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The Planar *k*-Centra Location Problem

Siddarth R. Kotian[1], Claribel Bonilla[2] and Trevor S. Hale*,[3]

[1]*Department of Industrial and Systems Engineering, Ohio University, USA*

[2]*Department of Industrial and Systems Engineering, University of San Diego, USA*

[3]*Department of Management, Marketing, and Business Administration, University of Houston - Downtown, USA*

Abstract: The objective of this research is to develop a procedure that will solve, contingent on a necessary and sufficient optimality condition, the planar *k*-centra single facility location problem with Euclidean distances. The planar *k*-centra location problem seeks to find a location that minimizes the sum of the Euclidean distances to the *k* furthest existing facilities. The intrigue herein lies in the inability to define the *k* furthest away *a priori* of determining the associated median location. Hence, an iterative procedure is developed that can lead to optimal solutions but is subject to degeneracy. Results suggest that this degeneracy is instantiation specific to the *k*-centra location problem.

INTRODUCTION

The objective of this study is to solve the planar *k*-centra single facility problem with Euclidean distances. The problem statement is to locate a single service point that minimizes the sum of the distances to the *k*-farthest demand points out of a set of given points. A *k*-centra problem is a location analysis problem, which encloses attributes from both the center as well as the median approaches. However, this is not a linear combination. In a *k*-centra problem, a set value of *k* is predefined and is used in the evaluation. If $k = 2$ (or possibly 3) then the corresponding problem becomes the associated 1-center problem and when $k = n$ the corresponding problem becomes the associated 1-median problem, thus enclosing both approaches. It also provides the decision maker with a more flexible tool. A real life example for the *k*-centra problem could be the location of a warehouse to satisfy distant customers. The advantage here over the minimax (center) solution for a similar problem is that minimax solutions are governed by one or two outlier locations. So, for example, in the case of an isolated unit located a significant distance from, say, a cluster formation of other units, the minimax problem applied to service all of the units is reduced to the minimization of the distance of the one particular unit. The *k*-centra location problem, on the other hand, considers the *k* largest distances thus encompassing more of the available data before deciding on the solution.

PROPOSED APPROACH

The solution procedure proposed herein is a step-by-step methodology that incorporates Weiszfeld's algorithm [1]. Starting with the minimax location solution, the distance from the radius of the solution would be reduced until *k* points lie outside the circle and $(n - k)$ points lie inside the circle as demonstrated in Fig. (**1**) for $k = 12$. Then, Weiszfeld's algorithm is applied to the *k* points outside the circle. The solution is then tested for optimality, and if the original *k* points are still the *k* furthest away from the median location, the solution is optimal. If not, this procedure is repeated again until either optimality or degeneracy is obtained. This research also proposes ways for dealing with degenerate problem instantiations.

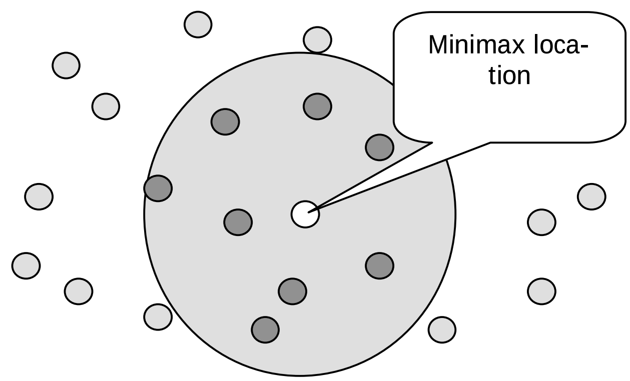

Fig. (1). Circle containing $(n - k)$ existing locations.

EXAMPLE PROBLEM TYPES

Three example problems, each of a different type are evaluated in this study to help validate the model as well as to show the efficacy of the approach. The problems were derived from examples in real life scenarios. The objective function remains the same for all the example problems, which is to locate a single service facility to best serve the *k* furthest away customers. The first problem considered is a typical unplanned city problem with random existing customer locations as shown in Fig. (**2**).

The next example problem depicted in Fig. (**3**) considers is a planned layout where customers are located in the annular region surrounding a central facility. The central facility could be a historic landmark or a city center.

