0.0328	0.0000
Total Population: 75 to 79 years	6,863	0.0109	8,892	0.0125	0.70426	0.0016	596,563	0.0283	642,681	0.0283	0.0000
Total Population: 80 to 84 years	3,695	0.0059	5,985	0.0084	0.60571	0.0025	388,930	0.0184	418,979	0.0184	0.0000
Total Population: 85 years and over	2,634	0.0042	4,711	0.0066	0.68643	0.0024	380,140	0.0180	409,501	0.0180	0.0000
	626,932	1.0000	710,231	1.0000		0.2115	21,113,483	1	22,745,143	1.0000	0.0000
S						0.10573					0.00000

Figs. (**2** and **3**) show the age distribution of Alaska in 2010 and in 2470, when it reaches stability

As another example, consider the United States. As was the case with Alaska, the projection is launched with CCRs taken over the 2000-2010 period that are held constant from the launch year to a year where ***S*** = 0 (relative to the preceding year in the projection cycle). Stability occurs at the year 2380. Table **3** displays this by showing the information at the launch period, 2000-2010, and the information at the period where stability is reached, 2330-2340. ***S*** = 5.65% at the launch year of 2010; by 2380, ***S*** = 0.00%.

Fig. (**4**) provides the change in ***S*** from 2010 to 2340 for the United States. Unlike Alaska, the path to stability is neither monotonic nor linear: It initially increases, "bounces around" a bit, then decreases substantially before its decrease slows considerably, which starts around 2160. From 2160 to 2340, ***S*** moves incrementally to zero. Fig. (**5**) shows the graph of ln(***S***) relative to time

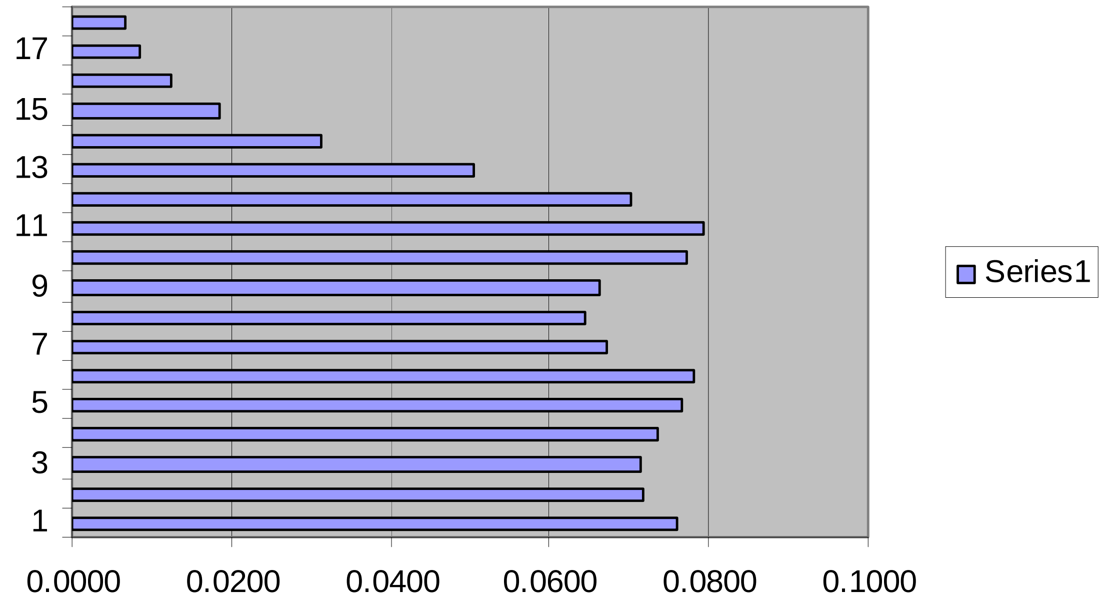

Fig. (2). Age Distribution of Alaska in 2010.

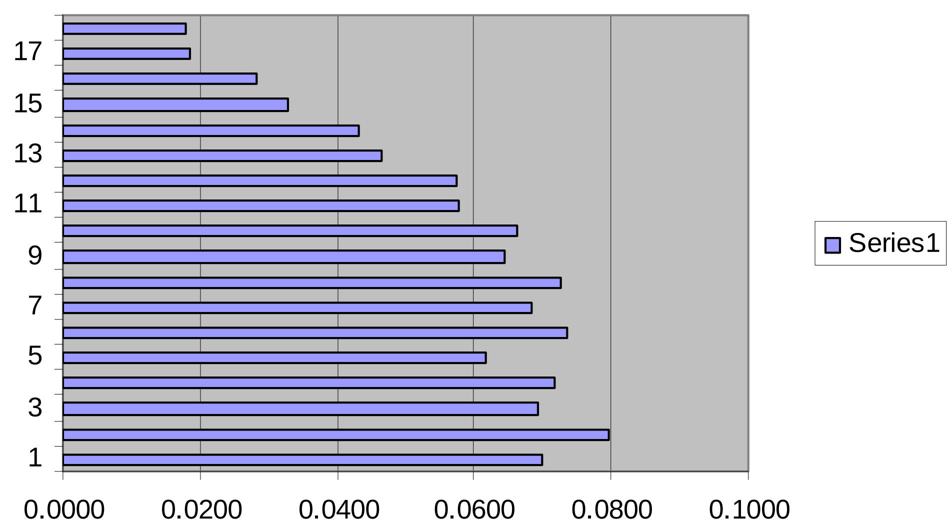

Fig. (3). Age Distribution of Alaska in 2470.

