

Stiffness of a Foldable Tower for Wind Turbine

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Abstract: With the shortage of fuel in the world, wind power becomes one of the most potential energy today. For the demand for more energy from wind turbine, the tower is tending to be designed much higher than before to capture the wind with faster speed. However, the higher the tower is the more possible the catastrophe of a whole wind turbine structure will occur when hurricanes come. This paper proposes a foldable tower for wind turbine to avoid the disasters brought about by strong wind in the high-level sky. The tower can be folded when hurricanes come and can be deployed fully to acquire more wind power in high sky in the normal cases. Firstly, the geometry design of a deployable structure is presented in details by discussing the deployable ratio. Then, it focuses on the statics, stiffness analysis of this structure. The equivalent stiffness of the foldable tower is expressed as a function of the primary design parameters and the deployable angle. What must be pointed out is that the result comprehensively expresses the dependence of the stiffness on the deployable angle and the stiffness of the fully deployed case. This is particularly useful for the applications in wind turbine.

Keywords: Foldable tower, geometry design, statics, stiffness analysis.

1. INTRODUCTION

Wind turbine, which is also called wind-generator set, is a kind of equipment that transforms wind energy into electrical energy. Wind turbine is a both ancient and modern machine. In terms of its age, windmill has been used for more than 3,000 years and from as early as the thirteenth century, horizontal-axis wind mill has been an integral part of rural economy [1]. However, with the advent of cheap fossil-fuelled engines, wind energy and wind turbine fell into disuse for a very long time. During this time until 1970s, only a few scientists such as Professor Hunter in German and corporations such as Electricite De France had made some finite advances in the development of wind turbine. But during the latest decades of the 20th century, things have completely changed. The forecast of the fuel shortage in the near future combined with the negative environmental impacts caused by the use of the traditional electricity production methods forced all those involved in the energy production field to start exploring new directions in energy production [2]. In 1997, the *Commission of Europe Union* published its white paper calling for 12 percent of the gross energy demand of Europe union to be contributed from renewable energy in 2010 [1]. Wind energy is the widely acknowledged renewable and clear energy source. The demand for sustainable energy rekindles the enthusiasm of the investigation of wind energy and wind turbine. As a result, wind turbine technology has developed a lot during these years, with a continuous increase in the rotor diameter and a doubling speed of the wind it makes use of, both contributing to an increase in energy output [1]. That's why

wind energy is called a new kind of energy even though it has a long history of several thousand years.

Now that wind energy is in great demand in the world today, technological improvement should be done to increase its energy output. The power law $V_2/V_1 = Z_2/Z_1^n$ is often used for height projection of wind profiles, with V_1 and V_2 being in m/s , Z_1 and Z_2 being in m , the exponent n sometimes is taken as depending on surface conditions or on atmospheric stability, (The power law profile for wind speed is shown here to be consistent with the observed height variation of Weibull wind speed probability distribution functions which have been found to fit the observed wind speed distributions, at least the above relevant threshold wind speeds [3]). It can be deduced that the wind turbine can obtain higher wind speed with the increase of height. In that case, the design of the higher wind turbine tower shows its importance.

Wind generator tower is used for carrying cabin and rotor, the primary components of a wind turbine. According to the discussion above, high tower has its advantages because the higher the tower is from the ground, the larger the wind speed will be on its top. Modern million-watt wind turbine has a tower as high as from 40 to 80 meters. The tallest wind turbine in the world is 205 meters (Its axis is 160 meters above the ground.) [4]. There are many kinds of wind turbine towers. It can be either a tubular tower or a lattice tower. The highest wind turbine mentioned has a lattice tower, but the most popular tower is a single tubular column [5]. Tubular tower ensures personal safety, because they can reach the top through an internal stair or lifter.

However, until now there is no foldable wind generator tower in the world. When strong wind comes, such as the hurricanes from the sea and the tornadoes, a high tower is liable to be ruptured, bended or even collapsed. Therefore the foldable tower is particularly necessary to be synthesized to settle these problems. The tower can be fully deployed to capture the wind energy when the wind is not strong enough.

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When the wind grows to be a destroyable hurricane, the tower can be folded completely and covered by a shelter to avoid the disaster.

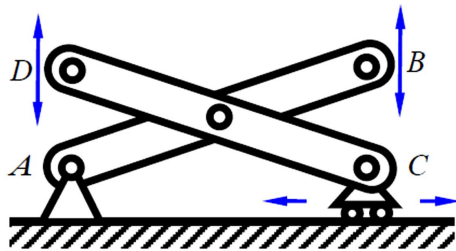
Deployable structures are prefabricated space frames consisting of straight bars linked together in the factory as a compact bundle, which can then be unfolded into large-span, load bearing structural shapes by simple articulation [6]. Deployable structures often offer significant advantage comparing to the conventional fixed structure for a wide spectrum of applications mainly because of their feature of transforming and adapting to changing needs [6]. The investigation of deployable structure has started since 1960s, and many kinds of deployable structures have been designed to be applied in all kinds of areas. For example, in aerospace industry, Christiansen, Ker, Fuentes, and Schneider created the TransHab shields which contain a positive attribution of flexible and deployable structures. The foam is compressed during launch to minimize volume and released on-orbit to deploy the bumpers of the shield [7]. In Biology area, the deployable structures are studied and these structures are classified into four groups, namely, planar, cylindrical, stiff and compliant [8]. What's more, some new methods have been worked out to design new foldable structures. For example, Wei *et al.* [9] proposed a method to create expandable structure for spatial objects. Gan and Pellegrino presented a numerical approach to the kinematic analysis of deployable structures forming a closed loop [10]. According to the discussion above, deployable structure is a good choice for the wind turbine tower, if it can be designed properly.

A good foldable tower should have artful structural design and a proper foldable ratio. What is more important, it should have enough strength and stiffness, namely, it should have the ability to prevent failure [11]. This paper is trying to achieve these goals.

2. GEOMETRY DESIGN OF THE FOLDABLE TOWER

2.1. Fundamental Structure Analysis

The simplest pantograph structure is a pair of links, AB and CD shown in Fig. (1), that are revolute jointed at their middle points. When one end of this structure, A , is pivot jointed with the fixed base and a second end, D , can make a determinate translation, this structure forms a scissor-like element (SLE) structure shown in Fig. (1).



A, B, C, D are hinges of this construction.

Fig. (1). A foldable SLE structure.

When a number of SLEs shown in Fig. (1) are connected one by one in series shown in Fig. (2), a foldable structure is

obtained. The top of the structure will move up and down when joint C_1 translates left and right along the guide on the fixed base.