*Address correspondence to this author at the Department of Management, Marketing, and Business Administration, University of Houston - Downtown, USA; Tel: +001-(713) 221-8262; Fax: +001-(713) 226-5226;
E-mail: halet@uhd.edu

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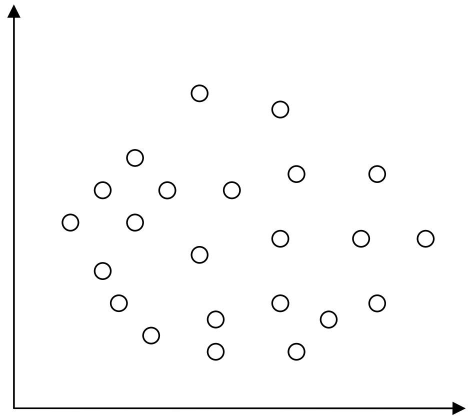

Fig. (2). An unplanned, random problem.

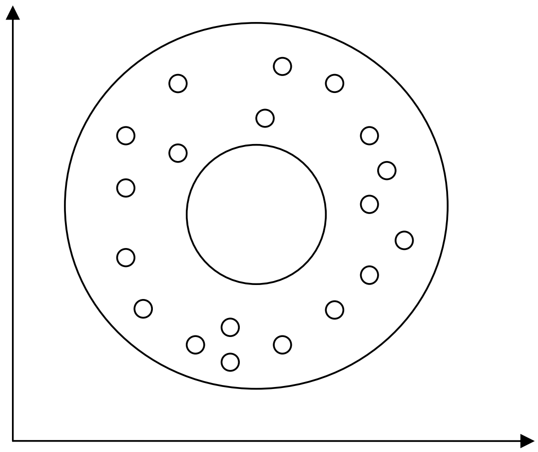

Fig. (3). An annular layout problem.

The final example considered herein is one where customer settlements are bunched together in isolated clusters all around the city. This example, depicted in Fig. (**4**) below, holds well in an industrial city where the workforce resides in the few available residential areas.

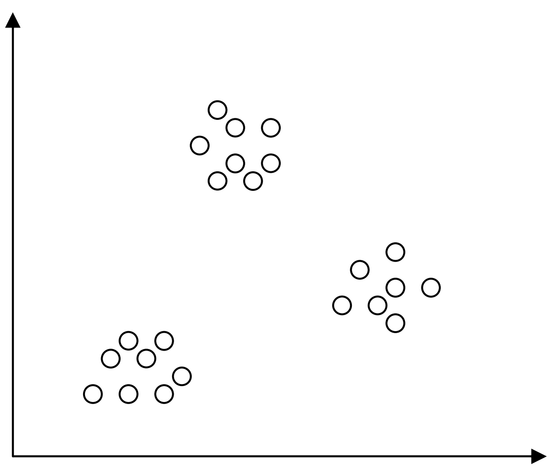

Fig. (4). A problem with location clusters.

ORGANIZATION OF STUDY

The organization of this research is as follows. The next section provides a literature review on citations specific to the *k*-centra location problem. The third section describes the methodology used to address the *k*-centra location problem. The fourth section concludes this paper with a brief summary as well as highlighting some open avenues for future research.

LITERATURE REVIEW

To wit, no research exists on a solution to the planar *k*-centra location problem. Indeed, it was this identified gap in the literature that led (at least in part) to this undertaking. Hence, this facility location literature review is relatively brief and will only address citations related to the *k*-centra location problem. For a large taxonomy and literature review on facility location in general, the reader is referred to Brandeau and Chiu [2]. Francis, McGinnis, and White [3] presented a survey paper in location analysis, which defined four classes of location problems and described algorithms to optimize them. For a comprehensive taxonomy of location science in general the reader is referred to Drezner and Hamacher [4]. For a more recent review of the facility location landscape, see Hale and Moberg [5].

