The Neutral Line Concept in Hot-Rolled Steel Cross-Sections

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Abstract: The present work deals with the effect of the thermally-induced residual stresses on the shape of a cross-section's neutral line for hot-rolled steel beams under combined loading conditions. In general, the neutral line of a beam cross-section is considered to be straight. In this study it is shown that, taking into account the existing thermal residual stresses, this line becomes a curve as far as the beam is subjected to pure bending or to combined loading that consists of a bending moment and an axial force.

Keywords: Steel sections, Hot-rolled sections, Neutral line, Thermal stresses, Residual stresses.

INTRODUCTION

The lifetime of a structural member is usually determined by the interaction between the defects within the member and the stress states to which it is subjected to, regardless of its material properties.

Contrarily to the externally applied loads, which can be readily and comprehensively introduced in the design of structural members, it is considerably more difficult to account for residual stresses in both the analysis and the design procedure.

This is true, since residual stresses vary significantly in magnitude, they are strongly non-linear (in distribution and nature), unpredictable and cannot be reliably measured.

Focusing on hot-rolled steel structural elements, the most common and simultaneously unavoidable type of residual stresses arising are the so called process-induced ones, which can be either flow-induced or thermally-induced, with the latter being the dominant component.

The mechanism of the development of thermally-induced residual stresses and their distribution for steel structural elements can be found in detail in numerous text-books, papers and reports (Huber and Beedle [1], Alpsten [2], Michaltsos [3], and others not quoted herein).

For the majority of widely used hot-rolled beams with sections as I-sections or orthogonal ones, this distribution is symmetric along the primary axes and thus the effect of such a self-equilibrated stress system is neglected either in elastic analysis or in elastoplastic one, mainly when simple (pure) loading conditions are specified, i.e. when the distribution of the stresses due to external loads is also symmetric.

However, this is not the case when combined loads are acting upon the component, as for beam-columns, moment frame beams, crane structures etc. Shifting of the neutral axis may cause a significant reduction of the moment resistance and subsequently of the overall bearing capacity of the member. With the continuing encouragement not only to optimize material performance, but simultaneously to minimize component weight, the primary advantages of steel (as for example ductility) must be utilized to the maximum extent.

In doing so, elastoplastic or plastic analysis ought to be used in the design of structural steel-work, and in order to provide reliable results, the effect of thermal residual stresses must be accounted for.

The existence of efficient non-destructive or semi-destructive techniques for measuring to a considerable depth the aforementioned stresses (e.g. neutral diffraction or crack compliance respectively), offers to engineering practice the capacity of a quantitative ascertainment of the amount of thermal residual stresses on a wide range of hot-rolled steel sections.

To the knowledge of the authors, however, there seems to be a lack of practice design guidelines and tools, based on solid theoretical background for taking into account the aforementioned stresses.

A limited number of publications exist in the relevant literature, but it deals mostly with built-up or complex structural components, as in Marcelin [4] or Dixit and Dixit [5]. The effect of thermal residual stresses on bearing capacity of hot rolled sections under either combined bending moment and axial force or, also, combined bending moment and shearing force is already proven [6, 7].

The present work aims to contribute to the specific topic mentioned above, and especially to deal with the effect of thermal-induced residual stresses on the form of the neutral line of hot-rolled sections under the action of bending moment and axial force. With the use of the elementary elastoplastic analysis, simple formulae are obtained and characteristic examples are presented.

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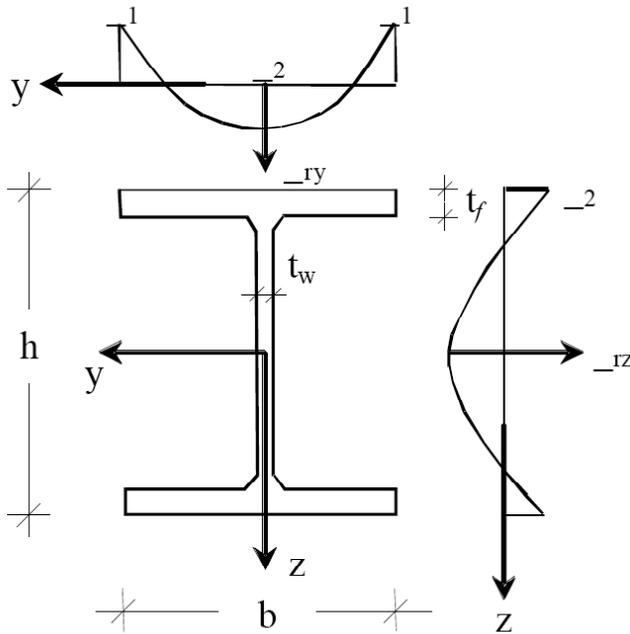
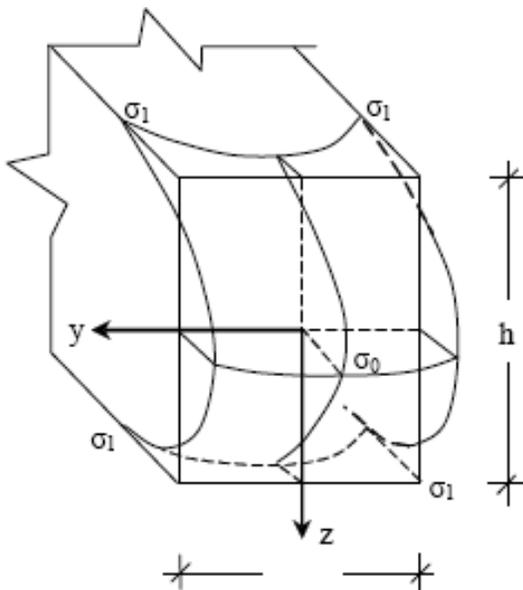**Fig. (1a).** Residual stresses in a steel I-section.