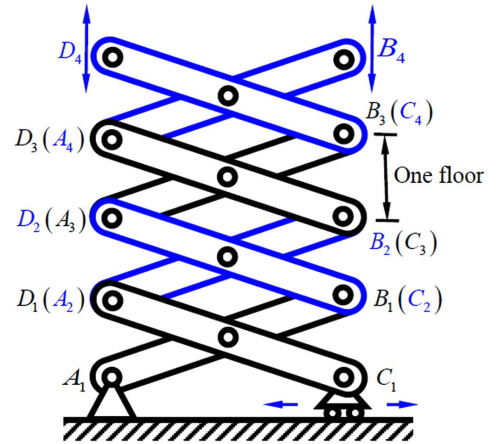
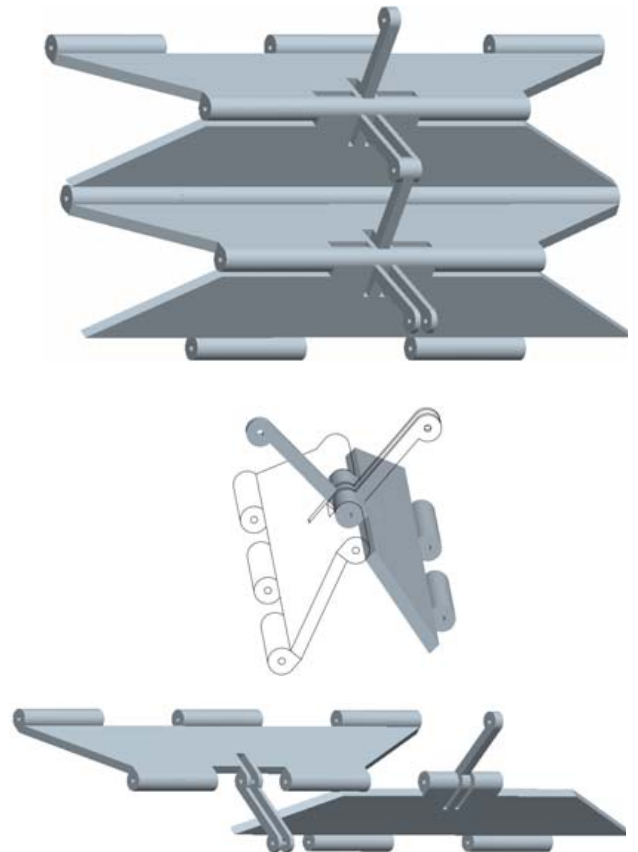


Fig. (2). A foldable structure.

However, the strength and stiffness of the foldable structure shown in Fig. (2) might not satisfy the needs of engineering. If half of each member is particularly designed to be thick plate, the lateral strength and stiffness of the whole structure will be improved greatly which is shown in Fig. (3).

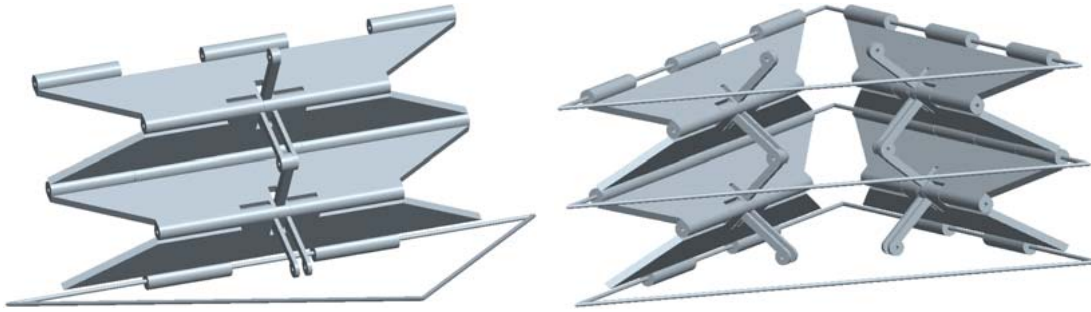


Top is the assembly of two levels, middle is the assembly of one level and bottom is a pair of members.

Fig. (3). Half of each member is expanded to be a thick plate.

Simultaneously, if two or more such structures are connected in parallel, the strength and stiffness will be increased a lot more. As is shown in Fig. (4) and Fig. (5), the whole

structure will become stronger if three such structures are connected through an equilateral triangle in each level correspondingly.



Left is one foldable structure, right is two foldable structures connected by an equilateral triangle in each level.

Fig. (4). Parallel structure scheme.

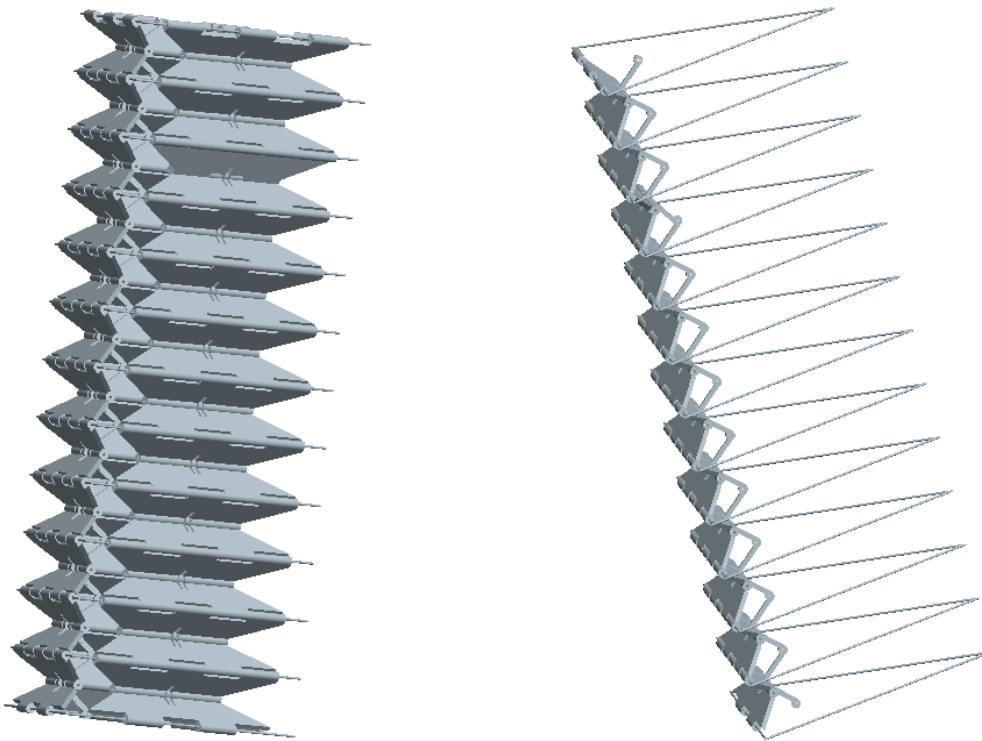
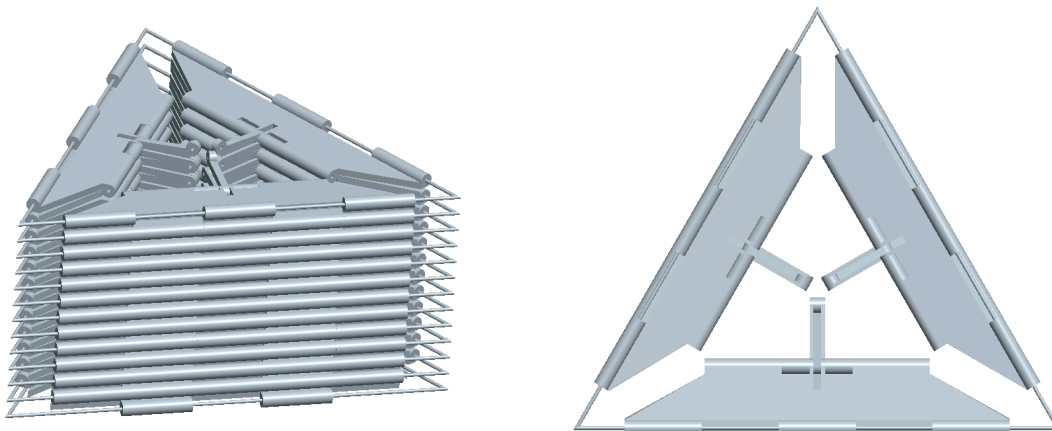


Fig. (5). The assembly of a foldable tower and its one branch.