Halpern [6] first introduced the "cent-dian" model as a parametric solution concept based on the bicriteria center/median model. Halpern modeled the problem in such a way that the inherent objective function characteristics of both the problems are considered while solving. The goal was to find an balance between efficiency (least-cost) and equity (worst-case). However, this particular method can sometimes fail to provide a solution to a discrete location problem mostly due to the limitations involved with direct combinations of two different functions.

In 2008, Drezner and Nickel [7] investigated a related problem in the ordered 1-median problem on a plane. Therein the authors present a novel and ingenious big-triangle/small-triangle approach.

Hansen, Labbe, and Thisse [8] introduced a variation of the cent-dian problem in the generalized center problem, which minimizes the difference between the maximum distance and the average distance. This model can be extended to formulate solutions for multiple facility location problems on a plane as well as on a network. This model can also be applied to discrete location problems.

To wit, the *k*-centra location problem was first formulated by Slater [9] in a regional academic conference pertaining to graph theory. The *k*-centra model combines both the center as well as the median concepts by minimization of the sum of the *k* largest distances. If $k = 2$ the model reduces to a standard center problem while with $k = n$ it becomes a standard median problem. Slater's work concentrated solely on the single facility location problem on a tree graph.

Peeters [10] studied the *k*-centrum model and introduced a full classification of the *k*-centrum criteria and some solution concepts for network based problems. He proposed two different variations on the median and the center functions each. The functions considered were the upper *k*-median where the sum of the *k* largest distances are minimized, lower *k*-median where the sum of the *k* smallest distances are minimized, upper *k*-center where the *k* largest distances are minimized, and lower *k*-center where the *k* smallest distances are minimized.

The *k*-centrum model is generally reserved for unweighted problems. However, significant research has been performed to show that satisfying the above criteria is not always necessary. Recently, Tamir [11] solved a weighted multiple facility *k*-centrum problem on paths and tree graphs using simple polynomial time algorithms. In this method, weights are assigned to all the distances from the new location to the existing locations and the distances are scaled accordingly.

Ogryczak and Zawadski [12] first introduced the conditional median method, which is an extension of the *k*-centrum concept when applied to weighted problems. Their paper proposes that a *k*-centrum problem can be evaluated for optimality by just considering only that specific part of the demand, which is in direct proportion to the existing largest distances. Thus this concept solves the objective function for the entire portion of the largest distances for a specified portion of the demand.

PROPOSED METHODOLOGY

The proposed methodology incorporates the well known Weiszfeld's algorithm. Weiszfeld's algorithm iteratively solves for the minisum location to the Weber problem as shown in (1) below.

$$\text{Minimize } f(x,y) = \sum w_i \sqrt{(x-a_i)^2 + (y-b_i)^2} \tag{1}$$

The solution methodology proposed in this research is an iterative heuristic method utilizing Weiszfeld's algorithm. The proposed solution steps are delineated below:

1. *Find the minimax solution for all the existing customer locations. This minimax solution is used as a starting point.*
2. *The next step is to find the distances of all existing locations from the minimax solution. The top k distances are considered to find the k farthest location points.*
3. *The next step is to use the minisum approach to evaluate the obtained k farthest points. Weiszfeld's algorithm is applied to solve the minisum aspect. After the first iteration, a new location point is obtained.*
4. *The next step is to check whether the same k points still are the points lying farthest away. If so, the solution obtained is the optimal k-centra location. This is a necessary and sufficient optimality condition. If not, reiterate by considering the new k-largest distances and the corresponding existing locations.*

The procedure is illustrated with a flowchart as shown in Fig. (**5**). Note that the algorithm may become degenerate and pivot between two or more non-optimal solution locations. This happens when the solution comes up with a different set of *k* points and this keeps repeating. Generally, it is observed that this degeneracy is due to a one or a few points being far away from the initial set of *k* points. The proposed solution is a semi-optimal solution, which neglects the coordinates of the rogue point(s).

As noted above, this approach was applied to solve three small examples problems each of a different type influenced by real life problems. Excel was used as a tool to solve the problems manually. All of the example problems were then also solved in MATLAB© to verify obtained solutions.

REAL LIFE LOCATION SCENARIOS

Three small location problems were solved to help validate this research using the *k*-centra methodology delineated in the previous section. The analogy used was real life are planning problems in cities. There are numerous examples of planned and unplanned city layouts. Older cities generally fall in the unplanned category while newer cities are methodically planned depending on population density, topographical features and business sectors.

