

# Dual-Hop Fixed Gain Relaying Transmissions with Semi-Blind in Asymmetric Multipath/Shadowing Fading Channels

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**Abstract:** In this paper, we investigate the end-to-end performance of a dual-hop fixed gain relaying system with semi-blind relay under asymmetric fading environments. In such environments, the wireless links of the considered system undergo asymmetric multipath/shadowing fading conditions, where one link is subject to only the Nakagami- $m$  fading, the other link is subject to the composite Nakagami-lognormal fading which is approximated by using mixture gamma fading model. First, the cumulative distribution function (CDF), the moment generating function (MGF) and the moments of the end-to-end signal-to-noise ratio (SNR) are derived under two asymmetric scenarios. Then, novel closed-form expressions of the outage probability, the average end-to-end SNR, the symbol error rate and the ergodic capacity for the dual-hop system are obtained based on the CDF and the MGF, respectively. Finally, some numerical and simulation results are shown and discussed to validate the accuracy of the analytical results under different scenarios, such as varying average SNR, fading parameters per hop, the choice of the semi-blind gain and the location of relaying nodes.

**Keywords:** Dual-hop relaying, fixed gain, composite fading channels, Mixture Gamma distribution, performance analysis.

## 1. INTRODUCTION

Wireless relaying transmissions have emerged as a promising technique for the high data-rate coverage required in the wireless communication networks [1]. In such networks, one source node communicates with one destination node through one or several intermediate nodes called relays. In the past few years, as an important relay transmission strategy, amplify-and-forward (AF) fixed gain systems have been widely investigated in terms of outage probability (OP), average bit/symbol error rate (ABER/ASER) and ergodic capacity for various system models and fading channel models, such as [2-5] and the references therein. This can be due to the fact that the fixed gain relay systems are not complicated and are easy to deploy which makes them more attractive from a practical viewpoint as compared to the variable gain AF relaying systems.

Recently, the interest in researching the asymmetric (or mixed) fading models has increased where all the single-hop links in wireless relaying systems experience different fading conditions. The authors in [6] and [7] first studied the end-to-end performance of dual-hop AF, relaying with both channel state information (CSI) based and fixed gain over mixed Rayleigh and Rician fading channels, respectively. After that, more cooperative relaying models are studied in mixed Rayleigh and Rician fading channels, such as [8, 9]. In [10], the authors analyzed the performance of dual-hop AF relaying in mixed Nakagami- $m$  and Rician fading channels. In [11], the authors studied the performance of a decode-and-forward cooperative system

under mixed Rayleigh and generalized Gamma fading channels. Performance analysis of dual-hop AF relaying systems is studied over mixed  $\Gamma$ - $\Gamma$   $\eta$ - $\mu$  and  $\Gamma$ - $\Gamma$   $\kappa$ - $\mu$  fading channels in [12]. The authors in [13] and [14] investigated the performance of the dual-hop fixed gain relaying system over composite multipath/shadowing fading channels, where the composite Nakagami- $m$ /lognormal (NL) distribution is approximated by using generalized-K (KG) distribution. Since the probability density function (PDF) of average signal-to-noise ratio (SNR) over KG fading channel includes modified Bessel functions, the outage probability and ASER in [13] include Meijer's G functions and some infinite-series representations. Some expressions remain complicated and intractable. In [14], the end-to-end moment generating function (MGF) is expressed by using Lommel function in the first scenario which the first hop undergoes in Rayleigh fading. Some approximations and bounds of the end-to-end MGF are derived in the second scenario which the first hop is subject to KG fading. To the best of our knowledge, some exact and simple closed-form expressions of the performance metric for the dual-hop fixed gain relaying system have not as yet been found over composite multipath/shadowing scenarios. Especially, no exact closed-form expression of the ergodic capacity for the dual-hop AF fixed gain relaying has been derived over any fading channels.

In [15], the authors developed an approach to approximate composite NL distribution by using mixture gamma (MG) distribution. This distribution avoids the above-mentioned problems, and some exact and simple results are obtained by adjusting the number of gamma distribution. In [16], the authors studied the end-to-end performance of dual-hop AF based CSI over composite NL fading channels using MG model, and found it is more precise and

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amenable to approximate the NL distribution by using MG model than KG model.

In this paper, we consider an asymmetric scenario of a dual-hop AF fixed-gain relaying system with semi-blind in a wireless propagation environment, where multipath fading, shadowing and the propagation path loss occur simultaneously. The semi-blind relaying only considers the statistical CSI of the first hop channels, and causes some performance degradation compared with the variable gain AF relaying. The S-R (first-hop) and the R-D (second-hop) links experience Nakagami-m or NL fading, where the NL fading model is approximated by using MG fading model. The main contribution of this paper is to first derive the statistics of the end-to-end SNR for the dual-hop fixed gain system, including the cumulative distribution function (CDF), the MGF and the moments. Then, some exact and simple closed-form expressions for the OP, the average SNR, the ASER, and the ergodic capacity are derived based on the analytical expressions of CDF and MGF, respectively. Also, two expressions for the parameter Z which describes the semi-blind relay gain is derived and discussed. With these results, we show various numerical and simulation results to confirm the accuracy of the proposed analysis under different conditions, such as varying average SNR, fading parameters per hop, the choice of the semi-blind gain and the location of relaying nodes.