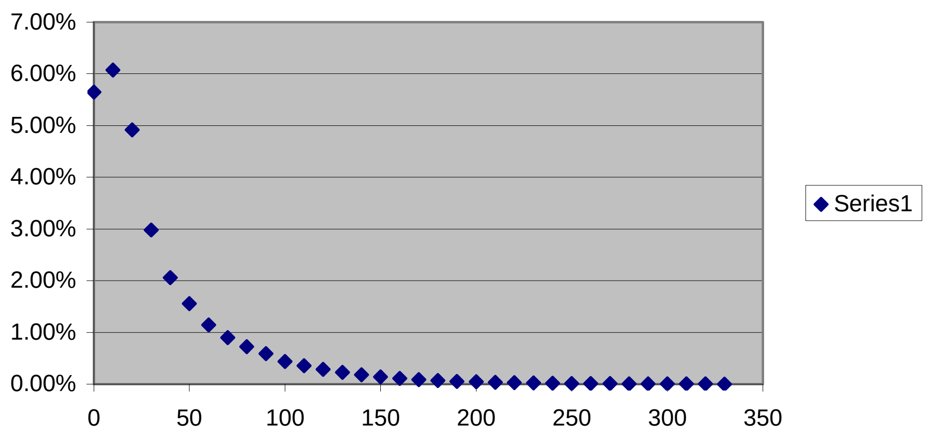

Fig. (4). Stability Index (***S***) over time (in Years) as the U.S. Population moves to Stability.

Table 3. The Population of the United States at Start (2000-10) and at Achieving Stability (2330-40)

	2000 Population	2000 Proportion by Age	2010 Population	2010 Proportion by Age	2000-2010 CCR	ABS Difference	Projected 2330	2330 Proportion By Age	Projected 2340	2340 Proportion by Age	ABS Difference
Total Population: 0 to 4 years	19,175,798	0.0681	20,201,362	0.0654	0.32245	0.0027	93,530,317	0.0629	98,079,782	0.0629	0.0000
Total Population: 5 to 9 years	20,549,505	0.0730	20,348,657	0.0659	0.32828	0.0071	95,120,900	0.0640	99,745,952	0.0640	0.0000
Total Population: 10 to 14 years	20,528,072	0.0729	20,677,194	0.0670	1.07830	0.0060	96,175,009	0.0647	100,853,404	0.0647	0.0000
Total Population: 15 to 19 years	20,219,890	0.0718	22,040,343	0.0714	1.07255	0.0005	97,285,512	0.0654	102,021,789	0.0654	0.0000
Total Population: 20 to 24 years	18,964,001	0.0674	21,585,999	0.0699	1.05154	0.0025	96,436,268	0.0649	101,131,448	0.0649	0.0000
Total Population: 25 to 29 years	19,381,336	0.0689	21,101,849	0.0683	1.04362	0.0005	96,819,753	0.0651	101,528,949	0.0651	0.0000
Total Population: 30 to 34 years	20,510,388	0.0729	19,962,099	0.0647	1.05263	0.0082	96,807,064	0.0651	101,511,824	0.0651	0.0000
Total Population: 35 to 39 years	22,706,664	0.0807	20,179,642	0.0654	1.04119	0.0153	96,132,719	0.0646	100,807,703	0.0646	0.0000
Total Population: 40 to 44 years	22,441,863	0.0797	20,890,964	0.0677	1.01856	0.0121	94,024,022	0.0632	98,603,346	0.0632	0.0000
Total Population: 45 to 49 years	20,092,404	0.0714	22,708,591	0.0736	1.00008	0.0022	91,674,594	0.0617	96,140,877	0.0617	0.0000

Table 3. cont….

	2000 Population	**2000 Proportion by Age**	**2010 Population**	**2010 Proportion by Age**	**2000-2010 CCR**	**ABS Difference**	**Projected 2330**	**2330 Proportion By Age**	**Projected 2340**	**2340 Proportion by Age**	**ABS Difference**
Total Population: 50 to 54 years	17,585,548	0.0625	22,298,125	0.0722	0.99360	0.0097	89,088,708	0.0599	93,421,807	0.0599	0.0000
Total Population: 55 to 59 years	13,469,237	0.0479	19,664,805	0.0637	0.97872	0.0158	85,568,496	0.0575	89,723,610	0.0575	0.0000
Total Population: 60 to 64 years	10,805,447	0.0384	16,817,924	0.0545	0.95635	0.0161	81,251,010	0.0546	85,199,911	0.0546	0.0000
Total Population: 65 to 69 years	9,533,545	0.0339	12,435,263	0.0403	0.92323	0.0064	75,328,846	0.0507	78,999,779	0.0507	0.0000
Total Population: 70 to 74 years	8,857,441	0.0315	9,278,166	0.0301	0.85866	0.0014	66,521,921	0.0447	69,766,698	0.0447	0.0000
Total Population: 75 to 79 years	7,415,813	0.0264	7,317,795	0.0237	0.76758	0.0026	55,138,893	0.0371	57,821,205	0.0371	0.0000
Total Population: 80 to 84 years	4,945,367	0.0176	5,743,327	0.0186	0.64842	0.0010	41,139,168	0.0277	43,134,032	0.0277	0.0000
Total Population: 85 years and over	4,239,587	0.0151	5,493,433	0.0178	0.74077	0.0027	38,954,558	0.0262	40,845,395	0.0262	0.0000
Total Population	281,421,906	1.0000	308,745,538	1.0000	1.09709	0.1130	1,486,997,759	1	1,559,337,512	1.0000	0.0000
S						0.05648					0.00002
GROWTH RATE			0.00927						0.00475		