UNPLANNED PROBLEM: RANDOM LOCATIONS

This is the most common layout seen in most cities all over the world. A single-facility planar location problem associated with such a layout presents one of the most complex and time-consuming problems in the field of location analysis. However, using the *k*-centra approach discussed in this thesis, only a selective few points are evaluated out of all the existing locations. Thus the computing time as well as complexity of the problem is reduced. The problem considered is a 15 point problem as shown in Table **1**. The problem is evaluated for different values of *k* (*k* = 1 to 15). The flowchart as shown in Fig. (**5**) is used as the solution guideline for evaluating the problem. The problem is first evaluated in Excel using manual iterations.

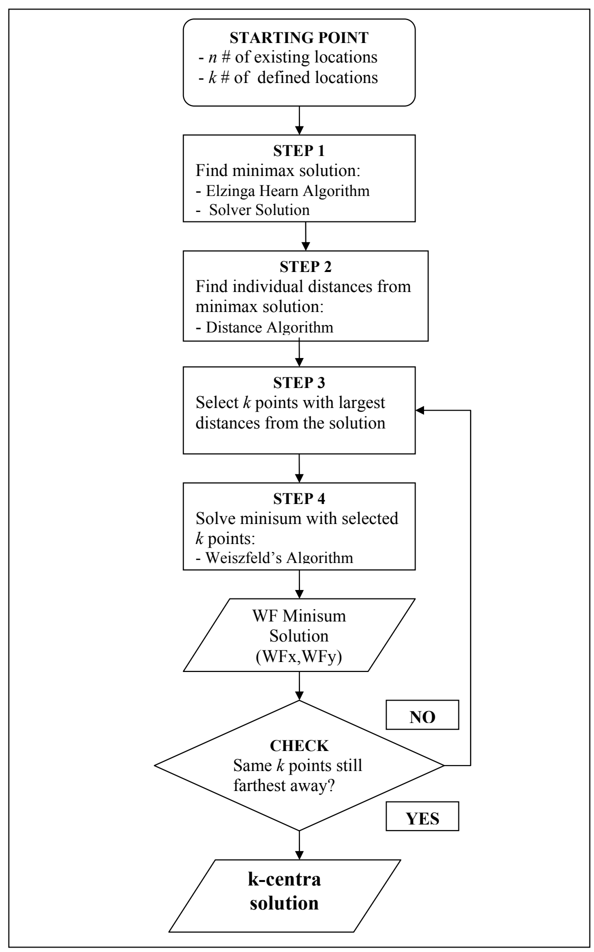

Fig. (5). Flowchart of *k*-centra solution approach.

The graphical representation is shown in Fig. (**6**). The aim is to locate a single service facility in the plane so as to minimize the sum of the distances to the *k* farthest points. The data required for this problem includes the coordinates for existing customer locations and a value for *k*. Reiterating, a *k* value could depend on the nature of the problem or it could be based on logical reasoning and demand flow.

Table 1. Unplanned Random Problem Coordinates

Point Number	*x*	*y*
1	2	2
2	2	4
3	5	2
4	3.5	4
5	7	4
6	9.5	4.5
7	10.5	2
8	11.5	5.5
9	9.5	8
10	7	6.5
11	5	6.5
12	2	9
13	5.5	8.5
14	8.5	9
15	11	9.5

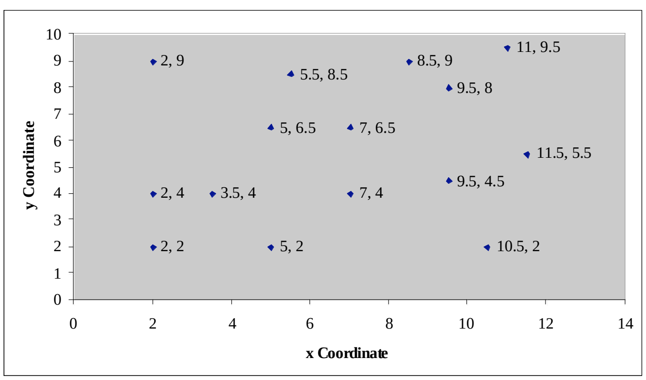

Fig. (6). Unplanned random problem coordinates.