## 2. SYSTEM AND CHANNEL MODELS

We consider a wireless dual-hop AF fixed gain relaying system in a mixed multipath/shadowing environment. The source node (S) communicates with the destination node (D) via a relaying node (R). The whole transmission is divided into two phases. In the first phase, S only transmits its signals to R, and in the second phase, R amplifies the received signals by a gain factor  $\beta$  and then forwards their amplified versions to D. Without loss of generality, we assume that the average powers of S and R are normalized to unity. If  $\beta$  is selected according to the fixed relay gain, which is defined as  $\beta^2 = 1/ZN_0\beta^2 = P_1/ZN_0$  as in [2]. Thus, the instantaneous end-to-end SNR,  $\gamma_{SRD}$ , at the destination can be expressed as in [2]

$$\gamma_{SRD} = \gamma_1\gamma_2/(\gamma_2 + Z) \quad (1)$$

where  $\gamma_i = \rho_i |h_i|^2$  is the instantaneous SNR of the  $i^{\text{th}}$ -hop link,  $|h_i|$  is the fading amplitude of the  $i^{\text{th}}$  hop link,  $i \in \{1, 2\}$ ,  $\rho_i = 1/N_0$  denotes the un-faded SNR,  $N_0$  is the power of the additive white Gaussian noise component,  $Z$  is a constant for a fixed gain  $\beta$ . Then,  $\bar{\gamma}_i = \rho \mathbf{E}[|h_i|^2] = \rho \Omega_i$  denotes the average SNR of the  $i^{\text{th}}$ -hop link,  $\mathbf{E}[\cdot]$  is the statistical expectation,  $\Omega_i$  denotes the deviation of  $|h_i|^2$ . Due to the capture of the path-loss effect, we define the local mean power  $\Omega_i = (d_0/d_i)^\varepsilon$ ,  $d_0$  denotes the distance between S and D,  $d_i$  is the distance of the  $i^{\text{th}}$  hop link, and  $\varepsilon$  is the path-loss exponent.

For (1), we consider two asymmetric scenarios. For the first asymmetric scenario denoted by S1, the first-hop link is subject to Nakagami-m fading and the second-hop link is subject to NL fading. In contrast, when the first-hop link undergoes NL fading and the second-hop link is

subject to Nakagami-m fading, this fading condition is identified as S2. Nakagami-m fading is more general and versatile, and covers a broad variety of multi-path fading. The proposed model can represent either an up or down link in a mobile communication network, such as a mobile station at the edge of the cell acts as S, another mobile (or fixed) station as R and a base station as D.

If the  $i^{\text{th}}$ -hop link experiences Nakagami-m fading,  $\gamma_i$  is a Gamma distributed variable with the PDF given by [17]

$$f_{\gamma_i}(\gamma) = A_i \gamma^{m_i-1} \exp(-B_i \gamma) \quad (2)$$

Where,  $A_i = B_i^{m_i} / \Gamma(m_i)$ ,  $B_i = m_i / \bar{\gamma}_i$ ,  $m_i$  is Nakagami-m fading parameter and is a integer. The CDF of  $\gamma_i$  can be obtained as:

$$F_{\gamma_i}(\gamma) = 1 - \Gamma(m_i, B_i \gamma) / \Gamma(m_i) \quad (3)$$

Where,  $\Gamma(\cdot, \cdot)$  is the upper incomplete gamma function.

When the  $i^{\text{th}}$ -hop link experiences NL fading,  $\gamma_i$  is a composite Gamma-lognormal distribution variable with the PDF given by [17]

$$f_{\gamma_i}(x) = \int_0^\infty \frac{m_i^m x^{m_i-1} \exp[-\frac{m_i x}{\rho y}]}{\Gamma(m_i)(\rho y)^{m_i}} \frac{1}{\rho y \sqrt{2\pi} \lambda_i y} \exp\left[-\frac{(\ln y - \mu_i)^2}{2\lambda_i^2}\right] dy \quad (4)$$

where  $\mu_i$  and  $\lambda_i$  are the mean and the standard deviation of lognormal shadowing, respectively,  $\lambda_i = (\ln 10/10)\sigma$ ,  $\mu_i = \ln \Omega_i$ ,  $\sigma$  denotes the standard deviation in dB.

Since a closed-form expression of (4) is not available in the open literature, the performance metrics of digital communication systems over NL distribution are intractable or difficult. Some approximations or simple forms of (4) have been paid great attention recently, such as KG,  $\mathcal{G}$  and MG fading models. Due to that KG and models include modified Bessel functions in PDFs, some expressions of the performance metrics still keep mathematical complications, and further approximations have to be adopted as in [19] and [20]. In order to avoid the above problems, we use MG distribution presented in [15] to approximate the composite NL distribution in this paper. Thus, the PDF of  $\gamma_i$  can be expressed as:

$$f_{\gamma_i}(x) = \sum_{j=1}^N T_j x^{m_i-1} \exp(-M_j x), \quad (5)$$

where  $a_j = 2m_i^m w_j \exp[-m_i(\sqrt{2}\lambda_{t_j} + \mu_i)] / \sqrt{\pi}\Gamma(m_i)$ ,  $a_j = 2m_i^m w_j \exp[-m_i(\sqrt{2}\lambda_{t_j} + \mu_i)] / \sqrt{\pi}\Gamma(m_i)$ ,  $M_j = b_j / \rho$ ,  $M_j = b_j / \rho$ ,  $T_j = c_j a_j / 2\rho^{m_i}$ ,  $b_j = m_i \exp[-(\sqrt{2}\lambda_{t_j} + \mu_i)]$ ,  $c_j = \sqrt{\pi} / \sum_{j=1}^N w_j$  is the normalization factor,  $t_j$  and  $w_j$  are weight factors and abscissas for Gaussian-Hermite integration,  $t_j$  and  $w_j$  for different  $N$  values are available in [21, Table (25.10)].