Fig. (1b). Residual stresses in an orthogonal steel cross-section.

EFFECT OF THERMALLY-INDUCED RESIDUAL STRESSES

The residual stresses are due to the way of treatment and production of the structural steel and they constitute a self-equilibrated stress state. Therefore, they are usually ignored in the analysis.

The distribution and magnitude of the thermal residual stresses can be readily taken form Alpsten [2] as well as from Eurocode 3 [8], and are shown in Fig. (**1a**) for a steel I-section and in Fig. (**1b**) for a solid orthogonal cross-section.

The I-Section

For the distribution shown in Fig. (**1a**), and taking into account the conditions:

$$\sigma_{ry}(0) = \sigma_2 , \quad \sigma_{ry}(\pm\frac{b}{2}) = \sigma_1 , \quad \sigma_{ry}(0) = \sigma_{rz}(\pm\frac{h}{2})$$

$$2t_f \int_{-b/2}^{b/2} \sigma_{ry}(y)dy + t_w \int_{-h/2}^{h/2} \sigma_{rz}(z)dz = 0$$

one obtains

$$\sigma_{ry}(y) = \frac{4(\sigma_1 - \sigma_2)}{b^2} \cdot y^2 + \sigma_2$$

$$\sigma_{rz}(z) = A \cdot z^2 + B \tag{1}$$

where

$$A = \frac{4A_f(\sigma_1 + 2\sigma_2) + 3\sigma_2(A_w - t_f t_w)}{t_w(2h^3 - 3ht_f^2 + t_f^3)}$$

$$B = \sigma_2 - \frac{h^2}{4}A$$

$$A_f = 2bt_f , \quad A_w \cong t_w h$$

The Solid Orthogonal Cross-Section

For the distribution showing in Fig. (**1b**), and taking into account the conditions:

$$\sigma_r\left(\pm\frac{b}{2}, \pm\frac{h}{2}\right) = \sigma_1 , \quad \sigma_r(0,0) = \sigma_o$$

$$\int_{-b/2}^{b/2} dy \int_{-h/2}^{h/2} \sigma_r(y,z)dz = 0$$

one can find

$$\sigma_r(y,z) = A(y^2 + z^2) + By^2z^2 + \Gamma \tag{2}$$

where

$$A = -\frac{2(\sigma_1 + 8\sigma_o)}{b^2 + h^2}$$

$$B = \frac{24(\sigma_1 + 2\sigma_o)}{b^2 h^2}$$

$$\Gamma = \sigma_o$$

As it is shown in the followings, the thermally-induced residual stresses cause a displacement of the neutral line of the cross-section in Fig. (**1a**), while for the one in Fig. (**1b**) the neutral line changes to a curve as well.

EXTERNAL LOADS AND RESIDUAL STRESSES

Let us consider now, a cross-section subjected to the action of a moment M_y and an axial force N_x.

As the above moment and axial force increase, the cross-section is strained and enters from the elastic region firstly into elastoplastic region and then into the plastic one - see Figs. (**2a**) to (**2c**).

It has been proved that the full plasticization of a cross-section cannot be realized from a theoretical point of view. A small elastic kernel always remains near the neutral line (in its final position) that behaves elastically, since any more
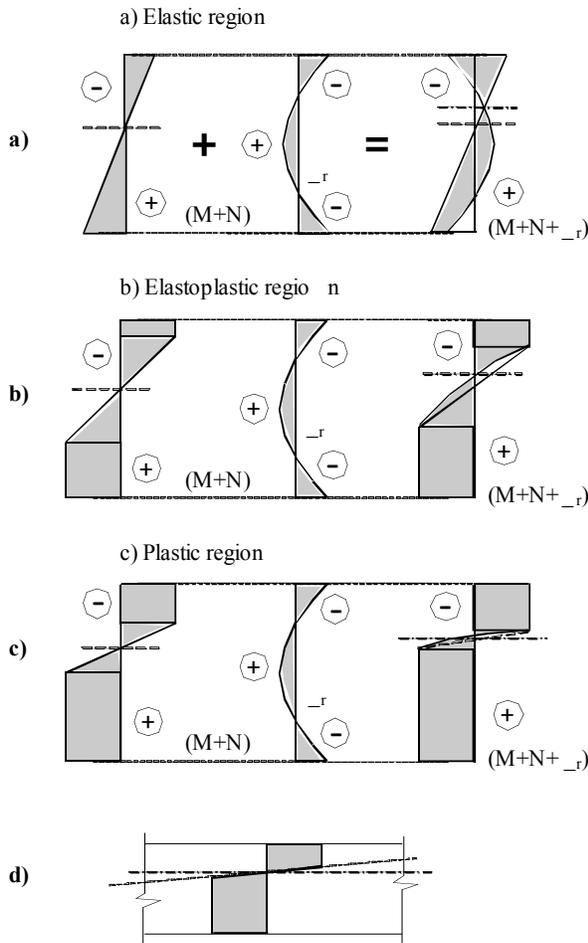
a) Elastic region

a) + (M+N) = (M+N+_r)

b) Elastoplastic regio   n

b) (M+N)   (M+N+_r)

c) Plastic region

c) (M+N)   (M+N+ _r)

d)

Fig. (2). Elastic – plastic zones due to bending.

plasticization involves an extremely large (theoretically infinite) deformation of the external fibers, which in fact cannot take place – see Fig. (**2d**).

This elastic kernel extends from 2% to 4% of the total cross-section area. Therefore, a situation like the one shown in Fig. (**2c**) will be considered as a fully plasticized cross-section.