Left is the isotropic view, right is the top view of the tower.

Fig. (6). The folded tower.

Theoretically, the tower can be completely folded. Fig. (6) shows the folded tower.

2.2. Calculation of Foldable Ratio

Foldable ratio is defined as the quotient of the maximum length to the minimum length of a foldable structure. It is an important quality for any foldable structures to have larger foldable ratio that can save more material to reach the same height and save the required space when completely folded. But the foldable ratio should not be very large because of the limitations of the structure design.

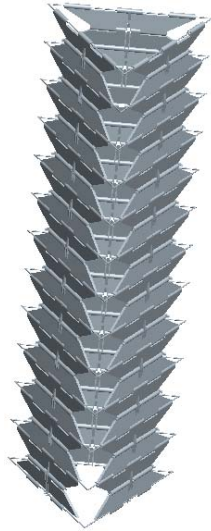


Fig. (7). The completely deployed tower.

Fig. (7) shows the case when the tower stretches to the maximum and Fig. (6) shows the case when it is completely folded. Assume the maximum angle is $2\theta_{max}$ and the minimum subtended angle of the two planes is $2\theta_{min} = 0$ in Fig. (8). According to the geometry relationship, $2\theta_{max}$ corresponds to the minimum of α .

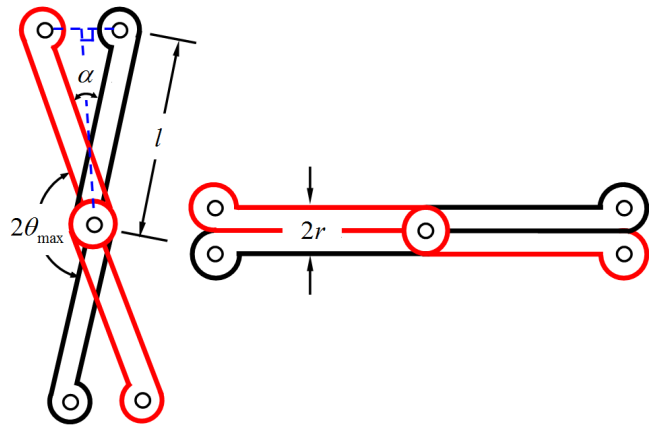


Fig. (8). The geometry position when the tower is completely deployed.

Theoretically,

$$\alpha_{min} = 2 \arcsin \frac{r}{l} \tag{1}$$

where r denotes the radius of the revolute joint and l represents the half length of each link.

Therefore,

$$2\theta_{max} = \pi - \alpha_{min} = \pi - 2 \arcsin \frac{r}{l} \tag{2}$$

As a result, the deployable ratio of the foldable tower is

$$q = \frac{2l \sin \theta_{max}}{2r} = \sqrt{\left(\frac{l}{r}\right)^2 - 1} \tag{3}$$

Equation (3) indicates that the deployable ratio of the tower is uniquely determined by the ratio of l to r . q will be as high as 19.97 if one select $\frac{l}{r} = \frac{2000\text{mm}}{100\text{mm}} = 20$ which is a very high ratio in engineering. That is to say, a folded tower of 4m high can be expanded to 79.88m. This kind of foldable tower can be effectively used in the wind turbine of offshore field for both providing the working condition for wind turbine and avoiding the hurricane's destroy by folding and covering a protective shelter.

3. STIFFNESS ANALYSIS OF THE FOLDABLE TOWER

3.1. Statics of the Foldable Tower

When the foldable tower works, it is deployed fully just as if it is a kind of normal fixed tower, staying in a state without moving. But the tower is subjected to the gravity of the wind turbine and the gravity of itself, so how can it be in a state of equilibrium at any poses including the pose, θ_{max} , shown in Fig. (8)? Therefore, the equivalent stiffness of the tower should be investigated.

Suppose that the bending moment, M , which is used to make the foldable tower in a state of equilibrium, when the subtended angle between AB and the horizontal plane is θ . Assume that the mass of every link is m , the length of each link is $2l$ as shown in Fig. (8). Suppose the bending moment M is exerted to joint A shown in Fig. (9). According to the principle of virtual power, one obtains:

$$M\delta\theta - \sum_{i=1}^n (2mg)\delta[(2i-1)l \sin \theta] - P\delta(2nl \sin \theta) = 0 \tag{4}$$

where $\delta(\cdot)$ represents the variation of “.”.

Therefore, one immediately gains

$$M = 2(n^2mg + nP)l \cos \theta \tag{5}$$

The start value of moment needed to unfold the tower is $M = 2(n^2mg + nP)l \cos \theta_{min}$ and then the tower is deployed in a constant velocity. When θ changes from θ_{min} to θ_{max} , the whole work of M is:

$$\begin{aligned}
 W &= \int_{\theta_{\min}}^{\theta_{\max}} M d\theta \\
 &= l(6n^2 mg + 2nP) \int_{\theta_{\min}}^{\theta_{\max}} \cos\theta d\theta \\
 &= l(6n^2 mg + 2nP)(\sin\theta_{\max} - \sin\theta_{\min})
 \end{aligned}
 \tag{6}$$

where W denotes the work done to the foldable tower by the torque M .

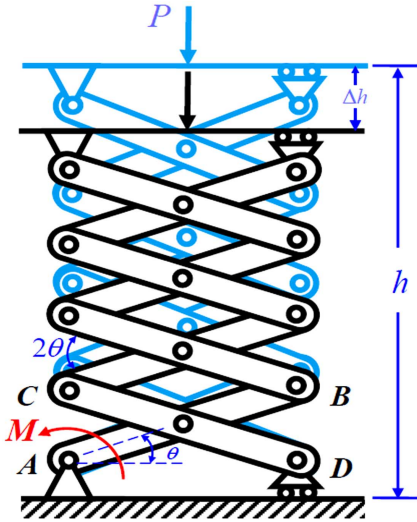


Fig. (9). Statics model.

3.2. Equivalent Stiffness of the Foldable Tower

When a load, P , shown in Fig. (9), is exerted on the top of the foldable tower, the whole structure is subjected to a compression and therefore its height will surely reduce a distance, denoted by Δh . Each link will be deformed correspondingly, compressed and bended when it subjects to the bending moment and axial force, which is shown in Fig. (10).

