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More Realistic than Anticipated: A Classical Forest-Fire Model from Statistical Physics Captures Real Fire Shapes

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Abstract: The quantitative study of wildfire data world wide revealed that wildfires exhibit power-law like frequency-area distributions. Although models exist to predict the spread of a specific fire, there is as yet no agreement on the mechanism which drives wildfire systems on the landscape scale. A classical model in this context is the Drossel-Schwabl cellular automaton (DS-FFM) which robustly produces a power-law like frequency-area statistic for fire sizes. This model originated in statistical physics where it was used to illustrate the concept of self-organized criticality. A conjecture has been made in the literature that this model is not able to produce the spatial patterns of actual wildfires and hence is of no ecological significance. We test this conjecture by comparing the shape of simulated fires in the DS-FFM to those of 68 fires in the boreal forests of Alberta, Canada. Our results suggest that, contrary to the conjecture, the Drossel-Schwabl model performs well in producing realistic fire shapes. It can hence not be excluded as a candidate mechanism behind wildfire systems. We do show, however, that the performance depends on the size of the fire. Best results are obtained for fires of 400-2,000 ha. Very large fires of 2,000-20,000 ha and smaller fires of 20-200 ha differ from the simulated burn scars in the distribution and median size of islands of unburnt vegetation. Nevertheless, the overall fit remains good even for these size classes.

INTRODUCTION

Forest fires world-wide exhibit power-law like frequency-area distributions over up to five orders of magnitude [1-4]. Several attempts have been made at providing an explanation for this behavior ranging from the theory of stochastic processes [5] to cellular automaton models [1, 6]. Insight into the controlling mechanisms is of fundamental importance in disturbance ecology [7], in addressing the question of predictability of large fire events [1], and in determining the sensitivity of fire regimes to climate change.

The most parsimonious model reproducing power-law like frequency-area distributions was developed in physics: the Drossel-Schwabl forest fire model (DS-FFM; [8,9]). However, the occurrence of power laws is a poor indicator of the true underlying mechanisms because power laws can be generated by many different mechanisms [10]. It would thus be desirable to test whether the DS-FFM reproduces further patterns observed in real forest-fire ecosystems [11]. Caldarelli *et al*. [12] found that the DS-FFM was not able to reproduce fractal shapes in the clusters of three real forest fires in Italy. This seems to indicate that the DS-FFM has no ecological significance, but it might still be worthwhile to test the model with data from more than three fires and regarding a broader range of structural properties of burnt areas, the burn scars.

Here we test the Drossel-Schwabl model by comparing simulated fires to those of 68 fires in the boreal forests of Alberta, Canada. The data was gathered by Eberhart and Woodard [13] and have been used for the validation of an ecologically motivated fire model [6]. We will show that the DS-FFM is more realistic than it seems and should also be considered by fire ecologists, who so far largely ignored this model (but see [1]). Our analysis also shows that a re-interpretation of the model's basic spatial unit, the grid cell, adds ecological significance to the model without changing its structure and dynamics.

METHODS

The DS-FFM

The DS-FFM model ([8,9]) is defined on a two-dimensional grid of length L. It is a cellular automaton in which every cell changes its state according to its own state and the states of its four neighbors. Every cell can be either 'empty', occupied by a 'tree', or 'burning'. The state of each cell is adjusted in every step of the simulation according the following four transition rules:

1 A burning 'tree' turns into an 'empty' space.

2 If at least one direct neighbor of a 'tree' is burning, it will burn in the next step.

3 A 'tree' can start to burn with probability f even if none of its neighbors are burning.

4 An 'empty' cell will be occupied by a tree in the next step with probability p.

The grid is initialized at random with an arbitrary number of trees. If iterated long enough, the system reaches a quasi-stationary state in which the average tree density, p_t, and fire density p_f are constant [14-16]. A necessary condition for the occurrence of power-law like distributions of fire sizes is a

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double separation of time scales such that $p/f \to \infty$ and $p \to 0$. It is extremely unlikely that two fires burn at the same time under this limit. The average number of trees that re-grow between two lightnings is then proportional to p/f so that the dynamics of the model depends on p/f rather than p and f separately [14, 17]. The model can hence be simulated in the following way [17,18]:

1 Choose a site at random. If empty, proceed with rule 2. If occupied by a tree, determine the forest cluster associated with it and evaluate its structural properties. Set all cluster sites to empty, i.e. the entire cluster is burnt.