THE ELASTIC REGION

The I-Section

Ignoring the thermal residual stresses, the position of the neutral line is given by the expression:

$$z_o = -\frac{I_b \cdot N_x}{A_b \cdot M_y} \tag{3}$$

where M_y and N_x are the moment and the axial force and I_b and A_b are the cross-section's moment of inertia and area, respectively.

Taking into account the thermal residual stresses, we have:

$$\sigma_x = \frac{M_y}{I_b}z + \frac{N_x}{A_b} + \sigma_{rz} \tag{4}$$

For the determination of the neutral line position will be: $\sigma_x=0$, or because of Eq. (1)

$$\frac{M_y}{I_b}z_n + \frac{N_x}{A_b} + A \cdot z_n^2 + B = 0$$

and finally:

$$z_n = \frac{1}{2 \cdot A} \cdot \left(-\frac{M_y}{I_b} + \sqrt{\left(\frac{M_y}{I_b}\right)^2 - 4 \cdot A \cdot \left(B + \frac{N_x}{A_b}\right)} \right) \tag{5}$$

In Fig. (**3**) one can see the old (z_o) and the new position (z_n) of the neutral line as well as the corresponding diagrams of σ_x for the case of a bending moment acting alone and of a simultaneous action of a bending moment and an axial force.

The Solid Orthogonal Cross-Section

Ignoring the thermal residual stresses, the position of the neutral axis is given also by the expression (3), while taking into accent the thermal residual stresses we get the expression (4). By setting $\sigma_x=0$ and because of Eq.(2) we obtain:

$$\frac{M_y}{I_b}z_n + \frac{N_x}{A_b} + A \cdot (y^2 + z_n^2) + B\,y^2z^2 + \Gamma = 0$$

which finally concludes to the following expression:

$$z_n = \frac{1}{2 \cdot (A + B \cdot y^2)} \cdot \left(-\frac{M_y}{I_b} + \right.$$
$$\left. + \sqrt{\left(\frac{M_y}{I_b}\right)^2 - 4 \cdot (A + B \cdot y^2) \cdot \left(A \cdot y^2 + \Gamma + \frac{N_x}{A_b}\right)} \right) \tag{6}$$

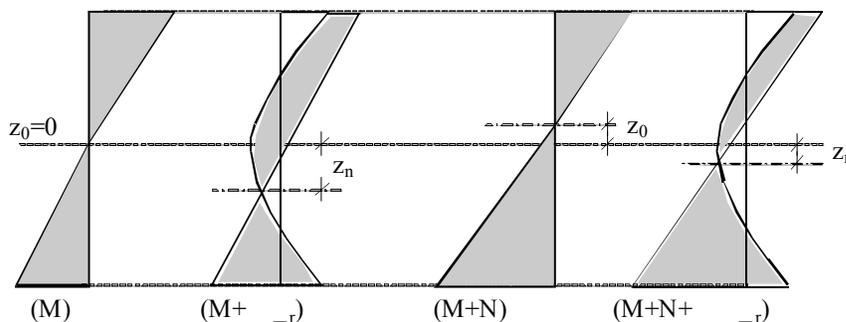$z_0=0$ z_n z_0 z_n

(M)   (M+ _r)   (M+N)   (M+N+ _r)

Fig. (3). Stress distribution diagrams.
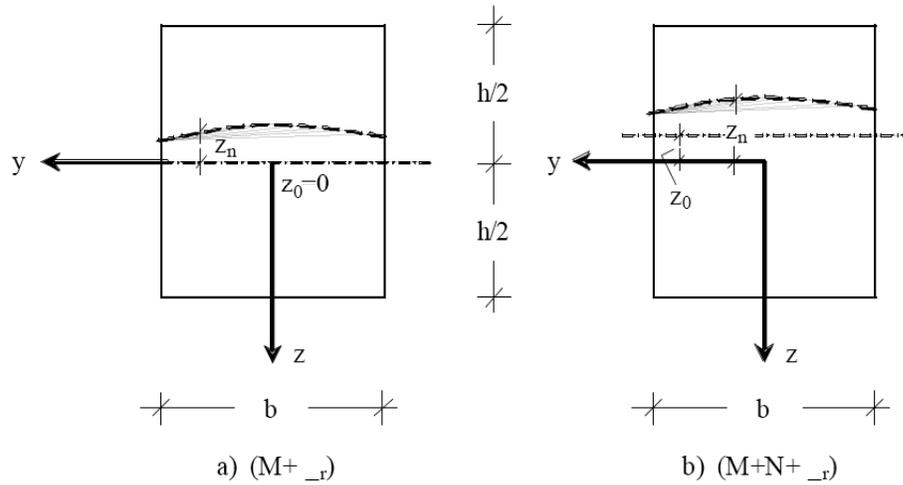
a) (M+ _r) b) (M+N+ _r)

Fig. (4). Neutral line positions due to bending, axial and thermal stresses.
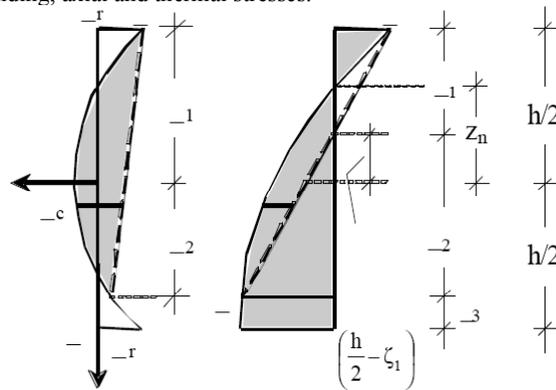

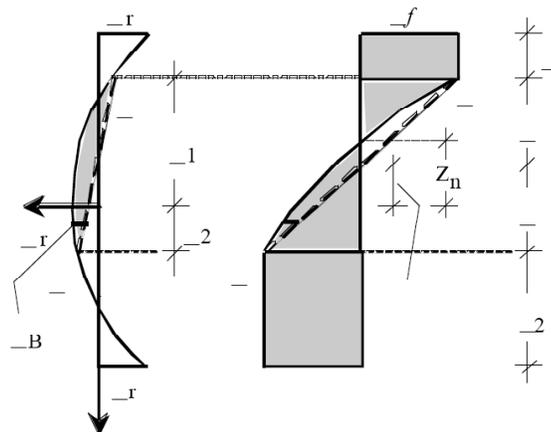**Fig. (5a).** Neutral line position with plastic region at the bottom.