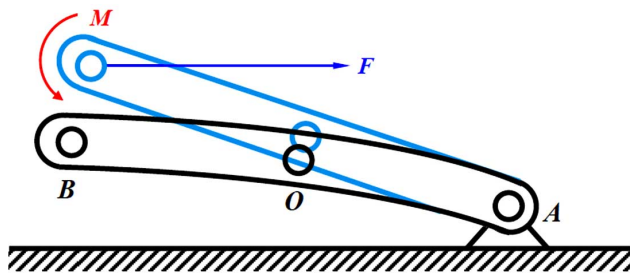


Fig. (10). Deformations of a link under a bending moment and a force.

When a specified force, P , is exerted on the top of the tower, the whole deformation Δh can be calculated. According to Hooke's law, the equivalent linear elastic coefficient k can be equivalently expressed as

$$k = \frac{P}{\Delta h}
 \tag{7}$$

To simplify the analysis of the deformations, all links in the structure are treated as slender and long straight beams. The subtended angle between AB and the horizontal plane

shown in Fig. (11) is θ which indicates the deployable angle of the whole structure. Also suppose $l_{AB} = l_{CD} = 2l$. The external force, P , is exerted on the geometry center of the tower's top plane. Assume there are n layers in the tower. Let the top layer be the 1st layer, and the bottom layer is the n th one.

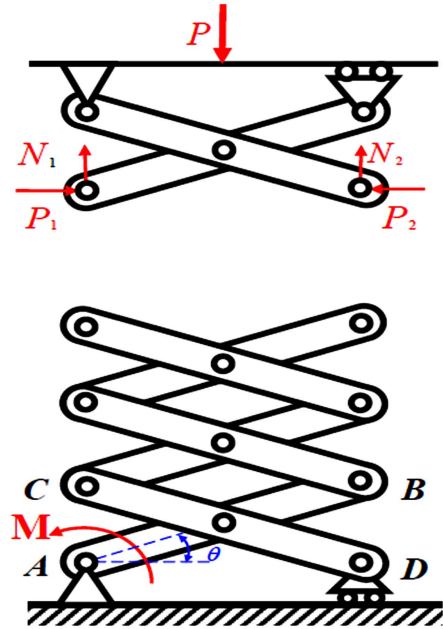


Fig. (11). Statics analysis of the first level.

First of all, the statics of the 1st layer in Fig. (12) can be analyzed as the free body system under static constraints. Because of the structural symmetry, one immediately finds that:

$$\begin{cases}
 N_1 = N_2 = \frac{P}{2} + mg \\
 P_1 = P_2
 \end{cases}
 \tag{8}$$

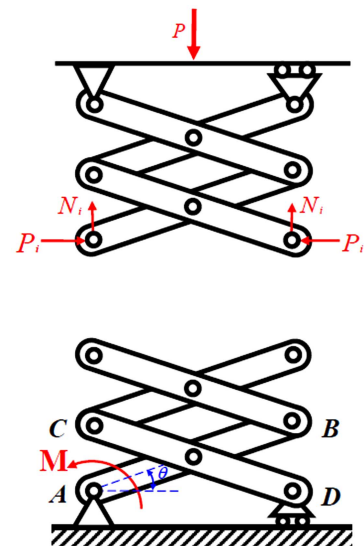


Fig. (12). The statics of the tower.

According to the principle of virtual power, there is

$$-P_2 \delta(2l \cos\theta) - P \delta(2l \sin\theta) - 2mg \delta(l \sin\theta) = 0
 \tag{9}$$

Substituting equation (8) into equation (9) yields

$$P_1 = P_2 = (P + mg) \cot \theta \quad (10)$$

Next, for the i^{th} layer where $i = 1, 2, \dots, n-1$.

$$N_i = \frac{1}{2} P + img \quad (11)$$

According to the principle of virtual power, one obtains:

$$-P_i \delta(2l \cos \theta) - P \delta(2il \sin \theta) - \sum_{j=1}^i 2mg \delta[(2j-1)l \sin \theta] = 0 \quad (12)$$

Simplifying equation (12) yields

$$P_i = i(P + img) \cot \theta \quad (13)$$

Associating equations (11) and (13) generates

$$\begin{cases} P_i = i(P + img) \cot \theta \\ N_i = \frac{P}{2} + img \end{cases} \quad (14)$$

From equation (14), one immediately obtains

$$\begin{cases} P_{i-1} = (i-1)[P + (i-1)mg] \cot \theta \\ N_{i-1} = \frac{P}{2} + (i-1)mg \end{cases} \quad (15)$$

Next, one can examine the statics of every link in the i^{th} ($i = 1, 2, \dots, n-1$) layer shown in Fig. (13).

$$-F_i + P_i + P_{i-1} = 0 \quad (16)$$

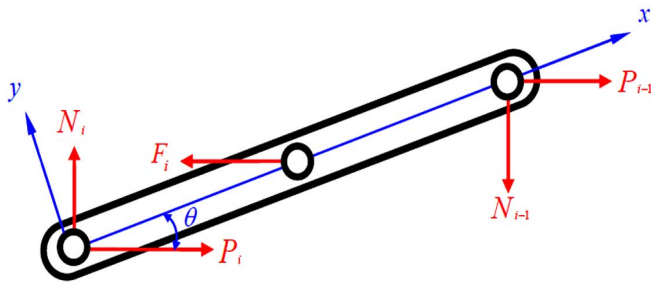


Fig. (13). The statics of a bar in the i^{th} floor.

Therefore,

$$F_i = P_i + P_{i-1} = [(2i-1)P + (2i^2 - 2i + 1)mg] \cot \theta \quad (17)$$

When $0 \leq x \leq l$, the bending moment, the axial and shearing forces of the link are

$$\begin{cases} M_z(x) = P_i x \sin \theta - N_i x \cos \theta + \int_0^x \frac{mg}{2l} \xi \cos \theta d\xi \\ = \left[\left(i - \frac{1}{2} \right) P + (i^2 - i) mg + \frac{mg}{4l} x \right] x \cos \theta \\ N_x(x) = P_i \cos \theta + N_i \sin \theta - \frac{mg}{2l} x \sin \theta \\ = \left(i \cot \theta \cos \theta + \frac{1}{2} \sin \theta \right) P + \left(i^2 \cot \theta \cos \theta + i \sin \theta - \frac{x}{2l} \sin \theta \right) mg \\ Q_y(x) = -P_i \sin \theta + N_i \cos \theta - \frac{mg}{2l} x \cos \theta \\ = - \left[\left(i - \frac{1}{2} \right) P - (i^2 - i) mg + \frac{mg}{2l} x \right] \cos \theta \end{cases} \quad (18a)$$

When $l \leq x \leq 2l$, the bending moment, the axial and shearing forces of the link are

$$\begin{cases} M_z(x) = P_{i-1} (2l-x) \sin \theta + N_{i-1} (2l-x) \cos \theta + \\ \int_0^{2l-x} \frac{mg}{2l} \xi \cos \theta d\xi = P \left(i - \frac{1}{2} \right) (2l-x) \cos \theta \\ + \left[(i^2 - i) + \frac{1}{4l} (2l-x) \right] (2l-x) mg \cos \theta \\ N_x(x) = -P_{i-1} \cos \theta + N_{i-1} \sin \theta - \frac{mg}{2l} x \sin \theta \\ = -(i-1) \left[P + (i-1) mg \right] \cot \theta \cos \theta \\ + \left(\frac{P}{2} + img \right) \sin \theta - \frac{mg}{2l} x \sin \theta \\ Q_y(x) = P_{i-1} \sin \theta + N_{i-1} \cos \theta - \frac{mg}{2l} x \cos \theta \\ = \left[\left(i - \frac{1}{2} \right) P + (i^2 - i + 1) mg \right] \cos \theta - \frac{mg}{2l} x \cos \theta \end{cases} \quad (18b)$$