The CDF of  $\gamma_i$  over MG fading can be obtained as:

$$F_{\gamma_i}(\gamma) = 1 - \sum_{j=1}^N R_j \Gamma(m_i, M_j \gamma), \quad (6)$$

where  $R_j = c_j a_j / 2b_j^m$ .

### 3. STATISTICS OF END-TO-END SNR

In this section, we first derive the closed-form CDF expressions of the end-to-end SNR for the dual-hop fixed gain system under S1 and S2, and then find their MGFs and the  $q^{\text{th}}$  moments based on the CDF expressions.

#### 3.1. CDF of end-to-end SNR

For the dual-hop fixed gain system, by using (1), the CDF of  $\gamma_{SRD}$  can be expressed as:

$$\begin{aligned} F_{\gamma_{SRD}}(x) &= \Pr(\gamma_{SRD} \leq x) \\ &= \int_0^\infty \Pr\left[\frac{\gamma_1 + y}{Z + y} \leq x \mid y\right] f_{\gamma_2}(y) dy \\ &= \int_0^\infty \Pr[\gamma_1 \leq (Z + y)x/y \mid y] f_{\gamma_2}(y) dy \end{aligned} \quad (7)$$

In the case of scenario S1, since the first-hop link is only subject to Nakagami-m fading,  $\gamma_1$  is a gamma distributed random variable. Thus, we can obtain the term  $\Pr[\bullet \mid \bullet]$  in (7) as:

$$\Pr[\gamma_1 \leq (Z + y)x/y \mid y] = 1 - \frac{\Gamma[m_1, B_1 x(Z + y)/y]}{\Gamma(m_1)}. \quad (8)$$

By using eq. (5) and eq. (8), and substituting them into (7), and with the help of the series expression of  $\Gamma(\bullet, \bullet)$  defined in [18, (8.352.2)], the binomial expansion defined in [18, (1.111)], and (3.471.9) in [18], after applying some algebraic manipulations, eq. (7) can be rewritten as:

$$\begin{aligned} F_{\gamma_{SRD-S1}}(x) &= 1 - \sum_{i=1}^N \sum_{j=0}^{m_i-1} \sum_{k=0}^j \frac{2C_k^j T_i Z^{(m_2-k+j)/2}}{M_i^{(m_2+k-j)/2} j!} (B_1 x)^{(m_2+k+j)/2} \\ &\quad \times \exp[-B_1 x] K_{m_2+k-j}[2\sqrt{xZM_i B_1}] \end{aligned} \quad (9)$$

where  $C_k^j = j! / [(j-k)! k!]$  is the binomial coefficients,  $K_\alpha(\bullet)$  is the second kind modified Bessel function of order  $\alpha$ .

Similarly, for the scenario S2, due to that the shadowed fading is considered in the first-hop link,  $\gamma_1$  is a gamma-lognormal distributed random variable. By using (6), we can obtain the term  $\Pr[\bullet \mid \bullet]$  in (7) as:

$$\Pr[\gamma_1 \leq (Z + y)x/y \mid y] = 1 - \sum_{i=1}^N R_i \Gamma\left[m_1, \frac{M_i x(Z + y)}{y}\right] \quad (10)$$

Similar as eq. (9), and substituting eq. (2) and (10) into eq. (7), the CDF of  $\gamma_{SRD}$  under the scenario S2 can be given by

$$\begin{aligned} F_{\gamma_{SRD-S2}}(x) &= 1 - \sum_{i=1}^N \sum_{j=0}^{m_i-1} \sum_{k=0}^j \frac{2A_k C_k^j \Gamma(m_1) Z^{(m_2-k+j)/2}}{j! B_2^{(m_2+k-j)/2} \Gamma(m_2) / R_i} (M_i x)^{(m_2+k+j)/2} \\ &\quad \times \exp[-M_i x] K_{m_2+k-j}[2\sqrt{xZM_i B_2}] \end{aligned} \quad (11)$$

#### 3.2. MGF of end-to-end SNR

Due to the fact that MGF is defined as Laplace transform of PDF, MGF can also be obtained by using CDF. By using the integration property of Laplace transform, the MGF of  $\gamma_{SRD}$  can be expressed as:

$$MGF_{\gamma_{SRD}}(s) = s \int_0^\infty \exp(-s\gamma) F_{\gamma_{SRD}}(\gamma) d\gamma. \quad (12)$$

In the case scenario of S1, by expressing  $K_\nu(2\sqrt{x}) = (1/2)G_{0,2}^{2,0}[x \mid_{\nu, 2-\nu}]$  and  $\exp(-x) = G_{0,1}^{1,0}[x \mid_0^-]$  as a Meijer's G function form defined in [22, (03.04.26.0009.01)] and [22, (01.03.26.0004.01)], and substituting eq. (9) into eq. (12), and with the help of (6.621.3) in [18], after applying some algebraic manipulations, the MGF of  $\gamma_{SRD}$  under the scenario S1 can be given by

$$MGF_{S1}(s) = 1 - \sum_{i=1}^N \sum_{j=0}^{m_i-1} \sum_{k=0}^j \frac{s C_k^j T_i M_i^{-(m_2+k)}}{j! Z^k (B_1 + s)} G_{1,2}^{2,1}\left[\frac{Z B_1 M_i}{B_1 + s} \mid_{m_2+k, j}^0\right] \quad (13)$$

where  $G[\bullet \mid \bullet]$  is the Meijer's G-function.