2 Choose $\theta = p/f$ sites at random and grow a tree at all of these sites which are empty. Employ rule 1.

which is how we implemented the DS-FFM in our simulations. Fig. (**1**) gives an idea what fire clusters in the model look like. We used a genetic algorithm GA [20; Appendix I] to search the parameter space of the DSM for configurations which deliver the best fits to the data. The robustness of the solutions is then demonstrated by a local analysis.

Data and Structural Properties

Detailed studies of structural properties of real fire patterns are rare ([6, 12]). We use data obtained for 68 fires in boreal forest of Alberta that burnt without human intervention [13]. The fires were grouped in size classes of (1) 20–40 ha, (2) 41–200 ha, (3) 20–400 ha, (4) 401–2,000 ha and (5) 2,001–20,000 ha. Structural properties studied by Eberhart and Woordard [13] and Ratz [6] were fire shape as well as number and size of islands of unburnt vegetation within fires. We evaluated the same measures in the DS-FFM:

Total Island Area

The area of unburnt islands relative to the area a_0 enclosed by the outer perimeter of the fire. Let a_b be the area which was actually burned, then

$$\text{Total unburned area} = 1 - \frac{a_b}{a_0}$$

Shape Index

The ratio of the outer perimeter p_o of the fire to the perimeter of a circle which encloses an equal area a_o.

$$\text{Shape index} = \frac{p_o}{2\sqrt{\pi a_0}}$$

Edge Index

The sum of the outer perimeter and the perimeter of all enclosed islands, p_w, is compared to the perimeter of a circle which encloses an area equal to the burned area a_b.

$$\text{Edge index} = \frac{p_w}{2\sqrt{\pi a_b}}$$

Island Size (MIS)

The median island size per fire was recorded and averaged for each size class.

Number of Islands (NI)

The number of islands per 100 ha of burned area.

An island within a burned cluster of the forest-fire model was defined as a coherent area of tree cells surrounded by empty (recently burned) cells. The algorithms used to determine these quantities from the simulation data are provided in pseudo-code notation in the Appendix II.

The Simulations

Simulations were carried out with L=400 (i.e., 16,000 grid cells) and absorbing boundary conditions. A number of 15,000 time steps was discarded at the beginning of each simulation in order to ensure that the system has reached the steady state [19].

To calibrate the spatial scale of the model, i.e. the size a of a grid cell in ha, we utilize the observation that in the data a minimum fire size of 40–100 ha was necessary for a fire to enclose at least one island [6, 13]. For each of 15 values of θ, 3,000 fires were analyzed to determine the relation between fire size and the probability that the burnt area includes at least one unburnt island (Fig. **2**). Only fires which did not reach the grid boundary were analyzed. Nevertheless, we sampled 2000 fires for each of the five size classes.

We used a standard GA [20] in analyzing the structural properties of the fires to make sure that we were not looking at an isolated range of $\theta = p/f$ (see Appendix I). We chose a GA since they tend to perform well when the feedback information on performance includes variation [20], as is the case here where properties are calculated from several stochastic simulation runs. The genetic algorithm searched for pairs of (θ, a) which best fit all the structural measures in the range of θ in [50,..., 1500] and a, the size of a grid cell in ha, in [2, ...,13]. The goodness of fit was calculated based on the sum of relative deviations between the distributions of field and simulation data: Let $d \in D$ stand for the data generated in the model and $w \in W$ for those determined in Alberta, then the goodness of fit used is:

$$F(D,W) = \sum_M \sum_S \frac{|d_{m,s} - w_{m,s}|}{w_{m,s}} \quad (1)$$

where S denotes the set of size classes and M the set of measures considered. The GA used a population size of 30 individuals (representing parameter sets), a number of 800

