# The Optimal Design of the Scroll Profiles Based on Chaos PSO

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**Abstract:** The traditional design method of scroll profiles is single-objective optimization. The parameters are provided by meeting the requirements of designing and machining. It is very difficult to get an optimal design scheme. In fact the design of scroll profiles is a multi-objective optimization problem. In this paper mathematical models of the scroll compressor are fully analyzed in theory. A cost function is designed and it not only guarantees the lower friction loss power and leakage loss power but also has higher efficiency compared with the existing methods. In a practical example, multi-objective optimization of the scroll profiles is realized based on Chaos PSO. The outcomes show that this design method is well developed. It is possible that this design method can be applied to other fields of study.

Keywords: Chaos, PSO, Scroll compressor, Scroll profiles, Optimization.

# **1. INTRODUCTION**

The scroll type compressor (STC), one kind of positive displacement compressor, has been well-developed due to its advantages of higher efficiency, quiet operation and good reliability. The scroll compressors are widely employed in residential and commercial air-conditioning, refrigeration and heat-pump applications, as well as automotive air-conditioning. Many theoretical and experimental studies have introduced and verified detailed mathematic models for STCs, including those of Morishita *et al.*, [1], who derived the geometric parameters, the equations of motion and the dynamics of the scroll compressor. Chen *et al.*, [2,3] presented a comprehensive model combining a detailed compression process model with a detailed overall compressor model.

From a technical point of view, Etemad and Nieter [4] provided a simple and easily understood optimization design approach to evaluate the effect of three relevant physical parameters on manufacturing, design limitations and energy losses for STC, and Ooi [5] presented a design optimization algorithm coupled with a mathematical model of the rolling piston compressor by employing a multi-variable, direct-search, constrained optimization technique. Although these studies could provide some guides for STC optimization design algorithm and design procedure, but the detail optimum design to put in practice were not demonstrated.

Tseng [6] proposes a family design procedure that combines the optimization method with the STC simulation package for using in STC commercial product development to balance cost, manufacturing effectiveness and product - performance. Liu [7] proposes developing a design optimiza tion procedure by integrating the parametric STC model with an optimum solver.

The parameters optimization of static and dynamic scroll plate is the main way to improve efficiency. The parameters are modified one by one in traditional methods [8]. Particle Swarm Optimization (PSO) is a kind of random optimization algorithm based on swarm intelligence. PSO is an easy and fast algorithm that can solve some multi-objective optimization problems [9].

Performance of PSO is very sensitive to properly setting parameters. Several attempts have been made to improve the performance of basic PSO, which were classified by Niu *et al.*, [10] as follows: i. Adjusting the parameters in standard PSO [11-13]. ii. Designing different population topologies [14-17]. iii. Combining PSO with other search techniques [18-20]. iv. Incorporating bio-inspired mechanisms into the basic PSO [21-25]. v. Utilizing a multi-population scheme instead of single-population scheme of the basic PSO [26-28].

This paper proposes a simple STC design method that combines with the optimization method. Mathematical models of scroll compressor are analyzed completely in theory. A cost function is designed and it not only guarantees the lower friction loss power and leakage loss power but also has higher efficiency compared with the existing methods. A chaos PSO is applied to optimize scroll profiles geometric parameters. This paper improves the basic PSO by introducing of chaos algorithm. This method improves the convergence rate of PSO while keeping the model as simple as possible. The outcomes show that this design method is well developed.

The rest of this paper is organized as follows. Section 2 discusses the mathematical model of scroll compressor. Section 3 analyses chaos PSO algorithm. Section 4 optimizes geometric parameters of scroll profiles, and Section 5 reaches a conclusion.

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## 2. MATH MODEL ANALYSIS

The theory of involutes of circle is mature and can be machined easily by numerical control. Involutes of circle are usually used as scroll profiles. The basic parameters of scroll plate are shown as Fig. (1).



Fig. (1). The basic parameters.

Using Morishitaetal's derivation for the STC analytical model [1], the relations of the basic parameters are:

$$P=2\pi a \tag{1}$$

 $t=2a\alpha$  (2)

Where *P* is scroll pitch and *a* is the radius of basic circle and *t* is the thickness of scroll wall and  $\alpha$  is the initial angle of involutes.

The stroke volume Vs is obtained as follows:

$$V_s = \pi P(P-2t)(2n-1)h \tag{3}$$

Where  $V_s$  is the stroke volume of scroll compressor and n is the turn number of scrolls and h is highness of scroll wall.