Similar as eq. (13), and substituting eq. (11) into eq. (12), the MGF of  $\gamma_{SRD}$  under the scenario S2 can be given by

$$MGF_{S2}(s) = 1 - \sum_{i=1}^N \sum_{j=0}^{m_i-1} \sum_{k=0}^j \frac{s C_k^j A_2 R_i \Gamma(m_1) Z^{-k}}{j! B_2^{(m_2+k)} (M_i + s)} G_{1,2}^{2,1}\left[\frac{Z B_2 M_i}{M_i + s} \mid_{m_2+k, j}^0\right] \quad (14)$$

#### 3.3. The $q^{\text{th}}$ Moment of end-to-end SNR

The  $q^{\text{th}}$  moment of the end-to-end SNR can be derived by using CDF as:

$$\mu(\gamma^q) = q \int_0^\infty \gamma^{q-1} [1 - F_{\gamma_{SRD}}(\gamma)] d\gamma \quad (15)$$

Similar to eq. (13), and substituting eq. (9) into eq. (15), the  $q^{\text{th}}$  moment of  $\gamma_{SRD}$  under the scenario S1 can be given by

$$\mu_{S1}(\gamma^q) = \sum_{i=1}^N \sum_{j=0}^{m_i-1} \sum_{k=0}^j \frac{q C_k^j T_i Z^{-k}}{j! M_i^{(m_2+k)} B_1^q} G_{1,2}^{2,1}[Z M_i \mid_{m_2+k, j}^{1-q}]. \quad (16)$$

Then, the  $q^{\text{th}}$  moment of  $\gamma_{SRD}$  under the scenario S2 can be given by

$$\mu_{S2}(\gamma^q) = \sum_{i=1}^N \sum_{j=0}^{m_i-1} \sum_{k=0}^j \frac{q C_k^j A_2 R_i Z^{-k}}{j! B_2^{(m_2+k)} M_i^q} G_{1,2}^{2,1}[Z B_2 \mid_{m_2+k, j}^{1-q}]. \quad (17)$$

### 4. PERFORMANCE ANALYSIS

In this section, based on the statistics of the end-to-end SNR in section 3, we derived the closed-form expressions of the OP, the average SNR, the ASER and the ergodic capacity over mixed fading channels, respectively. Then, the selective strategies of the semi-blind gain are discussed.

#### 4.1. Outage Probability

The OP is an important performance metric that is commonly used to characterize a digital communication system. It is defined as the probability that the instantaneous end-to-end SNR falls below a given threshold ( $\gamma_{th}$ ), this is,

$P_{out} = \int_0^{\gamma_{th}} f_{\gamma}(\gamma) d\gamma$ , where  $f_{\gamma}(\gamma)$  is the PDF of the instantaneous SNR. Using eq. (9) and eq. (11), a uniform expression of the OP under both S1 and S2 can be obtained as:

$$P_{out} = F_{\gamma_{SRD}}(\gamma_{th}) \quad (18)$$

#### 4.2. Average end-to-end SNR and Amount of Fading

The average end-to-end SNR is a useful performance measure serving as an excellent indicator of the overall system's fidelity. Therefore, the average end-to-end SNRs for the dual-hop fixed gain system can be obtained by setting  $q=1$  in eq. (16) and (17) under both S1 and S2, respectively.

The amount of fading (AoF) is a unified measure of the severity of fading, which is typically independent of the average fading power and is defined as:

$AoF = \mu(\gamma^2)/\mu^2(\gamma) - 1$  in [17]. The AoFs of dual-hop fixed gain relaying system can be obtained by setting  $q=1$  and 2 in (16) and (17) under both S1 and S2, respectively.

#### 4.3. Average Symbol Error Rate

Using the MGF-based approach, we can obtain the closed-form expression of ABER/SER of the above dual-hop systems over mixed fading channels. For many coherent demodulation schemes, the ASER for  $M$ -ary phase-shift keying signals ( $M$ -PSK) can be given by [17]

$$P_{e-MPSK} = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_{\gamma_{SRD}} \left( \frac{g_M}{\sin^2 \theta} \right) d\theta \quad (19)$$

where  $g_M = \sin^2(\pi/M)$ . Thus, the ASER of MPSK for dual-hop fixed gain system can be numerically evaluated by substituting eq. (13) and eq. (14) into eq. (19) under both S1 and S2. These can be done with some elementary numerical integration techniques.

#### 4.4. Ergodic Capacity

For a dual-hop AF system with fixed gain, the ergodic capacity can be obtained as:

$$\bar{C} = \Delta E \left[ \ln(1 + \gamma_{SRD}) \right] \quad (20)$$

where  $\Delta = 1/2 \ln 2$ , the factor 1/2 accounts for the fact that the transmission process takes place in two orthogonal channels.

Since an exact closed-form expression in (20) over mixed fading channels is not mathematically tractable by directly using a traditional approach (i.e., finding the PDF of  $\gamma_{SRD}$ ), we restructure eq. (20) as in [23]

$$\bar{C} = \Delta \left\{ \underbrace{E[\ln(1 + (1 + \gamma_1)\gamma_2/Z)]}_{\bar{C}_1} - \underbrace{E[\ln(1 + \gamma_2/Z)]}_{\bar{C}_2} \right\} \quad (21)$$

In order to find the closed-form expression of (21), we must obtain the closed-form expressions of  $\bar{C}_1$  and  $\bar{C}_2$ . Here, we let  $X = \gamma_2/Z$ ,  $Y = 1 + \gamma_1$ ,  $U = XY$ . Thus, the key problem is to find the PDFs of variables  $X$ ,  $Y$  and  $U$ , respectively.