Fig. (5b). Neutral line position with top and bottom plastic regions.

Fig. (**4**) shows the new position of the neutral line for the case of a pure moment (Fig. **4a**) and of a simultaneous action of a moment and an axial force (Fig. **4b**).

We discern that, except the change of its position, the neutral line changes from a straight line into a curve.

ELASTOPLASTIC AND PLASTIC REGIONS

As the influence of the thermal residual stresses is more evident and indicative on solid orthogonal cross-sections, the following analysis focuses on this type of cross-sections.

Once a cross-section enters into the elastoplastic region, the distribution of stresses (including the thermal residual ones) will have one of the forms shown in Fig. (**5a**) or Fig. (**5b**) - see also Fig. (**2**).

The part ΓΔ of diagrams (5a) and (5b) is a curve having the same form with the part AB of the σ_r – diagram. This curve may be determined as follows:

$$Point\ A: \quad \sigma_{rA} = A \cdot (y^2 + \alpha_1^2) + B \cdot y^2 \alpha_1^2 + \Gamma \qquad \Big\} \qquad (7)$$
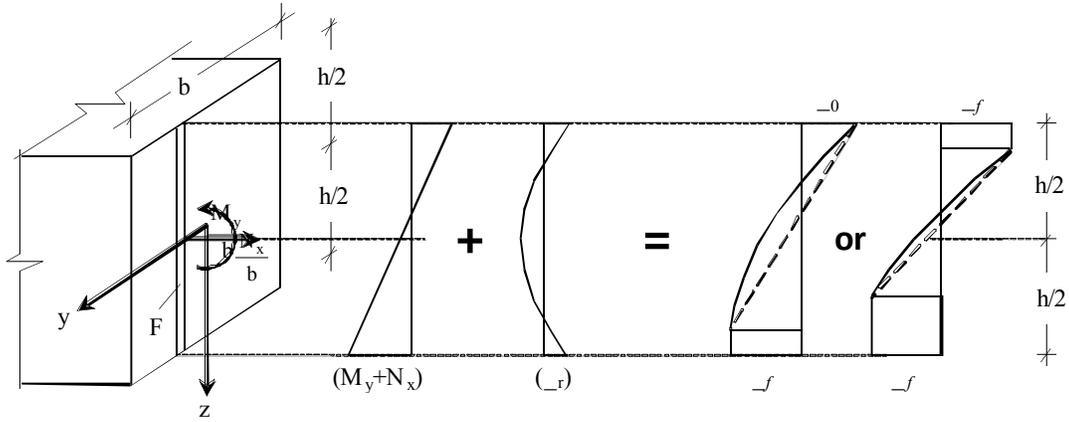$$Point\ B: \quad \sigma_{rB} = A \cdot (y^2 + \alpha_2^2) + B \cdot y^2 \alpha_2^2 + \Gamma$$

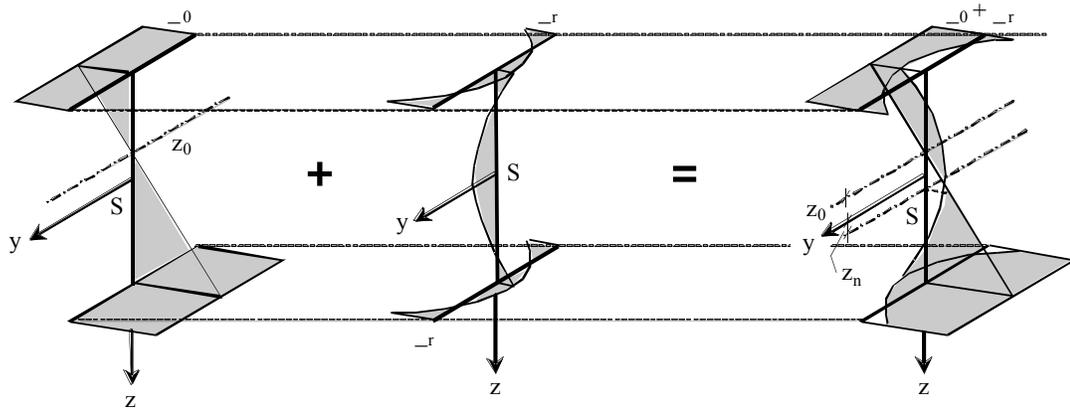**Fig. (6).** Stress distribution in a solid cross-section.

Fig. (7). Stress distribution in an I-section.