The first step in the methodology is to obtain the minimax location solution. Two different methods were used to obtain the minimax solution. The first method is a graphical solution based on the well known Elzinga-Hearn algorithm. The Elzinga-Hearn algorithm is a graphical method of solving the minimax objective function for a location problem. The method involved is the smallest circle covering method where the objective is to find the smallest possible circle which encompasses all the existing locations in the plane.

The minimax solution is obtained at $x = 6.5$ and $y = 5.75$. Excel's solver was also used to obtain the minimax solution and verify the results obtained by using the Elzinga-Hearn algorithm. The next step is to find the distance between the calculated minimax solution and the all the other existing location points. This provides the necessary starting point for the *k*-centra algorithm. The distances are then sorted in descending order as shown in Table **2**. The largest *k* distances are then considered for the particular *k*-centra problem of interest.

Table 2. Distances from Minimax Center Location

Point Number	Distance
1	5.857687
15	5.857687
12	5.550901
7	5.482928
8	5.006246
2	4.828302
3	4.038874
14	3.816084
9	3.75
4	3.473111
6	3.25
13	2.926175
5	1.820027
11	1.677051
10	0.901388

For example, consider $k = 6$. Points P1, P15, P12, P7, P8 and P2 are the points farthest away from the minimax solution (6.5, 5.75). Using the minimax solution as the starting solution, Weiszfeld's algorithm is then used to solve for the minisum location for these 6 points. This solution is found to be at x = 7.8638 and y = 6.2836 with an objective function value of 32.1273.

It is further observed that these same six points are indeed the six furthest away from the newly obtained minisum location. In this case, this solution is optimal for $k = 6$. Thus the 6-centra solution for this problem lies at x = 7.86 and y = 6.28. However, it was noticed that for $k = 5$, the algorithm proceeds into a degenerate loop and the solution points keep jumping from one solution space to the other. This degeneracy is investigated more fully below.

Fig. (**7**) shows a scatter plot depicting the minimax solution as the diamond and the 6-centra solution as the asterisk. Notice the proximity between the two solutions. It is premised that in real life applications, both solutions could be tendered as viable locations.

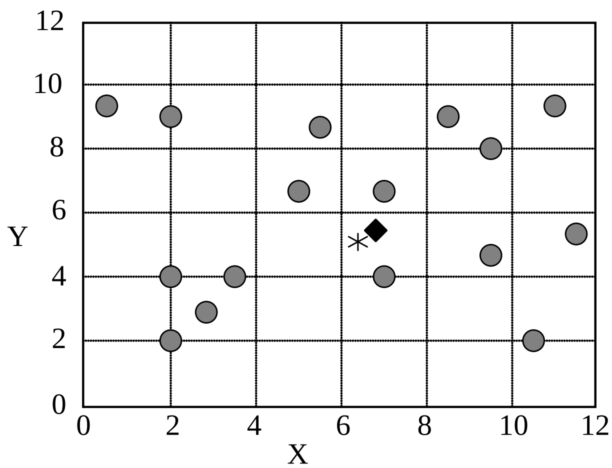

Fig. (7). k = 6, Unplanned city, random problem.

The solution plot for k = 5 is shown in Fig. (**8**). This is an example of a degenerate solution. The (same) minimax solution is shown with a diamond and the two, degenerate 5-centra "solutions" are shown with asterisks. The 5-centra solution alternates between these two points, (10.1, 6.75) and (3, 4.15). To resolve this matter, the objective function values can be calculated for each of the obtained solution points and the lowest objective function value obtained could be considered as the k-centra solution. This matter could also be resolved by including other parameters (e.g., the weights of the surrounding customer locations) to decide upon the appropriate location for the service facility.