In the asymmetric case of S1, with the help of eq. (2) and eq. (5), and using the variable transform method, the PDFs of  $X$  and  $Y$  can be obtained, respectively, as:

$$f_X(x) = \sum_{i=1}^N T_i Z^{m_i} x^{m_i-1} \exp(-ZM_i x) \quad (22)$$

$$f_Y(y) = A_i \exp(B_i)(y-1)^{m_i-1} \exp(-B_i y) \quad (23)$$

Then, by using eq. (22), the closed-form expression of  $\bar{C}_2$  can be written as:

$$\begin{aligned} \bar{C}_2 &= E[\ln(1+X)] = \int_0^{\infty} \ln(1+x) f_X(x) dx \\ &= \sum_{i=1}^N T_i Z^{m_i} \int_0^{\infty} x^{m_i-1} \ln(1+x) \exp(-ZM_i x) dx \end{aligned} \quad (24)$$

By expressing  $\ln(1+x) = G_{2,2}^{1,2}[x]_{1,0}^{1,1}$  in eq. (24) as a Meijer's G function defined in [22, (01.04.26.0003.01)], and using (07.34.21.0011.01) in [22], eq. (24) can be rewritten as:

$$\bar{C}_{2-S1} = \sum_{i=1}^N R_i G_{2,3}^{3,1}[ZM_i]_{m_i,0,0}^{0,1} \quad (25)$$

In the following, for the sake of finding the PDF of  $U$ , we let  $V=X$  as an auxiliary variable. Due to the fact of the independence between  $\gamma_1$  and  $\gamma_2$ ,  $X$  and  $Y$  are also independent of each other. Thus, by using Jacobian determinant, we can obtain the composite PDF of  $V$  and  $U$  as:

$$f_{UV}(u,v) = f_X(v) f_Y(u/v) / v \quad (26)$$

By using eq. (22) and (23), and the binomial expansion defined in [18] when  $m_i$  is integer, and with the aid of (6.621.3) in [18], the PDF of  $U$  can be written as:

$$\begin{aligned} f_U(u) &= \int_0^{\infty} f_{UV}(u,v) dv \\ &= \sum_{i=1}^N \sum_{j=0}^{m_i-1} \frac{2C_j^{m_i-1} T_i A_i (-1)^j Z^{(m_i+m_2-j)/2}}{(\exp(B_i))^{-1} (M_i/B_i)^{(m_2-m_i+j)/2}} \\ &\quad \times u^{(m_i+m_2-j)/2-1} K_{m_2-m_i+j} (2\sqrt{ZM_i B_i u}) \end{aligned} \quad (27)$$

By using eq. (27), similar as eq. (25), the closed-form expression of  $\bar{C}_1$  can be obtained as:

$$\bar{C}_{1-S1} = \sum_{i=1}^N \sum_{j=0}^{m_i-1} \frac{C_j^{m_i-1} T_i A_i \exp(B_i)}{(-1)^j B_i^{m_i-j} M_i^{m_2}} G_{2,4}^{4,1}[ZM_i B_i]_{m_2, m_i-j, 0, 0}^{0,1} \quad (28)$$

Then, by substituting eq. (25) and eq. (28) into eq. (21), we can obtain the exact closed-form expression of the ergodic capacity under S1.

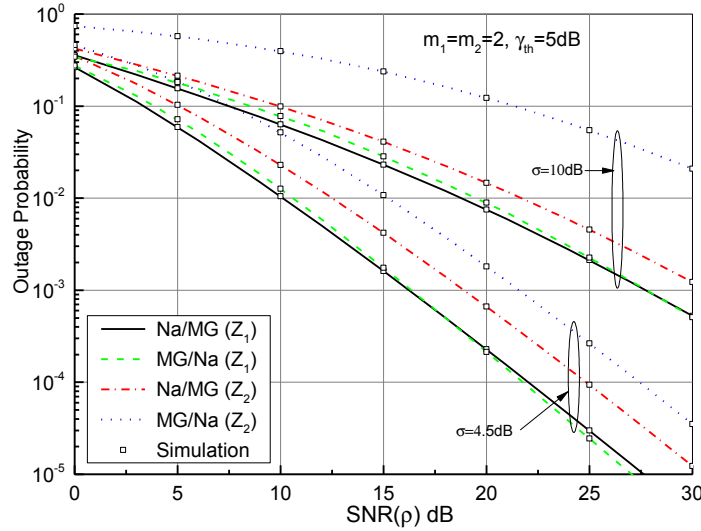


Fig. (1). Outage probability for the dual-hop fixed gain system versus the un-faded SNR ( $\rho$ ) under both S1 and S2 scenarios.

In the asymmetric case of S2, with the help of eq. (2) and eq. (5), and using the variable transform method, the PDFs of  $X$  and  $Y$  can be obtained, respectively, as:

$$f_x(x) = A_2 Z^{m_2} x^{m_2-1} \exp(-B_2 Zx) \quad (29)$$

$$f_y(y) = \sum_{i=1}^N T_i \exp(M_i)(y-1)^{m_1-1} \exp(-M_i y) \quad (30)$$

Similar as eq. (27), by using eq. (29) and eq. (30), the PDF of  $U$  in the asymmetric case of S2 can be written as:

$$f_U(u) = \sum_{i=1}^N \sum_{j=0}^{m_1-1} \frac{2C_j^{m_1-1} T_i A_2 (-1)^j Z^{(m_1+m_2-j)/2}}{(\exp(M_i))^{-1} (B_1/M_i)^{(m_2-m_1+j)/2}} \times u^{(m_1+m_2-j)/2-1} K_{m_2-m_1+j}(2\sqrt{ZM_i B_2} u) \quad (31)$$