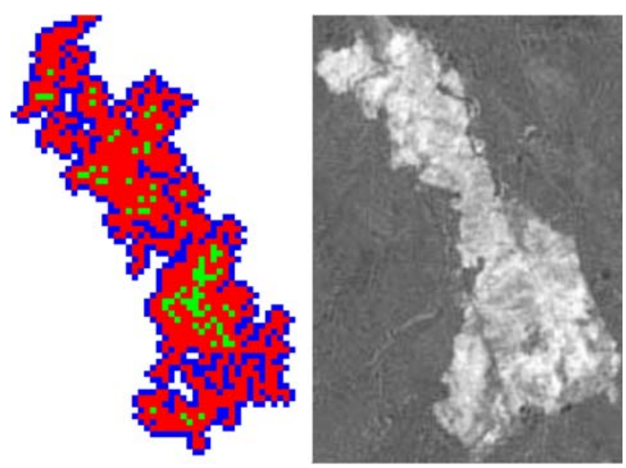

Fig. (1). A fire in the DS-FFM model (left, θ = 200). The rim of the fire is darkened, the unburnt islands are lightened up. White and dark gray enclosures are artifacts of the meandering rim. Satellite image of a fire [32] in a boreal forest (right).

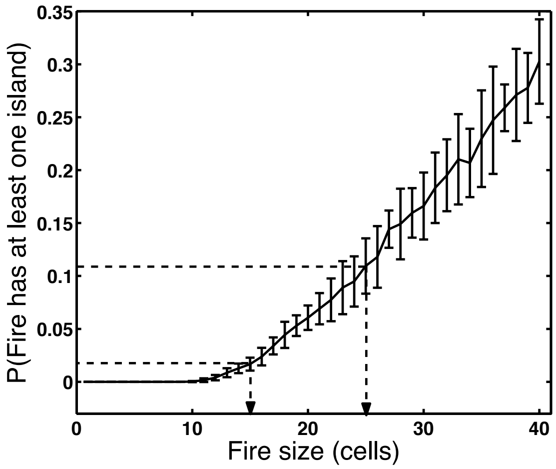

Fig. (2). The probability of a fire having at least one island in the forest-fire model versus fire size in cells. The arrows indicate the range used to determine the minimal size necessary to have at least one island.

individuals were evaluated in total. Each individual was run for 15,000 steps before taking 2,000 fire samples in a ten-step interval. The parameters were encoded in binary strings with random mutation (rate 0.05) and a crossover rate of 0.7 with roulette wheel selection [20].

RESULTS

Fires produced by the DS-FFM often seem to resemble real burn scars (Fig. **1**).

To calibrate the size of the grid cells, *a*, note that there can be no island in clusters smaller than eight cells since at least one cell has to be surrounded by others by definition. This leaves a maximum area per cell of $a_{max}=100/8 \approx 13$ ha since 100 ha is the upper limit in the data and eight is the smallest possible number of cells needed to enclose an island cell. Nevertheless, it is quite unlikely that an island will form directly in the center of a fire with exactly eight neighbors. We must hence determine the number of cells per fire at which an island typically appears [6]. This is arbitrary to some degree and hence we chose to use an interval in the necessary size, indicated by the arrows in Fig. (**2**), rather than just a point. Looking at the one-island probability reveals that approximately one percent of fires the size of 15 cells enclose at least one island as compared to ten percent in fires of size 25 (Fig. **2**). This suggests a lower range of $a \in [40/25, \dots ,100/15] \approx 2\text{-}7$ ha per cell for the model.