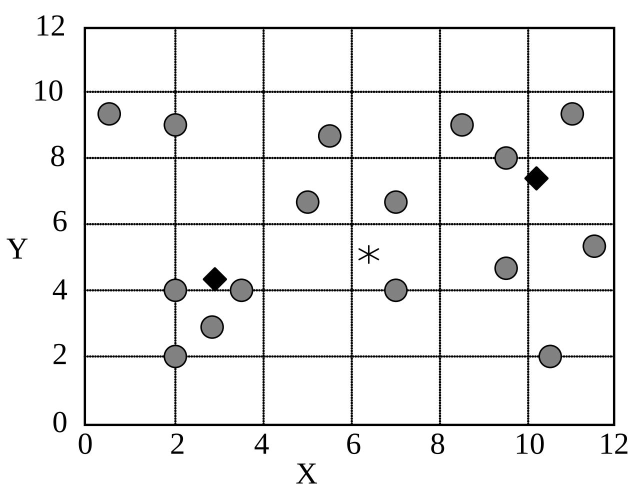

Fig. (8). k = 5, Unplanned city, random problem.

Indeed, the entire set of solutions obtained by using the k-centra methodology to solve the unplanned city, random location problem using Excel for all values of k are displayed in Table **3**.

It is observed (and interesting) that for all even values of k (k = 2, 4, 6, 8, 10, 12, and 14) as well as for k = n = 15, we get a singular optimal k-centra solution. An optimal k-centra solution implies that the point obtained after the first iteration still has the original k points as the k points lying farthest away. For all other k values, the problem becomes degenerate and the algorithm falls into a loop. This means that

Table 3. Excel solutions: Unplanned City, Random Problem

Method	x	y	Comments
Minimax	6.5	5.75	Optimal
k-centra (k = 2)	6.5	5.75	Optimal
k-centra (k = 3)	2.94573	4.87253	Degenerate
k-centra (k = 4)	6.38474	5.61015	Optimal
k-centra (k = 5)	10.0325	7.00555	Degenerate
k-centra (k = 6)	6.48704	5.29013	Optimal
k-centra (k = 7)	4.94579	4.44125	Degenerate
k-centra (k = 8)	6.58535	5.38736	Optimal
k-centra (k = 9)	6.99721	5.75654	Degenerate
k-centra (k = 10)	6.53427	5.52402	Optimal
k-centra (k = 11)	6.90182	5.39711	Degenerate
k-centra (k = 12)	6.73221	5.77254	Optimal
k-centra (k = 13)	6.77582	5.48388	Degenerate
k-centra (k = 14)	6.79665	5.55367	Optimal
Weiszfeld's (k = n)	6.74856	5.95321	Optimal

that the solution point obtained after all the iterations does not have the same k points lying farthest away as it started with. The solution point jumps around between 2 or 3 possible solution points never obtaining optimality.

PLANNED PROBLEM: ANNULAR LOCATIONS

This problem is analogous to a planned city where customers are located around a central city landmark, be it a school, a big employer, or a tourist attraction. The example problem considered here is again a 15-point problem and the problem is again solved for k = 1 to k = 15. The existing locations are graphically represented in Fig. (**9**) and listed in Table **4**.

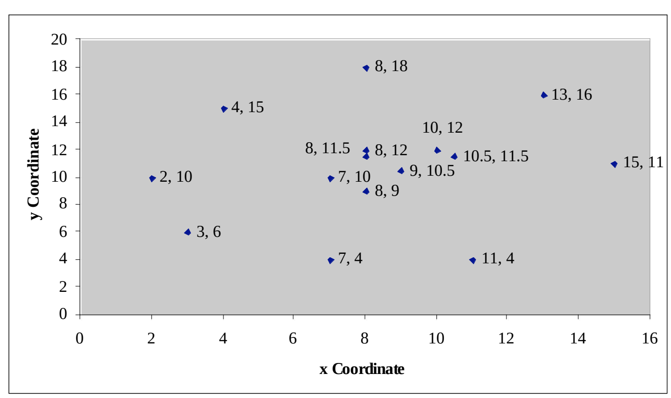

Fig. (9). Annular layout problem coordinates.