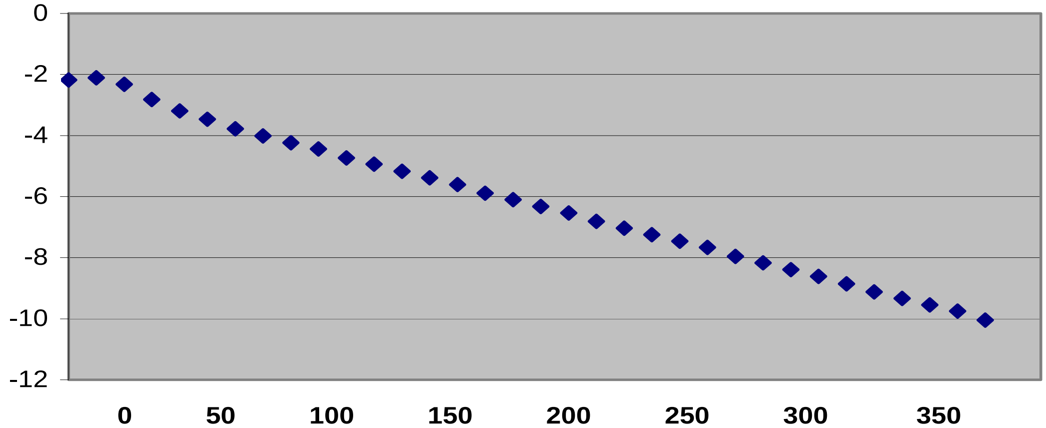

Fig. (5). The Natural Logarithm of ***S*** over time (in Years) as the U.S. Population moves to Stability.

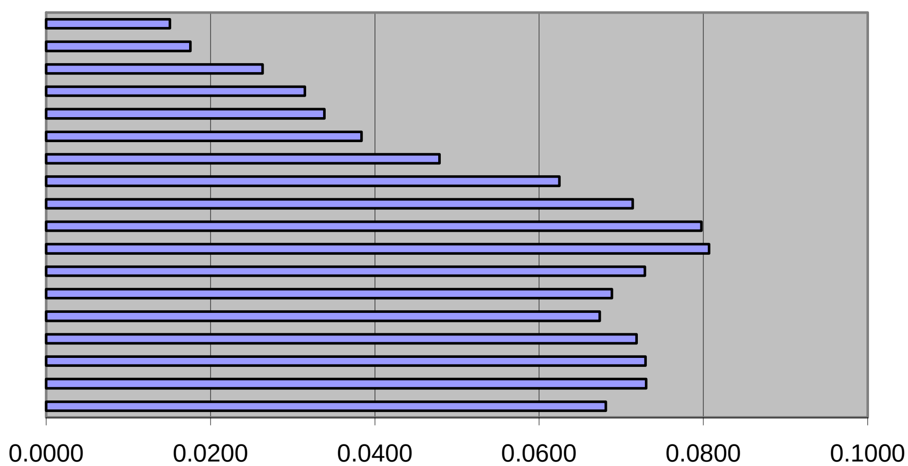

Fig. (6). Age Distribution of the U.S. in 2000.

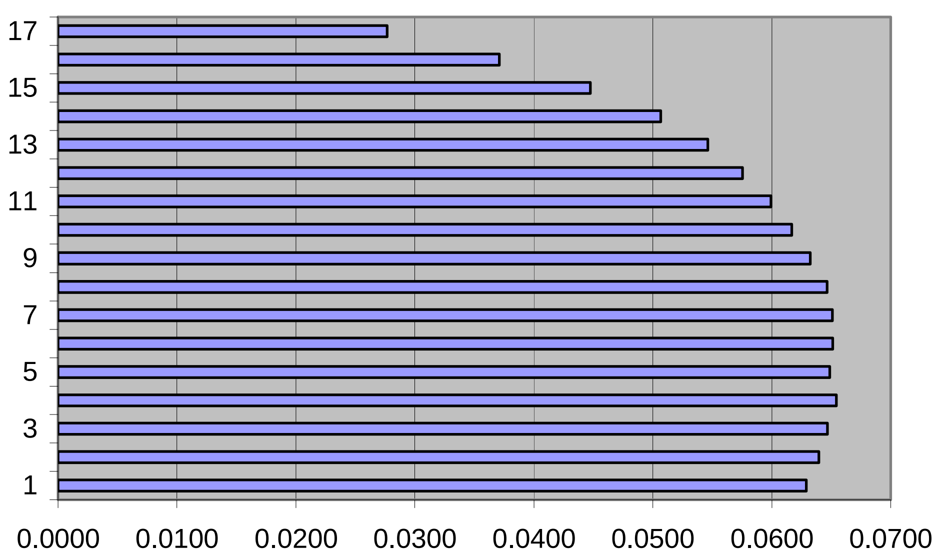

Fig. (7). Age Distribution of the U.S. in 2340.

Figs. (**6** and **7**) show the age distribution of the U.S. in 2000 and in 2340, when it reaches stability.

The final case study population is Whitman County, Washington. This population is of interest not only because it is growing but because it is heavily impacted by a "special population, namely students enrolled at Washington State University. In 2010, the total population of Whitman County was about 45,000. Students at Washington State University make up about half of this number. This can be seen in Fig. (**10**).