The equation of the straight line AB is:

$$\frac{\sigma_{AB} - \sigma_{rA}}{\sigma_{rA} - \sigma_{rB}} = \frac{z - \alpha_1}{\alpha_1 - \alpha_2} \quad \text{or}$$

$$\sigma_{AB} = \frac{z - \alpha_1}{\alpha_1 - \alpha_2}(\sigma_{rA} - \sigma_{rB}) + \sigma_{rA} =$$

$$= \frac{z - \alpha_1}{\alpha_1 - \alpha_2}(\alpha_1^2 - \alpha_2^2)(A + By^2) + A(y^2 + \alpha_1^2) + By^2\alpha_1^2 + \Gamma$$

or finally:

$$\sigma_{AB} = (z - \alpha_1)(\alpha_1 + \alpha_2)(A + By^2) + \\ + A(y^2 + \alpha_1^2) + By^2\alpha_1^2 + \Gamma \tag{8}$$

and the σ_c (from the hatched section of diagram σ_r) is given by the relation:

$$\acute{o}_c = \acute{o}_r - \acute{o}_{AB} =$$
$$= A(y^2 + z^2) + By^2z^2 + \acute{A} - (z - \acute{a}_1)(\acute{a}_1 + \acute{a}_2)(A + By^2) - $$
$$- A(y^2 + \acute{a}_1^2) - By^2\acute{a}_1^2 - \acute{A} =$$
$$= A(z^2 - \alpha_1^2) + By^2(z^2 - \alpha_1^2) - (z - \acute{a}_1)(\acute{a}_1 + \acute{a}_2)(A + By^2)$$

or finally:

$$\sigma_c = (z - \alpha_1)(z - \alpha_2)(A + By^2) \tag{9}$$

The case in Fig. (**5a**)

With the notations of Fig. (**5a**), we have:

$$\alpha_1 = \frac{h}{2}, \quad \alpha_2 = \zeta_1 + \zeta_2 - \frac{h}{2},$$

$$\zeta_3 = h - \zeta_1 - \zeta_2, \quad \sigma_o = \sigma_f \frac{\zeta_1}{\zeta_2} \tag{10}$$

therefore, the unknown ζ_1 and ζ_2 can be determined through the following conditions:

$$\int_{-h/2}^{h/2} \sigma \, dz = \frac{N_x}{b} \quad and \quad \int_{-h/2}^{h/2} \sigma z \, dz = \frac{M_y}{b} \tag{11}$$

The first condition gives:

$$\frac{N_x}{b} = \sigma_f(h - \zeta_1 - \zeta_2) + \frac{1}{2}\sigma_f\zeta_2 - \frac{1}{2}\sigma_f\zeta_1 + \int_{\alpha_1}^{\alpha_2} \sigma_c dz$$

or finally:

$$\sigma_f(h - \zeta_1 - \frac{\zeta_2}{2} - \frac{\zeta_1^2}{2\zeta_2}) + \frac{(\alpha_1 - \alpha_2)^3}{6}(A + By^2) = \frac{N_x}{b} \tag{12}$$

The second condition gives:

$$\frac{M_y}{b} = \frac{1}{2}\sigma_o\zeta_1(\frac{h}{2} - \frac{\zeta_1}{3}) + \frac{1}{2}\sigma_f\zeta_2(\frac{2\zeta_2}{3} - \frac{h}{2} + \zeta_1) + $$
$$+ \sigma_f\zeta_3(\frac{h}{2} - \frac{\zeta_3}{2}) + \int_{\alpha_1}^{\alpha_2} \sigma_c(z + \frac{h}{2} - \zeta_1)dz$$

or finally:

$$\sigma_f \left[\frac{\zeta_1^2}{2\zeta_2}\left(\frac{h}{2}-\frac{\zeta_1}{3}\right) + \frac{\zeta_2}{2}\left(\frac{2\zeta_2}{3}+\zeta_1-\frac{h}{2}\right) + \zeta_3\left(\frac{h}{2}-\frac{\zeta_3}{2}\right) \right] +$$

$$+ \frac{(\alpha_1+\alpha_2)(\alpha_1-\alpha_2)^3}{12}(A+By^2) +$$

$$+ (\frac{h}{2}-\zeta_1)\frac{(\alpha_1-\alpha_2)^3}{6}(A+By^2) = \frac{M_y}{b}$$

(13)

Equations (12) and (13) constitute a system, the solution of which gives the unknowns ζ_1 and ζ_2.

In order to define the position of the neutral line, we have to determine the equation of the curve $\Gamma\Delta$ (see Fig. **5a**).

The equation of the straight line $\overline{\Gamma\Delta}$ is given by the relation:

$$\frac{\sigma_{\Gamma\Delta}-\sigma_o}{\sigma_o-\sigma_f} = \frac{z-\alpha_1}{\alpha_1-\alpha_2}$$

or

$$\sigma_{\Gamma\Delta} = \frac{z-\alpha_1}{\alpha_1-\alpha_2}(\sigma_o-\sigma_f)+\sigma_o$$

and the equation of the curve $\Gamma\Delta$ is:

$$\sigma = \sigma_{\Gamma\Delta}+\sigma_c = \frac{z-\alpha_1}{\alpha_1-\alpha_2}(\sigma_o-\sigma_f)+$$

$$+\sigma_o+(z-\alpha_1)(z-\alpha_2)(A+By^2)$$

(14)

with the notations of Eq.(10).

The solution of the equation $\sigma=0$ gives the neutral line z_n for various values of y.

The case in Fig. (**5b**)

With the notations of Fig. (**5b**) we have:

$$\zeta = \frac{\alpha_1+\alpha_2}{2}, \quad \zeta_1 = \frac{h}{2}-\alpha_1,$$

$$\zeta_2 = \frac{h}{2}-\alpha_2, \quad \zeta_1-\zeta_2 = \alpha_1-\alpha_2$$

(15)

Where, the unknowns α_1 and α_2, can be determined by the conditions (11).