Several dynamic forces caused by gas pressure in chambers and acted on the orbiting scroll are considered. Those forces are classified as tangential, radial and axial ones. The tangential force(Ft) can be expressed as follows:

$$F_{t} = P_{s}ph\sum_{i=1}^{N} (2i - \frac{\theta}{\pi}) \cdot (\rho_{i} - \rho_{i+1}) \cdot$$

$$\tag{4}$$

Where  $p_s$  is atmospheric pressure and k is adiabatic exponent and  $\theta$  is the scroll turning angle.

$$\rho_i = \frac{P_1}{P_s} \tag{5}$$

The total length of leakage line can be calculated by the next formula.

$$L = 2Nh + \sum_{i=1}^{N} P(2\pi - \theta)$$
(6)

# **3. CHAOS PSO ALGORITHM**

# 3.1. Basic PSO Algorithm

PSO algorithm seeks the optimal value through iteration. The particle pursues two optimal values which are used to

renew it. One is a best solution found so far by a particle, namely the best individual value (pBest); the other is a best solution obtained so far by any particles in the population, namely the best global value (gBest). Particles update themselves with the formulas as follows:

$$V_{i} = V_{i-1} + c_{1} \cdot r_{1} \cdot (p_{b} - x_{i}) + c_{2} \cdot r_{2} \cdot (p_{g} - x_{i})$$
(7)

$$x_i = x_{i-1} + V_i \tag{8}$$

Where  $V_i$  is the present generational removal speed and  $V_{i-I}$  is the former generational removal speed and  $r_1$ ,  $r_2$  is a random figure ranging from 0 to 1 and  $c_1$ ,  $c_2$  is learning factor, generally 2 is assigned to them and  $x_i$  is the present generational location and  $x_{i-I}$  is the former generational location and  $p_b$  is the best individual location and  $p_g$  is the best global location and w is the factor of inertia.

The factor of inertia has great significance for the optimization performance. The larger w factor does good to skip the local extremely small point, but a smaller w factor does good to the algorithm convergence. Ordinarily, we use the formula as follows to modify it.

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{iter_{\max}} \cdot iter$$
<sup>(9)</sup>

Where *iter* is the number of iteration.

## 3.2. The Chaos Improvement of PSO Algorithm

In order to make a good efficiency of self-adapting manipulation, this paper divides the group into two subgroups according to the distinction of individual self-adapting values. Meanwhile, we can adopt different operations so as to sustain the diversity of the swarm and avoid getting involved in the local optimal value. The better adaptive particles are used to seek local optimum and speed up the algorithm convergence; the worse adaptive particles are used to seek global optimum during an earlier stage, and then skip local optimum during a later stage so as to avoid premature convergence. We can reach a compromise between global convergence and convergent velocity by means of making both local optimum and global optimum work at the same time.

N is supposed to be the number of particles, and the particle adaptive value at the i generation is  $f_i$ , and then the particle average adaptive value is:

$$f_{avg} = \frac{1}{n} \sum_{i=1}^{n} f_i$$
 (10)

The classified self-adapting method is applied to the particles whose adaptive value is  $f_i$  when the algorithm is involved in stagnation, as follows: (1)  $f_i$  is superior or equal to  $f_{avg}$ . These particles are the excellent ones in the swarm and they have become much closer to the global value. Consequently, their locations don't change. (2) $f_i$  is inferior to  $f_{avg}$ . These particles are the bad ones and their location would be changed by means of chaos initialization.

Chaos is a kind of nonlinear phenomenon which widely exists in nature. It has such characters as randomness, ergodicity, initialized condition sensibility, etc. By the way of chaos initializing the particles' locations, the random of PSO algorithm hasn't be changed, but both the diversity of the swarm and the ergodicity of search are improved. Initialization steps are as follows:

- (1) Produce  $1 \times n$  random digits ranging from 0 to 1,  $P_{1,n}$ , *n* is the number of variables in target function.
- (2) Assign the particles' location by chaos, mapped by Logistic function:

$$P_{i,n} = 4P_{i-1,n}(1 - P_{i-1,n}) \tag{11}$$

(i=2,3,...,m, m is the number of particles in the swarm),  $P_{i,n}$  is not a motionless point, if it is ,we shall add r/C, and C is a larger positive number.

(3) Chaos section ranging from 0 to 1 is mapped to the value section of the corresponding variables.

#### 3.3. Algorithm Description

The procedure of algorithm can be described as follows:

- (1) The locations, the velocity and inertia weight are initialized by chaos algorithm. The optimal individual location  $p_b$  and the optimal global location  $p_g$  are calculated.
- (2) Carry out operations as follows to each particle in the swarm:
  - Update the speed and location of particle based on formula (7) and formula(8);
  - <sup>(2)</sup> Calculate adaptive value  $f_i$ ;
  - (3) If its  $f_i$  is superior to the adaptive of  $p_b$ , then update the current location with the location of  $p_b$ ;
  - ④ If its  $f_i$  is superior to the adaptive of  $p_g$ , then update the current location with the location of  $p_g$ ;
- (3) Judge whether the terminated condition is satisfactory or not. if it is, carry out step(6) directly, otherwise do step(4);
- (4) Judge whether the algorithm is stagnant or not, if it is. carry out step(5) directly, otherwise do step(2);
- (5) Adopt classified self-adapting tactics according to the difference among particles' adaptive value, adjust the particles' location do step (2);
- (6) Output related information of  $p_g$  and end algorithm.