As was the case with Alaska and the United States, the projection is launched with CCRs taken over the 2000-2010

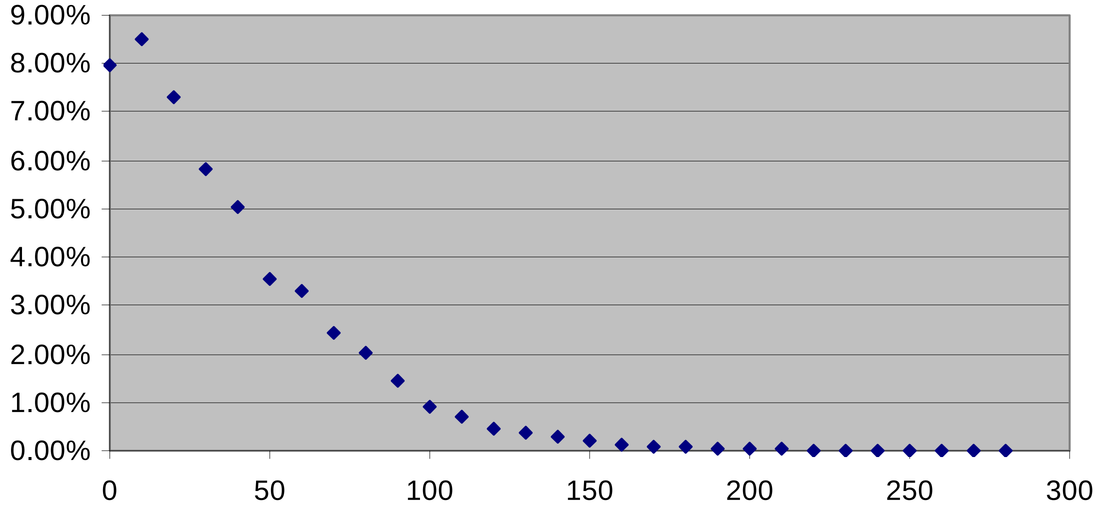

Fig. (8). Stability Index (***S***) over time (in Years) as the Whitman County Population moves to Stability.

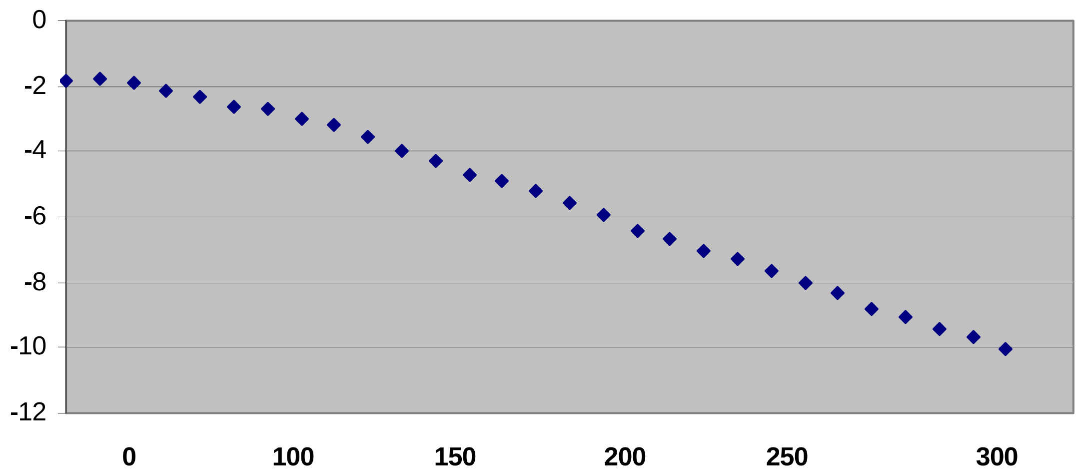

Fig. (9). The Natural Logarithm of ***S*** over time (in Years) as the Whitman County Population moves to Stability.

Fig. (10). Age Distribution of Whitman County in 2000.

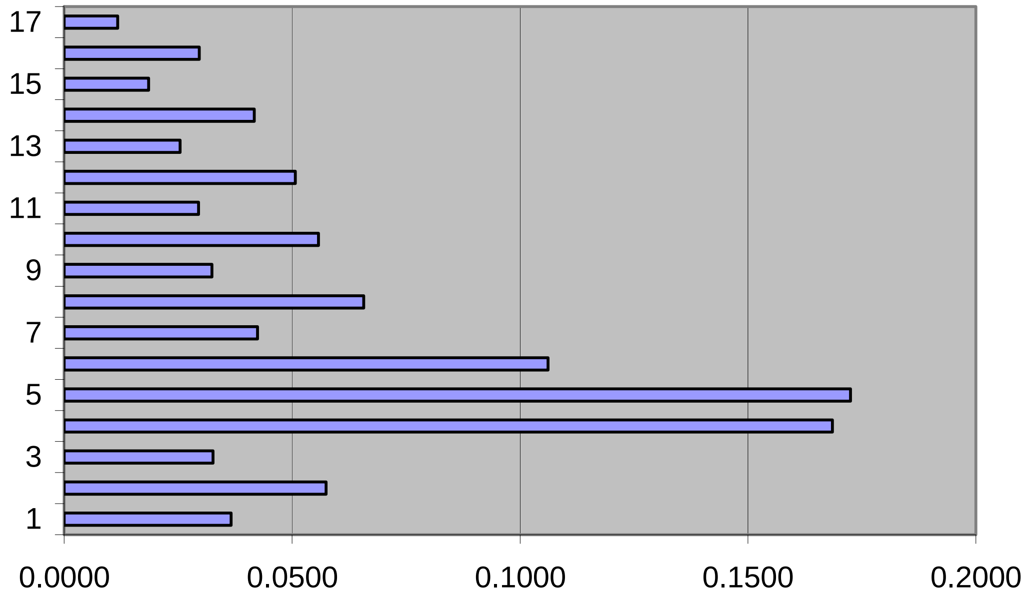

Fig. (11). Age Distribution of Whitman County in 2290.