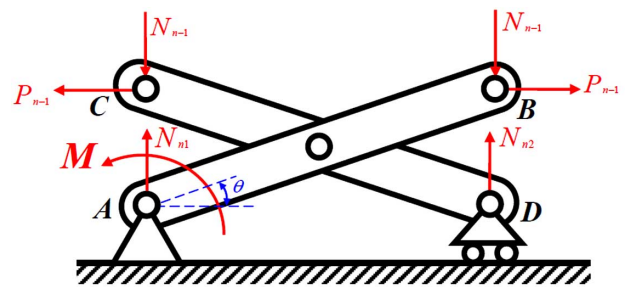


Fig. (14). The statics analysis for the bottom floor.

Finally, for the n^{th} layer (bottom layer), in line with the principle of virtual power, one has

$$M \delta \theta - 2N_{n-1} \delta(2l \sin \theta) + P_{n-1} \delta(2l \cos \theta) - 2mg \delta(l \sin \theta) = 0 \quad (19)$$

Because the equilibrium criteria of link AB are different from those of link CD , the statics of link CD (Fig. 16) should be established using the same method as well. The equilibrium conditions of link CD can be expressed as:

When $0 \leq x \leq l$, the bending moment, the axial and shearing forces of the link are

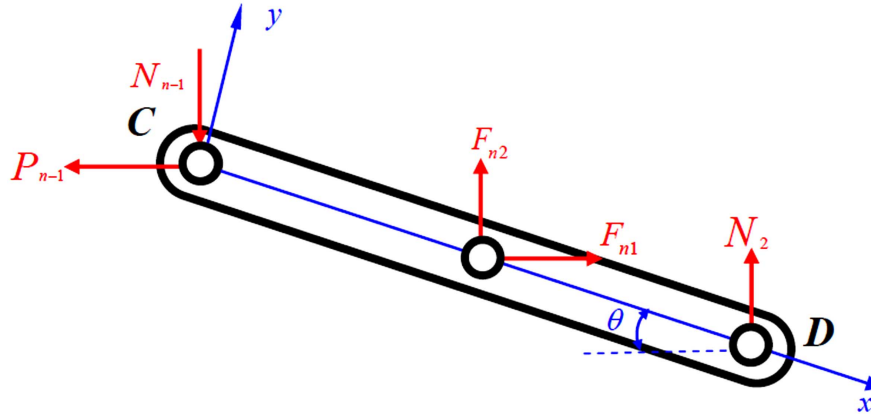


Fig. (16). Statics of link CD .

$$\left\{ \begin{aligned}
 M_z(x) &= N_{n-1}x \cos \theta + P_{n-1}x \sin \theta + \int_0^x \frac{mg}{2l} \xi \cos \theta d\xi \\
 &= \left[\frac{1}{2}P + (n-1)mg \right] x \cos \theta + (n-1) \left[P + (n-1)mg \right] x \cot \theta \sin \theta + \frac{mg}{4l} x^2 \cos \theta \\
 &= \left[\left(n - \frac{1}{2} \right) P + (n^2 - n)mg + \frac{mg}{4l} x \right] x \cos \theta \\
 N_x(x) &= N_{n-1} \sin \theta - P_{n-1} \cos \theta + \frac{mg}{2l} x \sin \theta \\
 &= \left[\frac{1}{2}P + (n-1)mg \right] \sin \theta - (n-1) \left[P + (n-1)mg \right] \cot \theta \cos \theta + \frac{mg}{2l} x \sin \theta \\
 Q_y(x) &= -N_{n-1} \cos \theta - P_{n-1} \sin \theta - \frac{mg}{2l} x \cos \theta \\
 &= - \left[\left(n - \frac{1}{2} \right) P + (n^2 - n)mg \right] \cos \theta - \frac{mg}{2l} x \cos \theta
 \end{aligned} \right. \tag{26a}$$

When $l \leq x \leq 2l$, the bending moment, the axial and shearing forces of the link are

$$\left\{ \begin{aligned}
 M_z(x) &= -N_{n2}(2l-x) \cos \theta - \int_0^{2l-x} \frac{mg}{2l} \xi \cos \theta d\xi = \left[\left(n - \frac{1}{2} \right) P + \left(n^2 - n + \frac{1}{2} - \frac{x}{4l} \right) mg \right] (2l-x) \cos \theta \\
 N_x(x) &= N_{n2} \sin \theta - \frac{mg}{2l} (2l-x) \sin \theta = - \left[\left(n - \frac{1}{2} \right) P + (n^2 - n + 1)mg - \frac{mg}{2l} x \right] \sin \theta \\
 Q_y(x) &= N_{n2} \cos \theta - \frac{mg}{2l} (2l-x) \cos \theta = - \left[\left(n - \frac{1}{2} \right) P + (n^2 - n + 1)mg - \frac{mg}{2l} x \right] \cos \theta
 \end{aligned} \right. \tag{26b}$$

Now the bending moment, the axial and shearing forces of all links of the structure are known. Therefore, the whole deformation Δh of the tower when the force P is exerted on the top can be obtained in line with the Castigliano's theorem

$$\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3, \tag{27}$$

where

$$\left\{ \Delta h_1 = \sum_{i=1}^{n-1} \left(\int_{2l}^{2l} \frac{N_x}{EA} \frac{\partial N_x}{\partial P} dx + \int_{2l}^{2l} \frac{M_z}{EI} \frac{\partial M_z}{\partial P} dx + \int_{2l}^{2l} \frac{KQ_y}{GA} \frac{\partial Q_y}{\partial P} dx \right) \right.$$

$$\left. \begin{cases} \Delta h_2 = \int_{AB} \left(\frac{N_x}{EA} \frac{\partial N_x}{\partial P} + \frac{M_z}{EI} \frac{\partial M_z}{\partial P} + \frac{KQ_y}{GA} \frac{\partial Q_y}{\partial P} \right) dx \\ \Delta h_3 = \int_{CD} \left(\frac{N_x}{EA} \frac{\partial N_x}{\partial P} + \frac{M_z}{EI} \frac{\partial M_z}{\partial P} + \frac{KQ_y}{GA} \frac{\partial Q_y}{\partial P} \right) dx \end{cases}$$

and where $K = \frac{6}{5}$ for rectangular cross-section link, $\frac{\partial(\cdot)}{\partial P}$ indicates the partial derivative of “.” with respect to force P , E represents the modulus of elasticity of the material; G represents shear modulus of elasticity of the material. And I represents the inertia moment of the transverse section of the beam; A represents the area of the transverse section of the link.