Table 4. Annular Layout Problem Coordinates

Point Number	*x*	*y*
1	2	10
2	3	6
3	7	4
4	11	4
5	15	11
6	13	16
7	8	18
8	4	15
9	7	10
10	8	9
11	10.5	11.5
12	9	10.5
13	10	12
14	8	12
15	8	11.5

The tabular results for the annular location problem solved in Excel are shown in Table **5** below.

Table 5. Excel Solutions: Annular Location Problem

Method	*x*	*y*	Comments
Minimax	8.45	10.8	Optimal
k-centra (*k* = 2)	6.5	5.75	Degenerate
k-centra (*k* = 3)	7.33482	9.35265	Degenerate
k-centra (*k* = 4)	7.24849	7.97259	Degenerate
k-centra (*k* = 5)	8.43329	9.62622	Degenerate
k-centra (*k* = 6)	9.6051	9.87137	Degenerate
k-centra (*k* = 7)	8.44544	9.89098	Degenerate
k-centra (*k* = 8)	7.8258	10.6031	Optimal
k-centra (*k* = 9)	8.58127	10.8565	Optimal
k-centra (*k* = 10)	8.91881	11.1285	Degenerate
k-centra (*k* = 11)	8.73425	10.701	Degenerate
k-centra (*k* = 12)	8.41566	10.5722	Optimal
k-centra (*k* = 13)	8.33592	10.8461	Optimal
k-centra (*k* = 14)	8.2086	11.2759	Optimal
Weiszfeld's (*k* = *n*)	8.4432	11.3161	Optimal

It is observed for this problem that the problem was degenerate for *k* = 2, 3, 4, 5, 6, 7, 10, and 11 and that it converged to a singular solution for all other values.

PLANNED PROBLEM: LOCATION CLUSTERS

The final example problem considered is a city problem with distributed location clusters. This is also a common layout for modern cities where the population is located in concentrated clusters. The concentrations could be developed residential areas or individual housing complexes. The existing customer locations are listed in Table **6**.

Table 6. Clustered Layout Problem Coordinates

Point Number	*x*	*y*
1	2	1
2	3	4
3	1	1
4	1	4
5	2	3
6	9	9
7	10	9
8	8	10
9	8	8
10	10	11
11	8	1
12	8	3
13	9	2
14	10	3
15	9	4

The graphical representation for the example clustered location problem is depicted in Fig. (**10**) below.

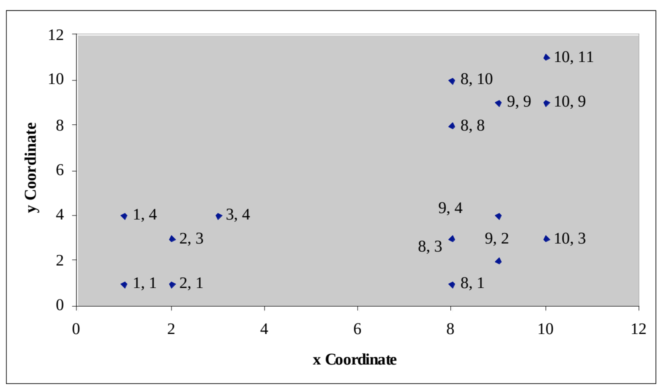

Fig. (10). Example of a clustered location problem.

The tabular results for the clustered location problem solved in Excel is shown in Table **7** below.

Table 7. Excel Solutions: Clustered Location Problem

Method	*x*	*y*	Comments
Minimax	5.4900	6.0000	Optimal
k-centra (*k* = 2)	5.4955	5.9950	Optimal
k-centra (*k* = 3)	4.2528	4.2198	Degenerate
k-centra (*k* = 4)	5.2992	3.3207	Degenerate
k-centra (*k* = 5)	6.3516	4.5916	Degenerate
k-centra (*k* = 6)	7.0189	4.3006	Degenerate
k-centra (*k* = 7)	9.4010	7.1000	Degenerate
k-centra (*k* = 8)	5.1135	5.8244	Degenerate
k-centra (*k* = 9)	6.0876	3.8864	Degenerate
k-centra (*k* = 10)	6.2231	5.0577	Optimal
k-centra (*k* = 11)	6.7038	5.0001	Degenerate
k-centra (*k* = 12)	6.3718	6.6343	Degenerate
k-centra (*k* = 13)	6.4110	4.9985	Optimal
k-centra (*k* = 14)	6.9987	4.8355	Optimal
Weiszfeld's (*k* = *n*)	7.3000	4.3569	Optimal

It is again interesting to note that the problems were degenerate for $k = 3, 4, 5, 6, 7, 8, 9, 11$, and 12 and converged to a singular solution for the other values of k.