The first condition gives:

$$\frac{N_x}{b} = \sigma_f(\zeta_2-\zeta_1)+\int_{\alpha_1}^{\alpha_2}\sigma_c dz$$

or finally:

$$\sigma_f(\zeta_2-\zeta_1)+\frac{(\alpha_1-\alpha_2)^3}{6}(A+By^2) = \frac{N_x}{b}$$

(16)

The second condition gives:

$$\frac{M_y}{b} = \sigma_f\zeta_1(\alpha_1+\frac{\zeta_1}{2})+\sigma_f\zeta_2(\alpha_2+\frac{\zeta_2}{2})+$$

$$+\sigma_f\frac{2\zeta^2}{3}+\int_{\alpha_1}^{\alpha_2}\sigma_c(z+\frac{h}{2}-\zeta-\zeta_1)dz$$

which concludes to:

$$\sigma_f\left[\alpha_1\zeta_1+\alpha_2\zeta_2+\frac{\zeta_1^2+\zeta_2^2}{2}+\frac{2\zeta^2}{3}\right]+$$

$$+\frac{(\alpha_1+\alpha_2)(\alpha1-\alpha_2)^3}{12}(A+By^2)+$$

$$+(\frac{h}{2}-\zeta-\zeta_1)\frac{(\alpha1-\alpha_2)^3}{6}(A+By^2) = \frac{M_y}{b}$$

(17)

Equations (16) and (17) constitute a system, the solution of which gives the unknowns α_1 and α_2.

The equation of the straight line $\overline{\Gamma\Delta}$ is given by the relation:

$$\frac{\sigma_{\Gamma\Delta}+\sigma_f}{-\sigma_f-\sigma_f} = \frac{z-\alpha_1}{\alpha_1-\alpha_2}$$

or

$$\sigma_{\Gamma\Delta} = -2\cdot\frac{z-\alpha_1}{\alpha_1-\alpha_2}\sigma_f-\sigma_f$$

and the equation that gives the curve $\Gamma\Delta$ is:

$$\sigma = \sigma_{\Gamma\Delta}+\sigma_c = \frac{2(z-\alpha_1)}{\alpha_1-\alpha_2}\sigma_f-\sigma_f +$$

$$+(z-\alpha_1)(z-\alpha_2)(A+By^2)$$

(18)

The solution of the equation $\sigma=0$ gives again the neutral line z_n for various values of y (See also Fig. (**6**)).

NUMERICAL RESULTS AND DISCUSSION

The Elastic Region

Let us consider an IPE200 standard steel profile with $\sigma_f = 3000 dN/cm^2$ which has the following properties: $h = 20\ cm$, $b = 10\ cm$, $t_f = 0.85\ cm$, $t_w = 0.56\ cm$, $I_y = 1940\ cm^4$, $A_b = 38.50\ cm^2$, $A_w = 10.36\ cm^2$. We also assume that the thermal residual stresses have the form shown in Fig. (**1a**) with values $\sigma_1 = -400 dN/cm^2$, $\sigma_2 = 400 dN/cm^2$.

The cross-section is subjected to the moment $M_y=200000$ dNcm and an axial force $N_x=5000$ dN.

Ignoring at first the thermal residual stresses, the neutral line is found at:

$$z_o = -\frac{N_x/A_b}{M_y/I_y} = -\frac{5000\cdot 1940}{200000\cdot 38.50} = -1.259 cm$$

while the stresses σ_o and σ_u at the upper and lower fibers, respectively, are:

$$\sigma_{o,u} = \frac{M_y}{I_y}\cdot\left(\mp\frac{h}{2}\right)+\frac{N_x}{A_b} =$$

$$= \mp\frac{200\,000}{1940}\cdot 10+\frac{5000}{38.50} = \begin{cases} -901.05\ dN/cm^2 \\ 1160.79\quad " \end{cases}$$
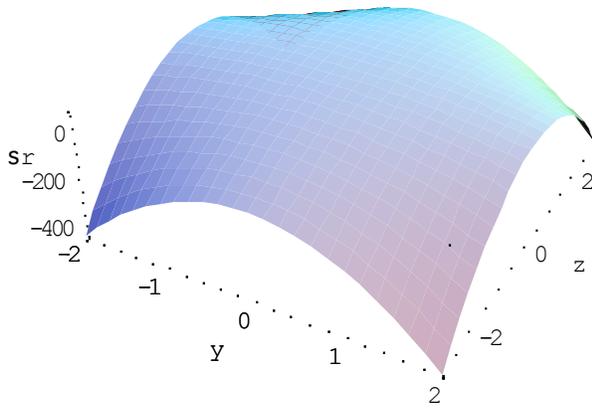
Applying Eq.(5), we obtain:
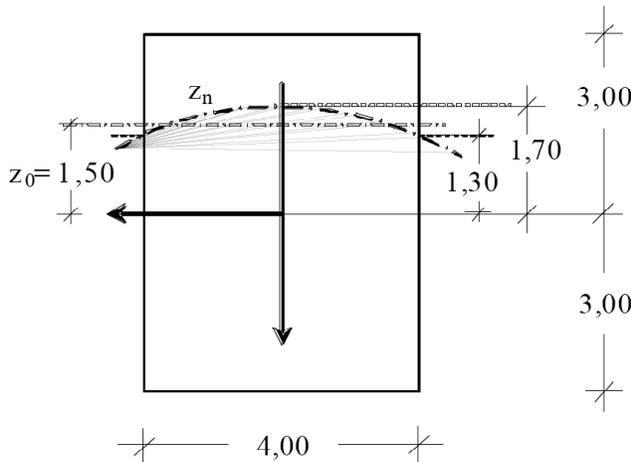
Fig. (8). Thermal stresses in a solid cross-section.

Fig. (9). Displaced position of the neutral line.

$z_n = +2.54 \ cm$.