Table 4. The Population of Whitman County, Washington at Start (2000-10) and at achieving Stability (2280-90)

	2000 Population	2000 Proportion by Age	2010 Population	2010 Proportion by Age	2000-2010 CCR	ABS Difference	Projected 2280	2280 Proportion by Age	Projected 2290	2290 Proportion by Age	ABS Difference
Total Population: 0 to 4 years	940	0.0463	1,978	0.0442	0.11408	0.0021	1,193,747,957,429	0.0366	2,546,285,089,708	0.0366	0.0000
Total Population: 5 to 9 years	971	0.0478	1,810	0.0404	0.26838	0.0074	1,874,211,939,120	0.0575	3,997,707,822,128	0.0575	0.0000
Total Population: 10 to 14 years	1,012	0.0498	1,789	0.0400	1.90319	0.0099	1,065,138,878,270	0.0327	2,271,930,953,021	0.0327	0.0000
Total Population: 15 to 19 years	2,696	0.1327	6,072	0.1356	6.25335	0.0029	5,494,407,269,480	0.1686	11,720,097,728,463	0.1686	0.0000
Total Population: 20 to 24 years	4,431	0.2181	11,394	0.2545	11.25889	0.0364	5,622,100,696,861	0.1725	11,992,284,959,494	0.1725	0.0000
Total Population: 25 to 29 years	1,368	0.0673	3,621	0.0809	1.34310	0.0135	3,459,708,398,795	0.1061	7,379,543,294,802	0.1061	0.0000
Total Population: 30 to 34 years	1,144	0.0563	2,324	0.0519	0.52449	0.0044	1,382,496,179,298	0.0424	2,948,716,321,260	0.0424	0.0000

Table 4. cont….

	2000 Population	2000 Proportion by Age	2010 Population	2010 Proportion by Age	2000-2010 CCR	ABS Difference	Projected 2280	2280 Proportion by Age	Projected 2290	2290 Proportion by Age	ABS Difference
Total Population: 35 to 39 years	1,106	0.0544	1,806	0.0403	1.32018	0.0141	2,141,213,337,187	0.0657	4,567,422,052,795	0.0657	0.0000
Total Population: 40 to 44 years	1,140	0.0561	1,864	0.0416	1.62937	0.0145	1,056,015,018,418	0.0324	2,252,598,669,765	0.0324	0.0000
Total Population: 45 to 49 years	1,013	0.0499	2,003	0.0447	1.81103	0.0051	1,817,937,131,839	0.0558	3,877,803,177,565	0.0558	0.0000
Total Population: 50 to 54 years	912	0.0449	2,212	0.0494	1.94035	0.0045	960,723,338,343	0.0295	2,049,039,667,317	0.0295	0.0000
Total Population: 55 to 59 years	766	0.0377	1,967	0.0439	1.94176	0.0062	1,654,956,562,279	0.0508	3,529,992,436,650	0.0508	0.0000
Total Population: 60 to 64 years	569	0.0280	1,679	0.0375	1.84101	0.0095	829,203,714,494	0.0254	1,768,700,093,287	0.0254	0.0000
Total Population: 65 to 69 years	472	0.0232	1,343	0.0300	1.75326	0.0068	1,360,103,723,348	0.0417	2,901,575,278,251	0.0417	0.0000
Total Population: 70 to 74 years	432	0.0213	885	0.0198	1.55536	0.0015	604,664,342,911	0.0186	1,289,710,522,544	0.0185	0.0000
Total Population: 75 to 79 years	454	0.0223	716	0.0160	1.51695	0.0064	967,400,112,467	0.0297	2,063,208,190,503	0.0297	0.0000
Total Population: 80 to 84 years	360	0.0177	584	0.0130	1.35185	0.0047	383,317,632,214	0.0118	817,416,611,713	0.0118	0.0000
Total Population: 85 years and over	529	0.0260	729	0.0163	1.60573	0.0098	728,019,832,900	0.0223	1,553,380,356,803	0.0223	0.0000
Total population	20,315	1.0000	44,776			0.1595	32,595,366,065,653		69,527,413,226,071		0.0000
S						**7.98%**					**0.00%**

Table 5. The Population of Whitman County, Washington at Start (2000-10) and at Achieving Stability (2410-20) when the USA CCRs are Applied

	2000 Population	2000 Proportion by Age	2010 Population	2010 Proportion by Age	2000-2010 CCR	ABS Difference	Projected 2410	2410 Proportion by Age	Projected 2420	2420 Proportion by Age	ABS Difference
Total Population: 0 to 4 years	940	0.0463	1,978	0.0442	0.11408	0.0021	26,582	0.0629	27,877	0.0629	0.0000
Total Population: 5 to 9 years	971	0.0478	1,810	0.0404	0.26838	0.0074	27,036	0.0640	28,349	0.0640	0.0000
Total Population: 10 to 14 years	1,012	0.0498	1,789	0.0400	1.90319	0.0099	27,338	0.0647	28,663	0.0647	0.0000
Total Population: 15 to 19 years	2,696	0.1327	6,072	0.1356	6.25335	0.0029	27,652	0.0654	28,998	0.0654	0.0000
Total Population: 20 to 24 years	4,431	0.2181	11,394	0.2545	11.25889	0.0364	27,407	0.0648	28,747	0.0649	0.0000
Total Population: 25 to 29 years	1,368	0.0673	3,621	0.0809	1.34310	0.0135	27,515	0.0651	28,858	0.0651	0.0000
Total Population: 30 to 34 years	1,144	0.0563	2,324	0.0519	0.52449	0.0044	27,515	0.0651	28,850	0.0651	0.0000
Total Population: 35 to 39 years	1,106	0.0544	1,806	0.0403	1.32018	0.0141	27,328	0.0647	28,649	0.0646	0.0000
Total Population: 40 to 44 years	1,140	0.0561	1,864	0.0416	1.62937	0.0145	26,727	0.0632	28,026	0.0632	0.0000
Total Population: 45 to 49 years	1,013	0.0499	2,003	0.0447	1.81103	0.0051	26,053	0.0616	27,330	0.0617	0.0000
Total Population: 50 to 54 years	912	0.0449	2,212	0.0494	1.94035	0.0045	25,316	0.0599	26,556	0.0599	0.0000
Total Population: 55 to 59 years	766	0.0377	1,967	0.0439	1.94176	0.0062	24,320	0.0575	25,498	0.0575	0.0000
Total Population: 60 to 64 years	569	0.0280	1,679	0.0375	1.84101	0.0095	23,100	0.0547	24,210	0.0546	0.0000
Total Population: 65 to 69 years	472	0.0232	1,343	0.0300	1.75326	0.0068	21,415	0.0507	22,453	0.0507	0.0000
Total Population: 70 to 74 years	432	0.0213	885	0.0198	1.55536	0.0015	18,904	0.0447	19,835	0.0448	0.0000
Total Population: 75 to 79 years	454	0.0223	716	0.0160	1.51695	0.0064	15,665	0.0371	16,438	0.0371	0.0000
Total Population: 80 to 84 years	360	0.0177	584	0.0130	1.35185	0.0047	11,692	0.0277	12,257	0.0277	0.0000
Total Population: 85 years and over	529	0.0260	729	0.0163	1.60573	0.0098	11,077	0.0262	11,604	0.0262	0.0000
	20,315	1.0000	44,776			0.1595	422,642	1.0000	443,199	1.0000	0.0002
S						7.98%	*S*				0.01%