We call this algorithm Chaos PSO algorithm, CPSO.

## 4. PARAMETERS OPTIMIZATION

## 4.1. Cost Function

In Fig. (2), mechanical frictional loss and leakage loss are the main parts of compressor loss power. Because the value is small, other loss power such as flux frictional power may be ignored.

Stroke volume is an important performance index of scroll compressor. It is directly related to the displacement of scroll compressor. At the same rotation speed of compressor, the compressor displacement increases correspondingly when the stroke volume increases. In the case of the same input power, designers will pursue bigger displacement and stroke volume. We may know from formula (3) that stroke volume can be optimized by changing the profile parameters. The goal of optimization is to increase the stroke volume.



Fig. (2). Composition of axis power.

When a scroll compressor runs, the dynamic scroll plate is affected by many forces such as the tangential gas force, the radial gas force and so on. The frictional loss of dynamic and static scroll plate are affected largely by the tangential gas force  $F_t$ . We may know from formula (4) that the tangential gas force  $F_t$  can be optimized by changing the profile parameters. The goal of optimization is to reduce the tangential gas force.

The leakage loss power is related to not only the gas character and the thermodynamic parameters but also the length of leakage line and leakage clearance. Leakage clearance can be guaranteed by machining and assemble process. The total length of leakage line is the most important fact affecting the leakage loss power. We may know from formula (6) that the total length of leakage line can be optimized by changing the profile parameters. The goal of optimization is to reduce the total length of leakage line.

Design of cost function is an important decision-making affecting optimal scheme. For scroll compressor optimization, the final target of optimization is less loss power and higher efficiency. The length of leakage line is the main factor affecting leakage power and the tangential gas force is the main factor affecting frictional power. In order to simplify the calculation in engineering application, the article designs a reasonable cost function. The cost function is provided as follows:

$$costfun=k_1L+k_2F_t+k_3(1/V_s)$$
(12)

Where  $k_1$  and  $k_2$  and  $k_3$  is weight coefficient. They represent the important degree of each performance indices. Our optimization target is to make the cost function become smallest.

### 4.2. Parameters Selection

The study of profiles parameters is very essential to the optimization of a scroll compressor. When given parameters are presented, a few unrelated parameters will appear in the compressor design process. Generally all geometric parameters can be fixed on by selecting three parameters. We selected *t*, *h* and *n* as the optimized parameters. The meaning of those parameters is shown in Table **1**. Those parameters should be confined in a small range according to design experience. The parameters in this document will be confined as follows: 3.5 < t < 6(mm), 10 < h < 50(mm), 2 < n < 4.5.

**Table 1. The Selected Parameters** 

Number	Name	Meaning	
1	n	the turn number of scrolls	
2	t	thickness of scroll wall	
3	h	highness of scroll wall	

## 4.3. Practical Example

We select CPSO as the optimization algorithm. Parameters of CPSO are set as follows: the size of swarm is 30,  $c_1=c_2=2$ ,  $w_{max}=0.9$ ,  $w_{min}=0.4$ , *iter=40*. Parameters of scroll compressor are set as follows:  $P_s=0.1MP$ ,  $P_d=0.77MP$ , k=1.31, n'=2900r/min, the angle  $\theta$  is a fixed value.

From Table **2**, we may find cost function value will become smaller and smaller. The optimal process is smooth and the parameters are convergent step by step.

Number	Parameters			Costfun
	Н	t	n	Costiun
1	21.5	4.1	3.395	1.87
2	24.5	4	3.365	1.38
3	23.5	3.95	3.375	1.13

**Table 2. Some Value of Cost Function** 

The speed of CPSO is faster than that of PSO in Fig. (3). CPSO isn't easy to be trapped in local optimization. It acquires a good effect that CPSO is applied in optimization of a scroll compressor. The optimal parameters were applied to draw the scroll profiles in Fig. (4). The basic circle hasn't



Fig. (3). Process of optimization.

been modified and the compressor has three compress chambers. We know from simulation other performance indices are good enough, too.



Fig. (4). Scroll profiles.

## **5. CONCLUSION**

This study has demonstrated a systematic and practical process for optimization of STC profiles parameters to meet the objective requirement of commercialization. Three important aspects of this research are summarized as follows:

- 1) The CPSO algorithm has the same character of simplicity and easy realization as PSO algorithm. It not only has the strong global optimal capability and the faster convergent velocity but also can avoid the earlier convergent problem of PSO algorithm. It has been proved that the CPSO algorithm in the application of scroll compressor is correct and efficient.
- 2) Based on the principle of improving efficiency, a simple cost function is proposed. In this cost function, friction and leakage of scroll compressor are completely considered. It is more practical than the earlier models.
- 3) A new design method of scroll compressor is brought forward by using CPSO. The less frictional loss and leakage loss and the higher efficiency may be attained by employing this method. The feasibility has been verified through the practical example. This new way has enough value in designing other scroll profiles.

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# **CONFLICT OF INTEREST**

None declared.

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