CONCLUSIONS

This objective of this thesis was to find a unique method to solve the planar single-facility *k*-centra problem with Euclidean distances. The method developed uses the iterative nature of Weiszfeld's method to selectively solve the minisum objective function for a set of predefined *k*-farthest away points. Three examples influenced by existing real-life location problems were solved manually in Excel and the results obtained were verified using a MATLAB code written for evaluating such problems. Further experimentation was conducted to check the accuracy and the speed of the *k*-centra solution as compared to the minisum solution.

ACCURACY OF THE *K*-CENTRA

In this research a *k*-centra method is proposed as an alternative approach to the minisum method. So, it was necessary to verify the accuracy of the solutions obtained with the *k*-centra approach. Five 15-point and 30-point problems were solved each for various values of k and were compared to the optimal minisum solution. It can be concluded that for an n-point problem, the objective function value decreases as k is increased from 1 to n. However, this is not a linear decrease. For a particular n-point problem, the percent increase from optimality drops steadily until k reaches a certain threshold percentage of n after which the drop is very gradual until the solution reaches complete optimality at $k = n$. There is a significant increase from optimality at lower values of k. Thus, the proposed *k*-centra approach provides the user with a good nearly optimal solution, provided the k value is selected appropriately.

SELECTING THE RIGHT *K* VALUE

It is absolutely necessary to select a good k value so that solution optimality is not completely sacrificed. A few problems were evaluated for 1% increase in optimality considering various values of n to obtain a general idea on selecting the right value of k. From the results obtained, it can be concluded that for smaller problems ($n < 30$), selecting a k value between 50% - 60% of n yields a good solution. For bigger problems ($n > 30$), a nearly optimal solution is obtained by selecting k between 40% - 50% of n. From the experimentation, it was also evident that the cutoff value of k (as a percent of n) to obtain 101% of optimality reduces from smaller to larger problems. However, after reaching a certain value of n, 101% of optimality is consistently obtained in the same range (40% - 50% of n).

COMPUTATION TIME FOR A *K*-CENTRA PROBLEM

The aim of this thesis was to find a good alternative methodology to the minisum method. A good alternative method should provide good results preferably using less computational time. The proposed *k*-centra method provides the user with a near optimal solution. However, it can be concluded that even though there is some reduction in the total computational time for the proposed *k*-centra method as compared to the minisum method, the time saved is negligible. Thus, the proposed *k*-centra methodology is not significantly faster than the traditional minisum method. This translates into a tradeoff between achieving complete optimality and computational time. The negligible reduction in computation time by using this method does not justify the deviation from optimality.

FUTURE RESEARCH

The proposed *k*-centra method discussed in this thesis has its drawbacks in terms of the probability of achieving a single optimal solution. Degeneracy is also an inherent problem with this particular method. These issues are easily handled by adding a few more parameters and constraints to the objective function. A few examples include addition of weights depending on demand, or frequency, terrain conditions, traffic issues and delivery cost considerations can easily negate sub-optimality and provide a single feasible solution.

Certain changes can be made to the proposed methodology so as to ensure that the computational time is reduced significantly as compared to a minisum method. Changes can include a different method to solve the Weiszfeld's loop, different selection criteria for k after every iteration, or perhaps different software used to evaluate the problem.

This research considers a single facility location problem. This method could be extended to more than a single facility problem depending on the size and nature of the problem. Another aspect is the consideration of rectilinear distance metric. This method can be modified to consider rectilinear distances by changing the objective function.

Finally, the anti *k*-centra problem is to locate so as to maximize the sum of the distances to the *k*-closest points out of the set of existing locations. Future research in this area could also be aimed at weighted or un-weighted instances of the anti *k*-centra problem.

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