Let us see now the influence of the thermal residual stresses on the neutral line and the resistant moments. Ignoring the thermal residual stresses, we have:

$$W_o = \frac{1940}{10 - 1.259} = 221.94 \ cm^3$$

$$W_u = \frac{1940}{10 + 1.259} = 172.31 \ cm^3$$

Taking into account the influence of the thermal residual stresses we have:

$$W_o = \frac{1940}{10 + 2.54} = 154.70 \ cm^3$$

$$W_u = \frac{1940}{10 - 2.54} = 260.05 \ cm^3$$

We consider next an orthogonal solid cross-section of dimensions b*h = 4x6 cm², and we consider in addition that the residual stresses have the form shown in Fig. (**8**) with

$\sigma_1 = -400 \ dN / cm^2 \ and \ \sigma_o = 100 \ dN / cm^2$.

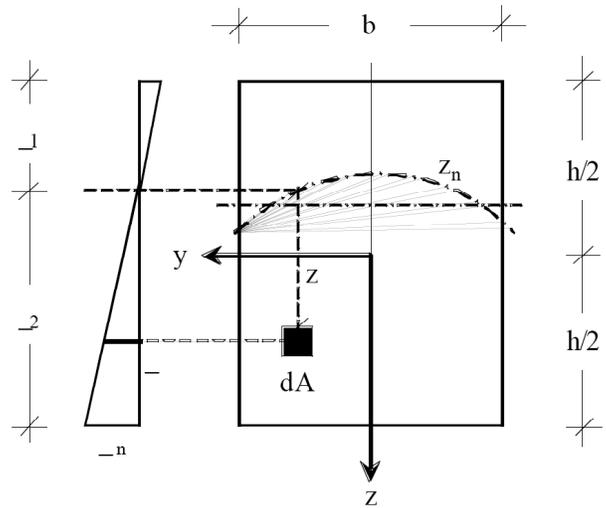For the above cross-section we have: $I_y = 72 \ cm^4, \ A_b = 24 \ cm^2$.

Fig. (10). The influence of thermal stresses in the form of neutral line.

Under the action of a moment M_y=24 000 dNcm and an axial force N_x=12 000 dN, we find that the neutral line z_o (without the influence of the thermal residual stresses) will be at the position:

$$z_o = -\frac{N_x / A_b}{M_y / I_y} = -\frac{12000 \cdot 72}{24000 \cdot 24} = -1.50 \ cm$$

Applying the Eq. (6), we find the curve of the neutral line z_n as it is shown in Fig. (**9**), where the neutral axis z_o (without the influence of the thermal residual stresses) is also shown.

Let us see now the influence of the thermal residual stresses on the neutral line and the resistant moments. Without the influence of σ_r we have:

$$M = \int_A \sigma \cdot z \ dA = \int_{A_1} \sigma_o \frac{z}{\zeta_1} z \ dA_1 + \int_{A_2} \sigma_u \frac{z}{\zeta_2} z \ dA_2 =$$

$$= b\sigma_u \left(\int_0^{\zeta_1} \frac{z^2}{\zeta_2} dz + \int_0^{\zeta_2} \frac{z^2}{\zeta_2} dz \right) = b\sigma_u \frac{\zeta_1^2 + \zeta_2^2}{3\zeta_2}$$

and finally we obtain:

$$W_u = b \cdot \frac{\zeta_1^2 + \zeta_2^2}{3\zeta_2} \quad and \quad W_o = b \cdot \frac{\zeta_1^2 + \zeta_2^2}{3\zeta_1}$$

Taking into account the influence of σ_r and through a similar process, we obtain:

$$W_u = \frac{1}{3} \int_{-b/2}^{b/2} \frac{\zeta_1^2 + \zeta_2^2}{\zeta_2} dy \quad and \quad W_o = \frac{1}{3} \int_{-b/2}^{b/2} \frac{\zeta_1^2 + \zeta_2^2}{\zeta_1} dy$$

From the above equations, without the influence of the thermal residual stresses we have: W_o=84 cm³ and W_u=28 cm³, while with the influence of the thermal residual stresses we have: W_o=93.17 cm³ and W_u=28.75 cm³ (See also Fig. (**10**)).

The Elasto-Plastic and Plastic Regions

Considering again the orthogonal solid cross-section, one can easily determine the plastic moment (the moment which
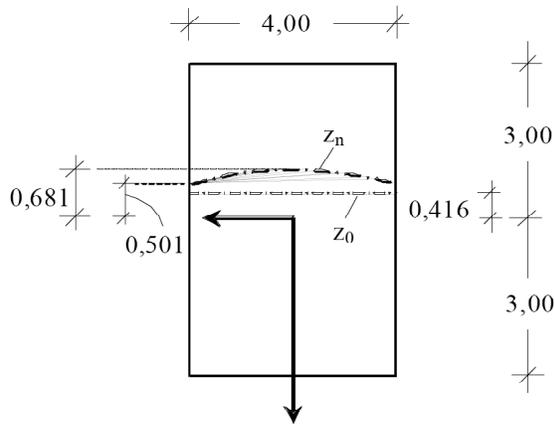
Fig. (11). Neutral line evolving to curve.

causes full plastification of the cross-section): $M_F = 108000$ dNcm.

Under the action of a moment $M_y = 80000$ dNcm and an axial force $N_x = 10000$ dN, and ignoring the stresses σ_r, the neutral axis will be at $z_o = -0.4167$ cm.

Considering the stress distribution σ_r shown in Fig. (7) and applying Eqs. (16), (17) and (18), we find that the neutral line becomes a curve as shown in Fig. (11).