The genetic algorithm revealed that the deviation between structural properties of model output and data was minimal for lower values of θ and *a* between four and eight ha/cell (Fig. **3**). Sensitivity to θ and *a* was not extremely strong, so that the following comparison of model output and field data, which was performed for the slightly suboptimal value of θ =200, can assumed to be robust regarding moderate uncertainties in θ and *a*.

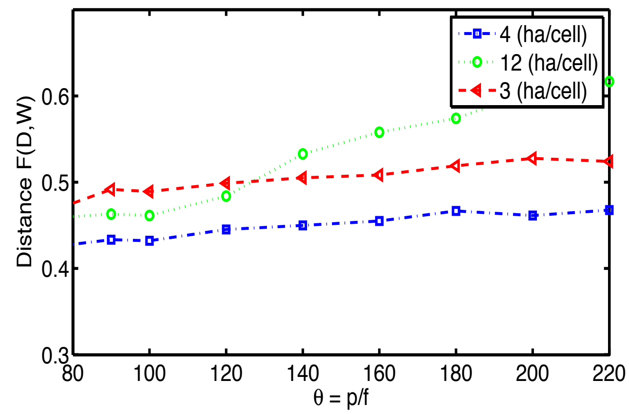

Fig. (3). Comparison of field and model data for Alberta using Eq. 1 for a range of θ and *a*. The closer the value is to zero, the better the fit. The best fit is attained for *a* between four and eight ha/cell.

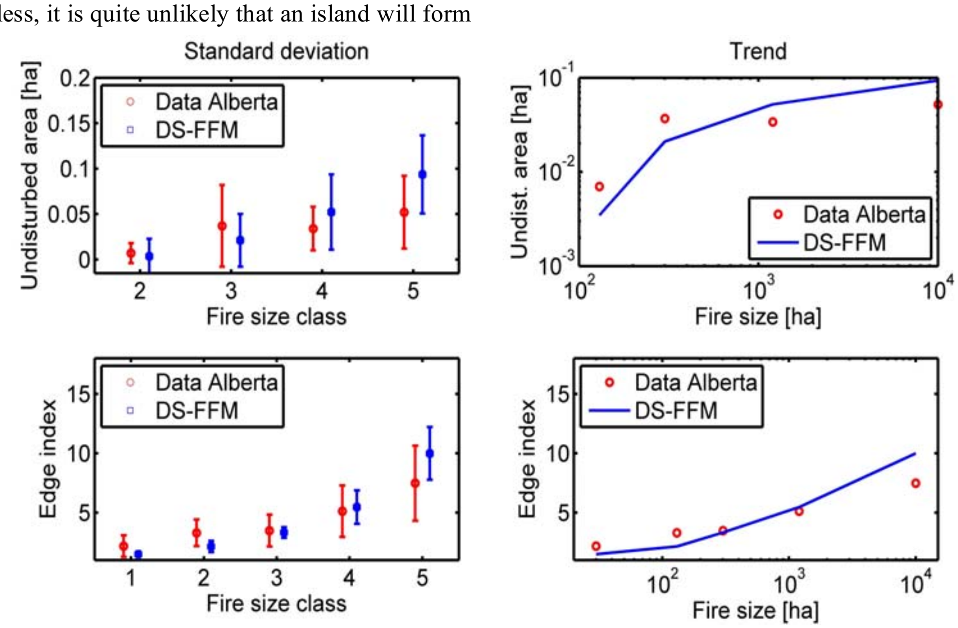

Fig. (4). The undisturbed area (upper left and right) and edge index (lower left and right) obtained for 2000 fires in each size class (θ=200). Both quantitative (overlap, left) and qualitative (trend, right) fit are good.

The qualitative and quantitative fit was excellent for the undisturbed area and edge index (Fig. **4**).