Thus, by using eq. (29) and eq. (31), after applying some algebraic manipulations, we can obtain the closed-form expressions of  $\bar{C}_1$  and  $\bar{C}_2$  under S2, respectively, then substituting them into (21), the ergodic capacity under S2 can be expressed as:

$$\bar{C}_{s2} = \Delta \left( \sum_{i=1}^N \sum_{j=0}^{m_1-1} \frac{C_j^{m_1-1} T_i A_2 \exp(M_i)}{(-1)^j M_i^{m_1-j} B_2^{m_2}} G_{2,4}^{4,1} [ZM_i B_2]_{m_2, m_1-j, 0, 0}^{0,1} \right) - G_{2,3}^{3,1} [ZB_2]_{m_2, 0, 0}^{0,1} / \Gamma(m_2) \quad (32)$$

4.5. The Choice of the Fixed Relay Gain

For the dual-hop fixed gain system using semi-blind relay, the relay gain is determined by the channel statistics at the first hop. In general, there are two major schemes to calculate the semi-blind gain as in [5].

In the first scheme, the fixed-gain relaying factor  $\beta$  is chosen equal to the average of CSI assisted gain as:

$$\beta^2 = \rho E\{1/(\gamma_1 + 1)\} = \rho \int_0^\infty f_{\gamma_1}(\gamma_1) / (\gamma_1 + 1) d\gamma_1 \quad (33)$$

Since the first-hop link undergoes Nakagami- $m$  fading in the case of scenario S1, the constant  $Z$  is given by

$$Z_{1-S1} = 1/[A_1 \exp(B_1) \Gamma(m_1) \Gamma(1-m_1, B_1)] \quad (34)$$

Similarly, since the first-hop link undergoes NL fading in the case of scenario S2, the constant  $Z$  is given by

$$Z_{1-S2} = 1/[\sum_{i=1}^N T_i \exp(M_i) \Gamma(m_1) \Gamma(1-m_1, M_i)] \quad (35)$$

In the second scheme, the fixed-gain relaying factor  $\beta$  is chosen as:

$$\beta^2 = \rho / (E\{\gamma_1\} + 1) = \rho / (\bar{\gamma}_1 + 1) \quad (36)$$

Thus, the constant  $Z$  is determined by the average SNR of the first-hop in the case of the scenarios of S1 and S2, and its uniform expression can be obtained as:

$$Z_2 = \bar{\gamma}_1 + 1 \quad (37)$$

where  $\bar{\gamma}_1 = \rho \Omega_1$  under S1, and  $\bar{\gamma}_1 = \sum_{i=1}^N R_i \Gamma(m_1 + 1) / M_i$  under S2.

5. NUMERICAL RESULTS AND DISCUSSION

In this section, we present some numerical and simulation results to evaluate the performance of the dual-hop fixed gain system using semi-blind schemes in mixed multipath/shadowing fading channels.

Fig. (1) illustrates the OP versus the un-faded SNR ( $\rho$ ) under both S1 (Na/MG) and S2 (MG/Na) scenarios where  $m_1=m_2=2$ , and  $\gamma_{th}=5\text{dB}$ . The impacts of the shadowing parameter ( $\sigma$ ) and the constant ( $Z$ ) on the OP are only considered. The impact of the multipath fading parameter ( $m$ ) on the OP can be found in [14]. In this case, a symmetric network geometry is assumed, this is,  $d_0=1, d_1=d_2=0.5, \epsilon=4, N=10$  for MG distribution.

As expected, it can be seen from Fig. (1) that the OP is improved as  $\rho$  increases, and decreases when the shadowing

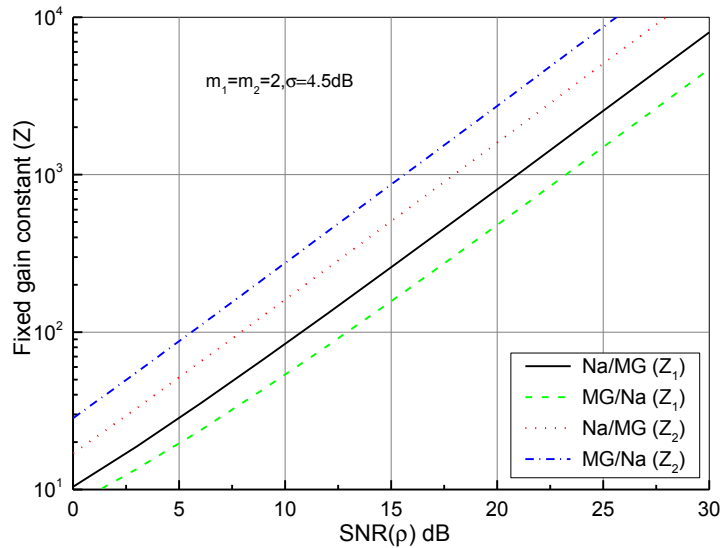


Fig. (2). Fixed-gain constant ( $Z$ ) versus the un-faded SNR ( $\rho$ ) under both S1 and S2 scenarios.