According to the Castigliano's theorem, one obtains

$$\begin{aligned} \Delta h_1 &= \sum_{i=1}^{n-1} \left(\int_{2l}^{2l} \frac{N_x}{EA} \frac{\partial N_x}{\partial P} dx + \int_{2l}^{2l} \frac{M_z}{EI} \frac{\partial M_z}{\partial P} dx + \int_{2l}^{2l} \frac{KQ_y}{GA} \frac{\partial Q_y}{\partial P} dx \right) \\ &= \frac{1}{EA} \sum_{i=1}^{n-1} \left(\int_0^l \left[\left(i \cot \theta \cos \theta + \frac{1}{2} \sin \theta \right) P + \left(i^2 \cot \theta \cos \theta + i \sin \theta - \frac{x}{2l} \sin \theta \right) mg \right] \left(i \cot \theta \cos \theta + \frac{1}{2} \sin \theta \right) dx \right. \\ &\quad \left. + \int_l^{2l} \left\{ -(i-1) \left[P + (i-1)mg \right] \cot \theta \cos \theta + \left(\frac{P}{2} + img \right) \sin \theta - \frac{mg}{2l} x \sin \theta \right\} \left[-(i-1) \cot \theta \cos \theta + \frac{1}{2} \sin \theta \right] dx \right) \\ &\quad + \frac{1}{EI} \sum_{i=1}^{n-1} \left(\int_0^l \left[\left(i - \frac{1}{2} \right) P + \left(i^2 - i \right) mg + \frac{mg}{4l} x \right] \left(i - \frac{1}{2} \right) x^2 \cos^2 \theta dx \right. \\ &\quad \left. + \int_l^{2l} \left\{ \left(i - \frac{1}{2} \right) P + \left[\left(i^2 - i \right) + \frac{1}{4l} (2l-x) \right] mg \right\} \left(i - \frac{1}{2} \right) (2l-x)^2 \cos^2 \theta dx \right) \\ &\quad + \frac{K}{GA} \sum_{i=1}^{n-1} \left(\int_0^l \left[\left(i - \frac{1}{2} \right) P - \left(i^2 - i \right) mg + \frac{mg}{2l} x \right] \left(i - \frac{1}{2} \right) \cos^2 \theta dx \right. \\ &\quad \left. + \int_l^{2l} \left[\left(i - \frac{1}{2} \right) P + \left(i^2 - i + 1 \right) mg - \frac{mg}{2l} x \right] \left(i - \frac{1}{2} \right) \cos^2 \theta dx \right) \\ &= \frac{2l^3}{3EI} \sum_{i=1}^{n-1} \left[\left(i - \frac{1}{2} \right) P + \left(i^2 - i + \frac{3}{16} \right) mg \right] \left(i - \frac{1}{2} \right) \cos^2 \theta \\ &\quad + \frac{l}{EA} \sum_{i=1}^{n-1} \left[\left(i \cot \theta \cos \theta + \frac{1}{2} \sin \theta \right) P + \left(i^2 \cot \theta \cos \theta + i \sin \theta - \frac{1}{4} \sin \theta \right) mg \right] \left(i \cos \theta \cot \theta + \frac{1}{2} \sin \theta \right) \\ &\quad + \frac{l}{EA} \sum_{i=1}^{n-1} \left\{ \left[\left(1-i \right) \cot \theta \cos \theta + \frac{1}{2} \sin \theta \right] P + \left[\left(i - \frac{3}{4} \right) \sin \theta - \left(i-1 \right)^2 \cot \theta \cos \theta \right] mg \right\} \left[\left(1-i \right) \cos \theta \cot \theta + \frac{1}{2} \sin \theta \right] \\ &\quad + \frac{Kl}{GA} \sum_{i=1}^{n-1} \left[\left(2i-1 \right) P + \frac{1}{2} mg \right] \left(i - \frac{1}{2} \right) \cos^2 \theta \end{aligned}$$

$$\begin{aligned}
&= \frac{l^3}{EI} \left[\left(\frac{2}{9}n^3 - \frac{2}{3}n^2 + \frac{11}{18}n - \frac{1}{6} \right) P + \left(\frac{1}{6}n^4 - \frac{2}{3}n^3 + \frac{43}{48}n^2 - \frac{11}{24}n + \frac{1}{16} \right) mg \right] \cos^2 \theta \\
&+ \frac{l}{EA} \left\{ \left[\left(\frac{2}{3}n^3 - 2n^2 + \frac{7}{3}n - 1 \right) \cot^2 \theta \cos^2 \theta + (n-1) \cos^2 \theta + \left(\frac{1}{2}n - \frac{1}{2} \right) \sin^2 \theta \right] P \right. \\
&\quad \left. + \left[\left(\frac{1}{2}n^4 - 2n^3 + \frac{7}{2}n^2 - 3n + 1 \right) \cot^2 \theta \cos^2 \theta + \left(\frac{5}{4}n^2 - \frac{5}{2}n + \frac{5}{4} \right) \cos^2 \theta + \left(\frac{1}{2}n^2 - n + \frac{1}{2} \right) \sin^2 \theta \right] mg \right\} \\
&+ \frac{Kl}{GA} \left[\left(\frac{2}{3}n^3 - 2n^2 + \frac{11}{6}n - \frac{1}{2} \right) P + \left(\frac{1}{4}n^2 - \frac{1}{2}n + \frac{1}{4} \right) mg \right] \cos^2 \theta
\end{aligned} \tag{28a}$$