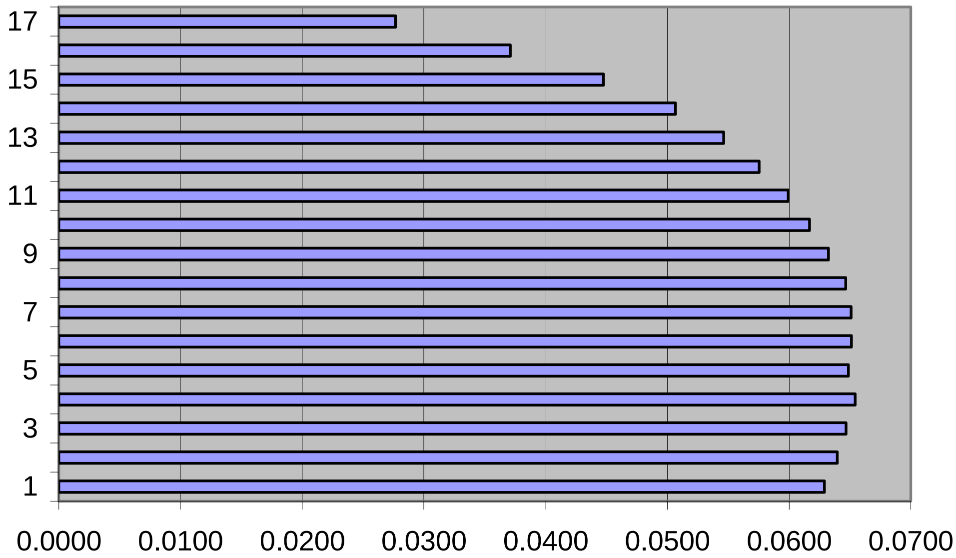

Fig. (12). Age Distribution of Whitman County in 2420 when USA CCRs are applied.

period, which are held constant from the launch year to a year where ***S*** = 0 (relative to the preceding year in the projection cycle). This occurs at the year 2290. Table **4** displays this by showing the information at the launch period, 2000-2010, and the information at the period where stability is reached, 2330-2340. ***S*** = .7.98% at the launch year of 2010; by 2290, ***S*** = .0.00%.

Fig. (**8**) provides the change in ***S*** from 2010 to 2290 for Whitman County. As was the case for the United States, the path to stability is neither monotonic nor linear: It initially increases, "bounces around" a bit, then decreases substantially before its decrease slows considerably, which starts around 2160. From 2160 to 2290, ***S*** moves incrementally to zero. Fig. (**9**) shows the graph of ln(***S***) relative to time.

Figs. (**10** and **11**) show the age distribution of Whitman County in 2000 and in 2290, when it reaches stability.

To examine the question of ergodicity, the 2000-2010 CCRs for the USA are applied to Whitman County, which reaches stability in 2420 using these CCRs. Table **5** contains the data while Fig. (**12**) shows the age distribution of Whitman County in 2240 when it reaches stability using the US CCRs. In comparing the age distribution found in Fig. (**12**) to that of the US (at stability) in 2340, it is clear that they are very similar, if not identical. To test this more rigorously, the differences were calculated and found to be essentially zero at each age group. In addition, the intrinsic growth rate of .00475 matches that of the US when it reaches stability. This confirms the idea that using CCRs to generate stable populations is consistent with ergodic theory. The test results are in Table **6**.

5. TIME TO STABILITY

One of the shortcomings of the analytic approach to a stable population is its inability to estimate the time required before a given population achieves stability. The CCR approach may offer a way to solve this problem in that a simple bivariate regression model was constructed using a sample of 18 U.S. states used in a different [24]. These states are shown in Exhibit **1**.

EXHIBIT 1

The 18 Counties Used in the Regression Analysis	
Pima County, AZ	Madison County, MS
Jefferson County, AR	Douglas County, NE
San Francisco County, CA	Bronx County, NY
Tulare County, CA	Rockland County, NY
Broward County, FL	Franklin County, OH
Lake County, IL	Multnomah County, OR
Black Hawk County, IA	Schuylkill County, PA
Calvert County, MD	Sevier County, TN
Hampden County, MA	Yakima County, WA

The regression model was constructed using one independent variable, the initial value of ***S***. The Dependent variable is time (in years) to "stability." Of course, there is more information available for a population at its time of launch (e.g., proportion of the population under 20 years of age, the initial rate of population change) that could be examined as potential independent variables in a multiple regression model. However, it seems obvious that since the larger the ***S*** score, the farther a population is from stability, the initial ***S*** score should serve as the starting point in a regression model. That is, the hypothesis is that there is an inverse relationship between initial ***S*** score and time to stability.