The number of islands per 100 ha is too low for all but the last size class yet still is within the variation of the data (Fig. **5**). The median island size is too low for small and medium sized fires, fits well for fires of size class 4 and is too low for large fires (Fig. **5**). The shape index fits best for small and medium fires and mirrors the increasing trend found in the data albeit being too large for fires of the last size class (20,000 ha and more).

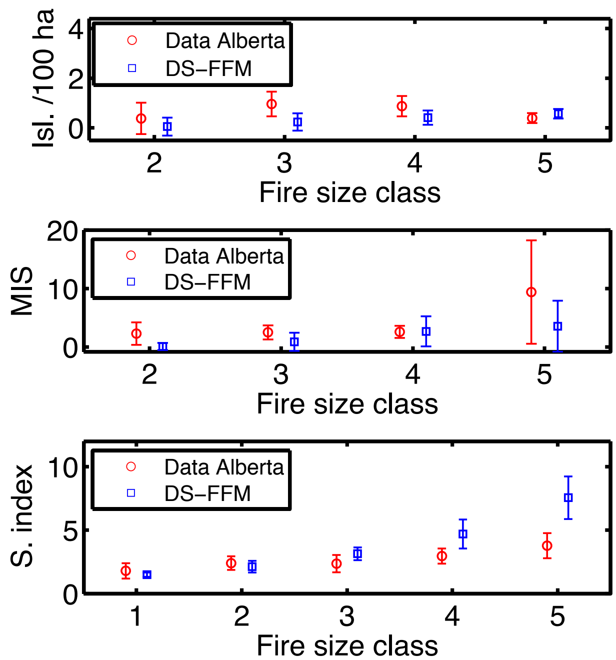

Fig. (5). The number of islands (NI), median island size (MIS) and shape index obtained for 2,000 fires in each size class (θ=200) in comparison to the same metrics taken from data in Alberta, Canada, [13].

DISCUSSION

Fire Shapes and the DS-FFM

We cannot support the conjecture of Caldarelli *et al*. [12] that the DS-FFM is unable to produce realistic fire shapes. The overall performance of the DS-FFM in reproducing data of 68 fires in Alberta is surprisingly good in general, although it depends on the fire size class. The match cannot be expected to be exact in a model as simple as the DS-FFM which ignores topography and weather influences. Nevertheless, this makes the qualitative agreement that we found even more astonishing.

The reason why Caldarelli *et al*. [12] were not able to match output of the DS-FFM to their data might be that they used data of three large fires (58, 60, and 156 km^2, respectively). Hence all three fires are comparable to those of the largest size class in the data set of Eberhart and Woodard [13], for which the match between data and model output was also limited in our analysis of structural properties. Large fires are affected stronger by changes in weather since they burn on the timescale of weeks rather than hours. A significant weather change is hence more likely to affect a larger fire. The DS-FM does not take this into account.

Nevertheless, we believe that the generally good fit is enough reason to warrant another look at the ecological interpretation of the DS-FFM. Note that this discussion does not focus on the role of self-organized criticality [1] in wildfire systems but only on the ecological assumptions which are implicitly contained in the DS-FFM.

The calibration of the model using the data of Eberhart and Woodard [13] made it necessary to introduce a reinterpretation of the original model. Whereas Drossel and Schwabl [8], who did not design their model for any ecological purpose, referred to grid cells occupied with forest as ‘trees’, our calibration to wildfire data showed that a grid cell should rather be associated with a forest stand of several hectare size. This finding has important implications for the ecological significance of the DS-FFM.

The DS-FFM from an Ecological Perspective

The DS-FFM seems to be overly unrealistic and thus of no relevance for fire and landscape ecology. Fire spread is deterministic, but recovery of vegetation is entirely stochastic, with ‘trees’ randomly popping up on empty grid cells. It would be more realistic to assume that fire spread is stochastic and that vegetation recovery includes a more deterministic component representing succession.