$$\begin{aligned}
\Delta h_2 &= \int_{AB} \left(\frac{N_x}{EA} \frac{\partial N_x}{\partial P} + \frac{M_z}{EI} \frac{\partial M_z}{\partial P} + \frac{KQ_y}{GA} \frac{\partial Q_y}{\partial P} \right) dx \\
&= \frac{1}{EA} \int_0^l \left[\left(n + \frac{1}{2} \right) P + (n^2 + n) mg - \frac{mg}{2l} x \right] \left[\left(n + \frac{1}{2} \right) \sin^2 \theta dx \right. \\
&\quad + \frac{1}{EA} \int_l^{2l} \left\{ - \left[(n-1)P + (n-1)^2 mg \right] \cot \theta \cos \theta + \left(\frac{1}{2}P + nmg - \frac{mg}{2l} x \right) \sin \theta \right\} \left[- (n-1) \cot \theta \cos \theta + \frac{1}{2} \sin \theta \right] dx \\
&\quad + \frac{1}{EI} \int_0^l \left\{ 2(nP + n^2 mg) l - \left[\left(n + \frac{1}{2} \right) P + (n^2 + n) mg \right] x + \frac{mg}{4l} x^2 \right\} \left[2nl - \left(n + \frac{1}{2} \right) x \right] \cos^2 \theta dx \\
&\quad + \frac{1}{EI} \int_l^{2l} \left[\left(n - \frac{1}{2} \right) P + \left(n^2 - n + \frac{1}{2} - \frac{x}{4l} \right) mg \right] \left(n - \frac{1}{2} \right) (2l - x)^2 \cos^2 \theta dx \\
&\quad + \frac{K}{GA} \int_0^l \left[\left(n + \frac{1}{2} \right) P + (n^2 + n) mg - \frac{mg}{2l} x \right] \left(n + \frac{1}{2} \right) \cos^2 \theta dx \\
&\quad + \frac{K}{GA} \int_l^{2l} \left[\left(n - \frac{1}{2} \right) P + (n^2 - n) mg + \frac{mg}{2l} (2l - x) \right] \left(n - \frac{1}{2} \right) \cos^2 \theta dx \\
&= \frac{l}{EA} \left[\left(n + \frac{1}{2} \right) P + \left(n^2 + n - \frac{1}{4} \right) mg \right] \left(n + \frac{1}{2} \right) \sin^2 \theta \\
&\quad + \frac{l}{EA} \left\{ \left[(1-n) \cot \theta \cos \theta + \frac{1}{2} \sin \theta \right] P + \left[\left(n - \frac{3}{4} \right) \sin \theta - (n-1)^2 \cot \theta \cos \theta \right] mg \right\} \left[(1-n) \cot \theta \cos \theta + \frac{1}{2} \sin \theta \right] \\
&\quad + \frac{l^3}{EI} \left[\left(\frac{7}{3}n^2 - \frac{2}{3}n + \frac{1}{12} \right) P + \left(\frac{7}{3}n^3 - n^2 + \frac{13}{48}n - \frac{1}{32} \right) mg \right] \cos^2 \theta \\
&\quad + \frac{l^3}{EI} \left[\left(\frac{1}{3}n^2 - \frac{1}{3}n + \frac{1}{12} \right) P + \left(\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{11}{48}n - \frac{1}{32} \right) mg \right] \cos^2 \theta \\
&\quad + \frac{Kl}{GA} \left[\left(2n^2 + \frac{1}{2} \right) P + \left(2n^3 + n - \frac{1}{4} \right) mg \right] \cos^2 \theta \\
&= \frac{l}{EA} \left\{ \left[\left(n^2 - 2n + 1 \right) \cot^2 \theta \cos^2 \theta + (1-n) \cos^2 \theta + \left(n^2 + n + \frac{1}{2} \right) \sin^2 \theta \right] P \right. \\
&\quad \left. + \left[\left(n^3 - 3n^2 + 3n - 1 \right) \cot^2 \theta \cos^2 \theta + \left(-\frac{3}{2}n^2 + \frac{11}{4}n - \frac{5}{4} \right) \cos^2 \theta + \left(n^3 + \frac{3}{2}n^2 + \frac{3}{4}n - \frac{1}{2} \right) \sin^2 \theta \right] mg \right\} \\
&\quad + \frac{l^3}{EI} \left[\left(\frac{8}{3}n^2 - n + \frac{1}{6} \right) P + \left(\frac{8}{3}n^3 - \frac{3}{2}n^2 + \frac{1}{2}n - \frac{1}{16} \right) mg \right] \cos^2 \theta \\
&\quad + \frac{Kl}{GA} \left[\left(2n^2 + \frac{1}{2} \right) P + \left(2n^3 + n - \frac{1}{4} \right) mg \right] \cos^2 \theta
\end{aligned} \tag{28b}$$

$$\begin{aligned}
\Delta h_3 &= \int_{CD} \left(\frac{N_x}{EA} \frac{\partial N_x}{\partial P} + \frac{M_z}{EI} \frac{\partial M_z}{\partial P} + \frac{KQ_y}{GA} \frac{\partial Q_y}{\partial P} \right) dx \\
&= \frac{1}{EA} \int_0^l \left\{ \left[\frac{1}{2} P + (n-1)mg \right] \sin \theta - (n-1) \left[P + (n-1)mg \right] \cot \theta \cos \theta + \frac{mg}{2l} x \sin \theta \right\} \left[\frac{1}{2} \sin \theta - (n-1) \cot \theta \cos \theta \right] dx \\
&\quad + \frac{1}{EA} \int_l^{2l} \left[\left(n - \frac{1}{2} \right) P + (n^2 - n + 1)mg - \frac{mg}{2l} x \right] \left(n - \frac{1}{2} \right) \sin^2 \theta dx \\
&\quad + \frac{1}{EI} \int_0^l \left[\left(n - \frac{1}{2} \right) P + (n^2 - n)mg + \frac{mg}{4l} x \right] \left(n - \frac{1}{2} \right) x^2 \cos^2 \theta dx \\
&\quad + \frac{1}{EI} \int_l^{2l} \left[\left(n - \frac{1}{2} \right) P + \left(n^2 - n + \frac{1}{2} - \frac{x}{4l} \right) mg \right] \left(n - \frac{1}{2} \right) (2l - x)^2 \cos^2 \theta dx \\
&\quad + \frac{K}{GA} \int_0^l \left\{ \left[\left(n - \frac{1}{2} \right) P + (n^2 - n)mg \right] + \frac{mg}{2l} x \right\} \left(n - \frac{1}{2} \right) \cos^2 \theta dx \\
&\quad + \frac{K}{GA} \int_l^{2l} \left[\left(n - \frac{1}{2} \right) P + (n^2 - n + 1)mg - \frac{mg}{2l} x \right] \left(n - \frac{1}{2} \right) \cos^2 \theta dx \\
&= \frac{l}{EA} \left\{ \left[\frac{1}{2} \sin \theta - (n-1) \cot \theta \cos \theta \right] P + \left[\left(n - \frac{3}{4} \right) \sin \theta - (n-1)^2 \cot \theta \cos \theta \right] mg \right\} \left[\frac{1}{2} \sin \theta - (n-1) \cot \theta \cos \theta \right] \\
&\quad + \frac{l}{EA} \left[\left(n - \frac{1}{2} \right) P + \left(n^2 - n + \frac{1}{4} \right) mg \right] \left(n - \frac{1}{2} \right) \sin^2 \theta \\
&\quad + \frac{l^3}{EI} \left[\left(\frac{2}{3} n - \frac{1}{3} \right) P + \left(\frac{2}{3} n^2 - \frac{2}{3} n + \frac{1}{8} \right) mg \right] \left(n - \frac{1}{2} \right) \cos^2 \theta \\
&\quad + \frac{Kl}{GA} \left[(2n-1)P + \left(2n^2 - 2n + \frac{1}{2} \right) mg \right] \left(n - \frac{1}{2} \right) \cos^2 \theta \\
&= \frac{l}{EA} \left\{ \left[\left(n^2 - 2n + 1 \right) \cot^2 \theta \cos^2 \theta + (1-n) \cos^2 \theta + \left(n^2 - n + \frac{1}{2} \right) \sin^2 \theta \right] P \right. \\
&\quad \left. + \left[\left(n^3 - 3n^2 + 3n - 1 \right) \cot^2 \theta \cos^2 \theta + \left(-\frac{3}{2} n^2 + \frac{11}{4} n - \frac{5}{4} \right) \cos^2 \theta + \left(n^3 - \frac{3}{2} n^2 + \frac{5}{4} n - \frac{1}{2} \right) \sin^2 \theta \right] mg \right\} \quad (28c) \\
&\quad + \frac{l^3}{EI} \left[\left(\frac{2}{3} n^2 - \frac{2}{3} n + \frac{1}{6} \right) P + \left(\frac{2}{3} n^3 - n^2 + \frac{11}{24} n - \frac{1}{16} \right) mg \right] \cos^2 \theta \\
&\quad + \frac{Kl}{GA} \left[\left(2n^2 - 2n + \frac{1}{2} \right) P + \left(2n^3 - 3n^2 + \frac{3}{2} n - \frac{1}{4} \right) mg \right] \cos^2 \theta
\end{aligned}$$