Population stability is measured "approximately" by selecting the time to stability defined as when ***S*** =0.01. That is, when only one percent of age distribution, the population at the preceding year needs to be re-allocated to match the age distribution of the population at the subsequent year. ***S*** =0.01 was selected because an examination of scatter plots

Table 6. Difference in Proportional Population by Age for the US At Stability and Whitman County at Stability using US CCRs

	Whitman County 2420 Proportion by Age	USA 2340 Proportion by Age	Difference
Total Population: 0 to 4 years	0.0629	0.0629	0.0000
Total Population: 5 to 9 years	0.0640	0.064	0.0000
Total Population: 10 to 14 years	0.0647	0.0647	0.0000
Total Population: 15 to 19 years	0.0654	0.0654	0.0000
Total Population: 20 to 24 years	0.0649	0.0649	0.0000
Total Population: 25 to 29 years	0.0651	0.0651	0.0000
Total Population: 30 to 34 years	0.0651	0.0651	0.0000
Total Population: 35 to 39 years	0.0646	0.0646	0.0000
Total Population: 40 to 44 years	0.0632	0.0632	0.0000
Total Population: 45 to 49 years	0.0617	0.0617	0.0000
Total Population: 50 to 54 years	0.0599	0.0599	0.0000
Total Population: 55 to 59 years	0.0575	0.0575	0.0000
Total Population: 60 to 64 years	0.0546	0.0546	0.0000
Total Population: 65 to 69 years	0.0507	0.0507	0.0000
Total Population: 70 to 74 years	0.0448	0.0447	0.0001
Total Population: 75 to 79 years	0.0371	0.0371	0.0000
Total Population: 80 to 84 years	0.0277	0.0277	0.0000
Total Population: 85 years and over	0.0262	0.0262	0.0000
SUM	1.0000	1.0000	0.0005

Table 7. Input Data For The Regression Model

Population	Initial Stability Index	Years to Stability, *S*=0.01
PIMA CO, AZ	0.06099	70
JEFFERSON CO, AR	0.06563	70
TULARE CO, CA	0.03966	80
BROWARD CO, FL	0.08147	110
LAKE CO, IL	0.08442	110
BLACK HAWK CO, IA	0.06886	100
CALVERT CO, MD	0.11430	130
HAMPDEN CO, MA	0.08246	110
MADISON CO, MS	0.07240	60
DOUGLAS CO, NE	0.05269	70
BRONX CO, NY	0.06185	120
ROCKLAND CO, NY	0.05063	70
FRANKLIN CO, OH	0.05076	70
MULTNOMAH CO, OR	0.06130	100

Table 7. cont…

Population	Initial Stability Index	Years to Stability, *S*=0.01
SCHUYLKILL CO, PA	0.06444	70
SEVIER CO, TN	0.05636	70
YAKIMA CO, WA	0.04223	60

Table 8. Estimated and Actual Years to *S* =0.01

Population	Initial *S*	Actual Years to *S* = 0.01	Estimated Years to *S* =0.01 Using the Regression Models	Difference (Estimate - Actual)	Percent Difference
Alaska	0.10573	120	123	3	2.50%
United States	0.0565	60	52	-8	-13.19%
Whitman County, WA	0.07903	90	71	-19	-20.56%

revealed that a long "tail" exists in going from ***S*** =0.01 to ***S*** =0.00 (see, e.g. Figs. **1**, **5**, or **9**). Because U.S. states are used, there are ten years between these two points in time.

The NCSS statistical system was used to build the regression model, an overview of which is given below. The input data used to build the regression model are found in Table **7**.

(YEARS TO ID =0.01) = 31.42 + (861.53*INITITAL_ID_SCORE)

(p= .047) (p = .0011)

$r^2 = .495$

Both the intercept and the partial regression coefficient for the initial ID score are statistically significant ($\alpha = 0.05$) and that the coefficient of determination suggests that the model explains 50 percent of the variation in years to approximate stability (***S*** =0.01). Provides a scatter plot of years to "approximate stability (***S*** =0.01) by the initial ***S*** score.

To get an idea of the accuracy of the model shown in the regression equation, it was used to estimate time to ***S*** =0.01 for the case study populations, Alaska, the United States, and Whitman County, Washington. Table **8** provides the results of this examination.

The estimates for the US and Whitman County are reasonably accurate, with error of -8 and -19 years respectively and the estimate for Alaska is very accurate in that the time to approximate stability at ***S*** =0.01 is estimated as 123 years and the actual number of years to ***S*** =0.01 is 120.

6. CONCLUSION

Cohort Change Ratios (CCRs) appear to us to be useful as a tool for examining the idea of a stable population, given the informal and non-rigorous examination found in this paper. Benefits of the CCR approach include the ability to easily deal with both sexes and all of the components of change, including migration. A drawback of the CCR approach is that one cannot easily assess the effect of each component of change since they are all effectively rolled into CCRs. However, we can add a new perspective on a stable population by noting that it has an invariant relative age structure and both a constant rate of growth and a constant set of Cohort Change Ratios. It may seem obvious that a constant set of CCRs would eventually yield a stable population, but the obvious has not yet been stated [2, 3, 6-11, 14-16, 25- 28].

A by-product of this paper is the Index of Stability (***S***). As noted earlier, there is no reason that our Index could not be used in the traditional approach, but a search revealed no mention of such an index in the literature. Calling upon the Index of Dissimilarity for this purpose appears to be a natural use for it.