And indeed, the forest fire model of Ratz [6], which is also a simple cellular automaton that ignores topography, weather, and details of a local forest stand’ s structure, is based on these assumptions: stochastic fire spread and deterministic succession, described as aging, of stands within a grid cell. Consequently, the Ratz model is frequently cited by fire and landscape ecologists [22-24]. A further model that is very similar to the Ratz model was developed by Peterson [25]. It includes the concept of ‘memory’, i.e. the memory of the last fire event. Ratz [6] showed that his model was able to reproduce structural properties of the 68 wildfires analyzed by Eberhart and Woodard [13] surprisingly well.

Here we followed Ratz [6] and compared the output of the DS-FFM to the same set of wildfires. We were able to show that despite the almost inverse assumptions on which DS-FFM and the Ratz model are based, the DS-FFM is also able to reproduce structural properties of real wildfires.

We found that a grid cell corresponds to several hectares. The question then is, when does an entire stand become susceptible to fire? Crown fires develop in forest stands in which there is a minimum bulk density in the crown space which allows fire to consume and spread in the crowns. It is known for boreal forests that this property is attained and stable after the first two or three decades after a fire [26-28].

In the DS-FFM, however, the probability of recovery, or becoming susceptible to fire, of a single cell is *p*, but *p* refers to the time scale of fire events. The random, or Poisson, process of recovery needs several iterations between fires in order to rebuild connected clusters in the burned area.

The time scales of fire events and vegetation recovery must hence be separated to make sense if compared to wildfire systems. If we do so we arrive at the model implementation proposed by Grassberger ([17], see above). The elementary time step of the model is now the time between sparks, quantified by f, rather than the time scales of fire spread. In the case of a fire event, fire spreads instantaneously on the cluster of connected fuel. The average size of such a fuel patch depends on the number of grid cells which recover between two sparks. This is because the DS-FFM reaches a quasi-equilibrium in which the average area consumed by fire is equal to the average area which regrows [8]. Fluctuations around this equilibrium, or long-term average, are large and produce the well-known power-law like frequency-area distributions of fires in the model.

The core message for landscape ecology here is: it is possible to create this type of frequency-area distribution of fires simply by simulating the dynamics of fuel patches. This is a qualitative rather than quantitative result, and indeed the exponent obtained for the DS-FFM (λ=–1.16) is too large if compared with those found in actual wildfire data ($|\lambda| \in [1.2 - 1.8]$; [1]). The model produces too many large fires. Nevertheless, this is qualitatively acceptable given the multitude of aspects of fire spread neglected in this model, of which the most prominent is weather and its influence on burning conditions.

The role of the connectivity of the fuel patch in wildfire spread is discussed in detail by Turner and Romme [29]. The degree to which the fuel mosaic can determine fire shape is discussed as a function limited by the burning conditions which are controlled by weather. If burning conditions are poor, the fuel patch is wet and fires die out rapidly. If the conditions are good, the fire will spread to consume the entire patch of connected fuel. Extreme burning conditions can lead to fires which spread irrespectable of the fuel mosaic [30].

The DS-FFM with separated time scales assumes good burning conditions in this sense. Our results suggest that the fire shapes as produced by the model fit best to medium sized fires of 400-2,000 ha. This leads to the following hypothesis: smaller and larger fires are stronger influenced by weather and topography. Smaller wildfires have more islands than expected by the fuel-connectivity model underlying the DS-FFM. Large fires have less, yet larger islands. Nevertheless, the total amount of unburnt area is as predicted by the DS-FFM in both cases.

CONCLUSION

We conclude that the forest fire model by Drossel and Schwabl [8] should no longer be ignored by fire and landscape ecologists. It can serve as a minimalistic null model. Since the model does not include too many assumptions and hypothesis, it offers the advantage of being analytically tractable. This allows making important general predictions, for example: a larger sparking rate leads to smaller fires on average if all other factors are held constant. This suggests the existence of a buffer mechanism which might lie behind the debated ineffectiveness of fire suppression efforts [31]. It is in this spirit in which we believe this model can be applied to wildfire systems with benefit.

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