Ignoring the influence of σ_r, we have:

$$M = \int_A \sigma z \, dA = 2\int_{A_\varepsilon} \sigma_f \frac{z^2}{\zeta} \, dA_\varepsilon +$$

$$+ b\sigma_f \left(\zeta_1 \frac{h_1 + \zeta}{2} + \zeta_2 \frac{h_2 + \zeta}{2} \right)$$

and since it is: $\zeta_1 = h_1 - \zeta$, and $\zeta_2 = h_2 - \zeta$ (see Fig. **12a,b**), we get:

$$M = 2b\sigma_f \left. \frac{z^3}{3\zeta} \right|_0^\zeta +$$

$$+ b\sigma_f \left(\frac{(h_1 + \zeta)(h_1 - \zeta)}{2} + \frac{(h_2 + \zeta)(h_2 - \zeta)}{2} \right) =$$

$$= \sigma_f b \left(\frac{h_1^2 + h_2^2}{2} - \frac{\zeta^2}{3} \right)$$

and: $W = \frac{b}{2}(h_1^2 + h_2^2) - \frac{b\zeta^2}{3}$

Taking into account the influence of σ_r it is valid that:
$$\sigma = \sigma_f \frac{z}{\zeta}$$

and, therefore:

$$M = \int_A \sigma \bar{z} \, dA =$$

$$= 2\int_{A_\varepsilon} \sigma_f \frac{\bar{z}^2}{\zeta} \, dA_\varepsilon + \sigma_f \int_{-b/2}^{b/2} \zeta_1 \frac{h_1 + \zeta}{2} \, dy +$$

$$+ \sigma_f \int_{-b/2}^{b/2} \zeta_2 \frac{h_2 + \zeta}{2} \, dy + \int_{-b/2}^{b/2} dy \int_{\alpha_1}^{\alpha_2} \sigma_c (z + \frac{h}{2} - h_1) dz$$

and since it is: $\zeta_1 = h_1 - \zeta$ and $\zeta_2 = h_2 - \zeta$, we obtain:

$$M = \sigma_f \int_{-b/2}^{b/2} \left(\frac{h_1^2 + h_2^2}{2} - \frac{\zeta^3}{3} \right) dy +$$

$$+ \int_{-b/2}^{b/2} \frac{(\alpha_1 + \alpha_2)(\alpha_1 - \alpha_2)^3}{12}(A + By^2) dy +$$

$$+ \int_{-b/2}^{b/2} (\frac{h}{2} - h_1) \frac{(\alpha_1 - \alpha_2)^3}{6}(A + By^2) dy =$$

$$= \sigma_f \int_{-b/2}^{b/2} \left(\frac{h_1^2 + h_2^2}{2} - \frac{\zeta^3}{3} + \frac{(\alpha_1 - \alpha_2)^3}{12}(\zeta_1 + \zeta_2 - 2h_1) \frac{A + By^2}{\sigma_f} \right) dy$$

or finally:

$$W = \int_{-b/2}^{b/2} \left(\begin{array}{c} \frac{h_1^2 + h_2^2}{2} - \frac{\zeta^3}{3} + \frac{(\alpha_1 - \alpha_2)^3}{12}(\zeta_1 + \zeta_2 - \\ -2h_1)\left(\frac{2k + 16\lambda}{b^2 + h^2} + \frac{24k + 48\lambda}{b^2 h^2} y^2 \right) \end{array} \right) dy$$

$$where: \quad k = \frac{\sigma_1}{\sigma_f} \quad and \quad \lambda = \frac{\sigma_o}{\sigma_f}$$

From the above expressions, without the influence of σ_r we get: $W = 28.0556$ cm^3 while with the influence of σ_r we get: $W = 23.7684$ cm^3.

CONCLUSIONS

This paper studies the influence of the distribution of the thermal residual stresses on the position as well as the form of the neutral line through a simple approach.
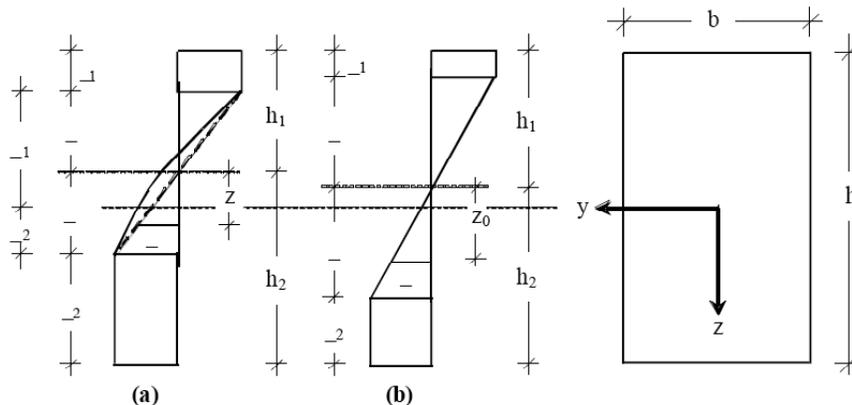

Fig. (12). Plastic regions and stress distributions in a solid cross-section.

A displacement of the neutral line (in doubly symmetric cross-sections) takes place in the case of simultaneous action of a bending moment and an axial force and only in the elastic or elastoplastic region, while the form and position of the neutral line in the plastic region is not affected.

This displacement is significant and ranges (for the case studies presented herein) from 5% to 30% of the height of an I-cross-section and from 2% to 10% for a solid orthogonal one.

A phenomenon that appears in solid orthogonal cross-sections is the evolution of the form of the neutral line which from a straight line becomes a curve.

The aforementioned alteration of the form and the displacement of the neutral line lead to the change of the corresponding resistant moments, which for the I-section of the case study amounts to 25% for the upper resistant moment and to 45% for the bottom one.

The corresponding change for the solid orthogonal cross-section studied herein amounts from 20 to 25%.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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