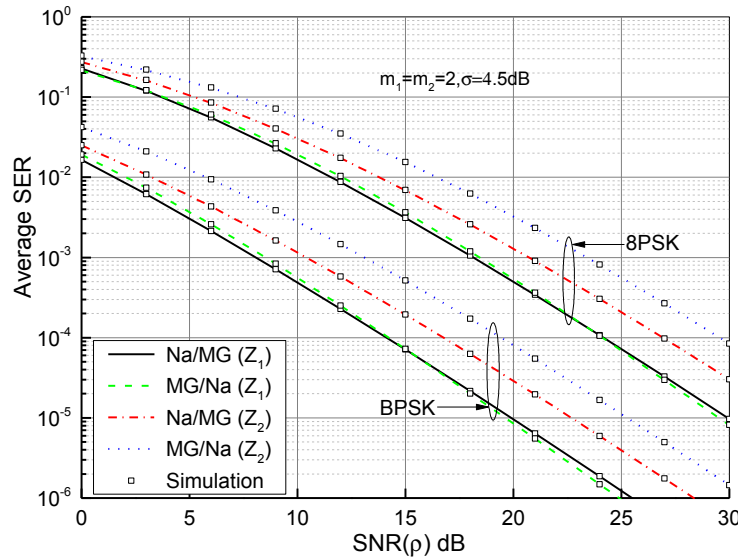


Fig. (3). Average SER of MPSK for the dual-hop fixed gain system versus the un-faded SNR ( $\rho$ ) under both S1 and S2 scenarios.

deviation ( $\sigma$ ) increases ( $\sigma=4.5\text{dB}\rightarrow 10\text{dB}$ ) under both S1 and S2. The impact of fading asymmetry on the OP, under the case of S1 shows better performance than S2 using  $Z_2$  in the entire range of  $\rho$ , and the performance of both cases using  $Z_1$  outperforms that using  $Z_2$ , whereas, the case under S1 is not always better than the one under S2 by using  $Z_1$ . These results can be explained from two points. It is a fact that for the dual-hop fixed gain system the end-to-end performance is dominated by the first-hop channel gain. The other fact is that the calculation of the fixed-gain in (33) and (36) differs in the position of the expectation operator. The case using  $Z_1$  averages the received signal-plus-noise power, and the case using  $Z_2$  only averages the received signal power, such that  $Z_2$  is always greater than  $Z_1$ . It can be proved from Fig. (2), and the value of  $Z_1$  under S2 (MG/Na) scenarios is minimum. Thus, it can be seen from Fig. (1) that the case under S2 using  $Z_1$  shows almost similar performance as the case under S1 in medium and high SNR. At the same time, the

simulation results in Fig. (1) coincide perfectly with the analytical results in (9) and (11), and verify the mathematical accuracy.

Fig. (3) shows the ASER of MPSK versus the un-faded SNR ( $\rho$ ) under both S1 and S2 scenarios where  $m_1=m_2=2$ , and  $\sigma=4.5\text{dB}$ . In this case, the symmetric network geometry is assumed as in Fig. (1). It can be seen from Fig. (3) that ASER of MPSK is improved as  $\rho$  increases. The impact of fading asymmetry on ASER shows agreement with the ones on the OP in Fig. (1). As expected, the performance of MPSK shows better when the value of  $M$  is smaller. At the same time, the simulation results still agree perfectly with the analytical results.

In Fig. (4), we show the impact of the relay location on the ABER of BPSK for the dual-hop fixed-gain system under both S1 and S2 scenarios where  $m_1=m_2=2$ . In this section, the asymmetric network geometry is examined where R is moved on a straight line between S and D,  $d_1$  denotes the

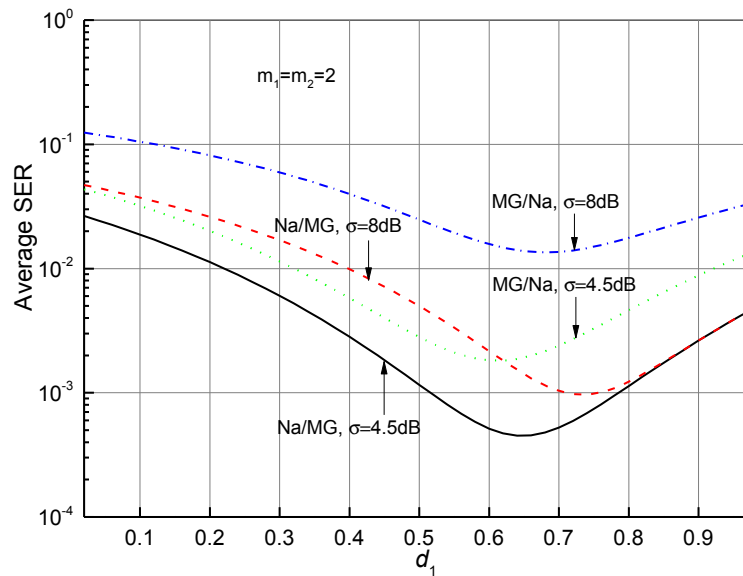


Fig. (4). Average SER of BPSK for the dual-hop fixed gain system versus  $d_1$  under both S1 and S2 scenarios, and  $Z_2$ .