In applying the US CCRs to Whitman County, when this population reached stability, its age distribution was the same as that found for the US when the latter reached stability. That is, as suggested by formal stable population theory, this case study shows that when a constant set of rates is applied to a given population, the initial age distribution is "forgotten" as the population becomes stable. This suggests that the CCR approach is consistent with theory.

The CCR approach appears to be sufficiently useful to warrant further investigation. In these studies, it appears that it would be useful to graph the Stability Index over the time it takes a given population to reach stability. The graphs presented here for the three case studies suggest a non-monotonic and non-linear path and similar results (not shown here) were found for the sample of 18 states. The work with the 18 states suggests that regression models may provide a way to overcome the major drawback associated with the analytic approach that time to stability cannot be found. The regression model suggests that for at least US states and counties, an initial ID score taken from two successive census counts (e.g., 2000 and 2010) can be placed into the regression model in order to estimate the number of years to approximate stability (***S*** =0.01). This finding suggests that the Index of Stability can be used with other initial condition variables and the years to stability as a basis for models that potentially could predict the years to stability, given an initial ***S***.

It may be the case that the regression model is accurate only within "families" of population dynamics. Here, the types of dynamics come to mind that are analogous to the Regional Model Life Tables and Stable Populations developed by Coale and Demeny [27]. If this is the case, then the different families would need to be identified and regression models specific to each family would need to be constructed using data from the populations with each family.

Another area for research is the use of CCRs in conjunction with ideas promulgated by Keyfitz [29] for examining stable processes across two (or more) interacting populations. Because it can deal with both sexes and migration quite handily, the CCR approach may be more tractable in regard to examining the path to stability in such populations.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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Declared none.

REFERENCES

[1] Swanson DA, Stephan GE. Glossary. In Siegel J and Swanson DA, Eds. The methods and materials of demography, 2nd ed. San Diego, CA: Elsevier Academic Press 2004; pp. 751-78.
[2] Coale AJ. The growth and structure of human populations: a mathematical investigation. Princeton, NJ: Princeton University Press 1972.
[3] Preston S, Heuveline P, Guillot M. Demography: measuring and modeling population processes. Malden, MA: Blackwell Publishing 2001.
[4] Lotka AJ. Relation between birth rates and death rates. Science (New Series) 1907; 26(653): 21-2.
[5] Dublin L. On the true rate of natural increase. J Am Stat Assoc 1925; 20: 305-39.
[6] Alho J, Spencer B. Statistical demography and forecasting. New York, NY: Springer 2005.
[7] Arthur B, Vaupel J. Some relationships in population dynamics. Popul Index 1984; 50(2): 214-26.
[8] Bennett N, Horuchi S. Mortality estimation from registered deaths in less developed countries. Demography 1984; 21(2): 217-33.
[9] Le Bras H. The nature of demography. Princeton, NJ: Princeton University Press 2008.
[10] Pollard A, Yusuf F, Pollard G. Demographic techniques. 3rd ed. New York, NY: Pergamon Press 1974.
[11] Popoff C, Judson D. Some methods of estimation for statistically underdeveloped areas. In Siegel J and Swanson DA, Eds. The methods and materials of demography, 2nd ed. San go, CA: Elsevier Academic Press 2004; pp. 603-41.
[12] Preston S, Coale AJ. Age structure, growth, attrition, and accession: a new synthesis. Popul Index 1982; 48: 217-59.
[13] Rogers A. Regional population projection models. Scientific Geography Series. Beverly Hills, CA: Sage Publications 1985; vol. 4.
[14] Schoen R. Modeling multigroup populations. New York, NY: Plenum Press 1985.
[15] Rogers A. Multiregional demography: principles, methods, and extensions. New York, NY: John Wiley and Sons 1995.
[16] Rogers A, Little J, Raymer J. The indirect estimation of migration. Dordrecht. The Netherlands: Springer 2010.
[17] Pollak R. A re-formulation of the two-sex problem. Demography 1986; 23: 247-59.
[18] Pollak R. Two sex demographic models. J Polit Econ 1990; 98: 399-420.
[19] Swanson DA, Tedrow L. Using cohort change ratios to estimate life expectancy in populations with negligible migration: a new approach. Canadian Studies in Popul 2012; 39: 83-90.
[20] United Nations. Methods for estimating adult mortality. New York, NY: Population Division, United Nations 2002.
[21] Hamilton CH, Perry J. A short method for projecting population by age from one decennial census to another. Soc Forces 1962; 41: 163-70.
[22] Smith S, Tayman J, Swanson DA. State and local population projections: methodology and analysis. New York, NY: Kluwer Academic/Plenum Publishers 2001.
[23] Swanson DA, Schlottmann A, Schmidt R. Forecasting the population of census tracts by age and sex: an example of the hamilton-perry method in action. Popul Res Policy Rev 2010; 29(1): 47-63.
[24] Swanson DA, Hough G. An evaluation of persons per household (pph) estimates generated by the American community survey: a demographic perspective. Popul Res Policy Rev 2012; 31: 235-66.
[25] Bacaër N. A short history of population dynamics. Dordrecht, The Netherlands: Springer 2011.
[26] Carey J. Applied demography for biologists: with special emphasis on insects. Oxford, England: Oxford University Press 1993.
[27] Coale AJ, Demeny P. Regional model life tables and stable populations. Princeton, NJ: Princeton University Press 1996.
[28] Land K, Rogers A. Multidimensional mathematical demography. San Diego, CA: Academic Press 1982.
[29] Keyfitz N. A general condition for stability in demographic processes. Can Stud Popul 1974; 1: 29-35.

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