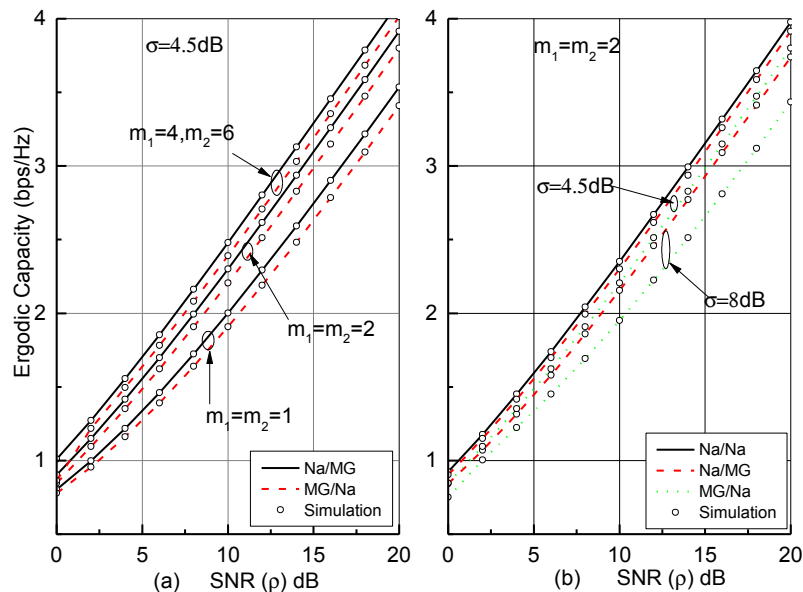


Fig. (5). Ergodic capacity for the dual-hop fixed gain system versus the un-faded SNR ( $\rho$ ) under both S1 and S2 scenarios.

distance between S and R,  $\rho=10\text{dB}$ ,  $N=10$  for MG distribution. From Fig. (4), it can be seen that the optimum performance under both S1 and S2 scenarios is located at the right of the middle, and the case under S1 is closer to D. These results can be caused by the first-hop fading gain and the path loss. It is due to the fact that the optimum performance often takes place when the first-hop and second-hop have a similar performance. For example, the first-hop fading gain is better than the second-hop under the case of S1, such that the first-hop needs more path loss than the second-hop. If the shadow deviation increases ( $\sigma=4.5\text{dB}\rightarrow\sigma=8\text{dB}$ ), the location of the optimum performance moves towards D. These results are helpful to the selection of relaying nodes in relaying networks.

Fig. (5) illustrates the ergodic capacity in (21) versus the un-faded SNR ( $\rho$ ) under different fading parameters, where  $d_1=d_2=0.5$ , and the constant  $Z_2$  is selected. As expected, the

ergodic capacity increases with increasing  $\rho$  from this figure. At the same time, it is clear that they match well between our exact analytical results and simulations over entire range of  $\rho$ . Fig. (5a) shows the impact of multipath parameters ( $m$ ) on ergodic capacity. It can be seen that the ergodic capacity increases with increasing  $m_i$  ( $i=1,2$ ), and that the capacity performance under S1 outperforms that under S2. It can be explained by the fact that the end-to-end performance is dominated by the channel gain of the first-hop for the dual-hop fixed gain system. Fig. (5b) shows the impact of shadowing ( $\sigma$ ) parameters on ergodic capacity. As expected, the ergodic capacity decreases with increasing  $\sigma_i$  under both S1 and S2.

Fig. (6) plots the AoF of the dual hop fixed gain system versus the un-faded SNR ( $\rho$ ), where  $m_1=m_2=2$ , and  $\sigma=4.5\text{dB}$ . It can be seen that the AoF changes slightly when  $\rho$  increases.

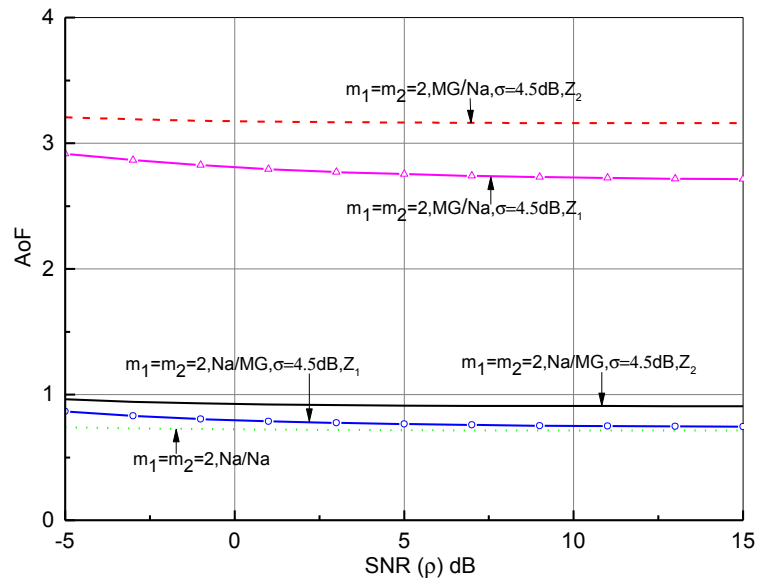


Fig. (6). AoF of the dual hop fixed gain system versus the un-faded SNR ( $\rho$ ) under both S1 and S2 scenarios.

es, or when the first-hop link undergoes Nakagami- $m$  fading. However, it changes largely when the first-hop link undergoes MG fading.

## CONCLUSION

In this paper, we investigate the end-to-end performance of a dual-hop fixed gain relaying system with semi-blind over mixed multipath/shadowing fading conditions, where the composite NL fading is approximated by using MG fading model. First, we derived the CDF, the MGF and the moments of the end-to-end SNR which is derived under S1 and S2 scenarios. The analytical expressions of CDF and MGF, novel closed-form expressions of the OP, the ASER and the ergodic capacity for the dual-hop system are derived, respectively. Finally, some numerical and simulation results are shown and discussed to validate the accuracy of the analytical results under different scenarios, such as varying average SNR, fading parameters per hop, the choice of the semi-blind gain and the location of relaying node. These works in this paper can be helpful to analyze the performance of cooperative relaying systems over composite fading channels in the future.

## CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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Declared none.

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