Substituting equations (28a), (28b) and (28c) into equation (27) yields

$$\begin{aligned} \Delta h &= \Delta h_1 + \Delta h_2 + \Delta h_3 \\ &= \frac{l^3}{EI} \left[\left(\frac{2}{9}n^3 + \frac{8}{3}n^2 - \frac{19}{18}n + \frac{1}{6} \right) P + \left(\frac{1}{6}n^4 + \frac{8}{3}n^3 - \frac{77}{48}n^2 + \frac{1}{2}n - \frac{1}{16} \right) mg \right] \cos^2 \theta \\ &\quad + \frac{l}{EA} \left\{ \left[\left(\frac{2}{3}n^3 - \frac{5}{3}n^2 + 1 \right) \cot^2 \theta \cos^2 \theta - (n-1) \cos^2 \theta + \left(2n^2 + \frac{1}{2}n + \frac{1}{2} \right) \sin^2 \theta \right] P \right. \\ &\quad \left. + \left[\left(\frac{1}{2}n^4 - \frac{5}{2}n^3 + 3n - 1 \right) \cot^2 \theta \cos^2 \theta + \left(-\frac{7}{4}n^2 + 3n - \frac{5}{4} \right) \cos^2 \theta + \left(2n^3 + \frac{1}{2}n^2 + n - \frac{1}{2} \right) \sin^2 \theta \right] mg \right\} \\ &\quad + \frac{6l}{5GA} \left[\left(\frac{2}{3}n^3 + 2n^2 - \frac{1}{6}n + \frac{1}{2} \right) P + \left(4n^3 - \frac{11}{4}n^2 + 2n - \frac{1}{4} \right) mg \right] \cos^2 \theta \end{aligned} \tag{29}$$

The equivalent elastic coefficient of the structure shown in Fig. (9) is

$$\begin{aligned} k_1(\theta) &= \frac{P}{\Delta h} \\ &= \frac{1}{\left\{ \frac{l^3}{EI} \left[\left(\frac{2}{9}n^3 + \frac{8}{3}n^2 - \frac{19}{18}n + \frac{1}{6} \right) + \left(\frac{1}{6}n^4 + \frac{8}{3}n^3 - \frac{77}{48}n^2 + \frac{1}{2}n - \frac{1}{16} \right) \frac{mg}{P} \right] \cos^2 \theta \right. \\ &\quad \left. + \frac{l}{EA} \left[\left(\frac{2}{3}n^3 - \frac{5}{3}n^2 + 1 \right) \cot^2 \theta \cos^2 \theta - (n-1) \cos^2 \theta + \left(2n^2 + \frac{1}{2}n + \frac{1}{2} \right) \sin^2 \theta \right] \right. \\ &\quad \left. + \left[\left(\frac{1}{2}n^4 - \frac{5}{2}n^3 + 3n - 1 \right) \cot^2 \theta \cos^2 \theta + \left(-\frac{7}{4}n^2 + 3n - \frac{5}{4} \right) \cos^2 \theta + \left(2n^3 + \frac{1}{2}n^2 + n - \frac{1}{2} \right) \sin^2 \theta \right] \frac{mg}{P} \right. \\ &\quad \left. + \frac{6l}{5GA} \left[\left(\frac{2}{3}n^3 + 2n^2 - \frac{1}{6}n + \frac{1}{2} \right) + \left(4n^3 - \frac{11}{4}n^2 + 2n - \frac{1}{4} \right) \frac{mg}{P} \right] \cos^2 \theta \right\}} \end{aligned} \tag{30}$$

In most engineering cases, there is the following approximation:

$$\frac{mg}{P} = \frac{2}{n} \tag{31}$$

Substituting equation (31) into equation (30) obtains

$$\begin{aligned} k_1(\theta) &= \frac{1}{\left\{ \frac{l^3}{EI} \left[\frac{5}{9}n^3 + 8n^2 - \frac{307}{72}n + \frac{7}{6} - \frac{1}{8n} \right] \cos^2 \theta \right. \\ &\quad \left. + \frac{l}{EA} \left[\left(\frac{5}{3}n^3 - \frac{20}{3}n^2 + 7 - \frac{2}{n} \right) \cot^2 \theta \cos^2 \theta - \left(\frac{9}{2}n - 7 + \frac{5}{2n} \right) \cos^2 \theta + \left(6n^2 + \frac{3}{2}n + \frac{5}{2} - \frac{1}{n} \right) \sin^2 \theta \right] \right. \\ &\quad \left. + \frac{6l}{5GA} \left(\frac{2}{3}n^3 + 10n^2 - \frac{17}{3}n + \frac{9}{2} - \frac{1}{2n} \right) \cos^2 \theta \right\}} \end{aligned} \tag{32}$$

Considering the structure of the foldable tower shown in Fig. (5) or Fig. (6), one can find that there are 3 structures of that in Fig. (9). Therefore, the ultimate equivalent stiffness of

the foldable tower will be immediately obtained from equation (32):

$$k(\theta) = \frac{3}{\left[\frac{l^3}{EI} \left[\frac{5}{9}n^3 + 8n^2 - \frac{307}{72}n + \frac{7}{6} - \frac{1}{8n} \right] \cos^2 \theta + \frac{l}{EA} \left[\left(\frac{5}{3}n^3 - \frac{20}{3}n^2 + 7 - \frac{2}{n} \right) \cot^2 \theta \cos^2 \theta - \left(\frac{9}{2}n - 7 + \frac{5}{2n} \right) \cos^2 \theta + \left(6n^2 + \frac{3}{2}n + \frac{5}{2} - \frac{1}{n} \right) \sin^2 \theta \right] + \frac{6l}{5GA} \left(\frac{2}{3}n^3 + 10n^2 - \frac{17}{3}n + \frac{9}{2} - \frac{1}{2n} \right) \cos^2 \theta \right]} \quad (33)$$

Because

$$k'(\theta) = \frac{3 \left[Q(1 + \sin^2 \theta) 2 \cot^3 \theta + R \sin 2\theta \right]}{S^2} \quad (34)$$

where

$$S = \frac{l^3}{EI} \left[\frac{5}{9}n^3 + 8n^2 - \frac{307}{72}n + \frac{7}{6} - \frac{1}{8n} \right] \cos^2 \theta + \frac{l}{EA} \left[\left(\frac{5}{3}n^3 - \frac{20}{3}n^2 + 7 - \frac{2}{n} \right) \cot^2 \theta \cos^2 \theta - \left(\frac{9}{2}n - 7 + \frac{5}{2n} \right) \cos^2 \theta + \left(6n^2 + \frac{3}{2}n + \frac{5}{2} - \frac{1}{n} \right) \sin^2 \theta \right] Q = \frac{2l}{EA} \left(\frac{5}{3}n^3 - \frac{20}{3}n^2 + 7 - \frac{2}{n} \right) + \frac{6l}{5GA} \left(\frac{2}{3}n^3 + 10n^2 - \frac{17}{3}n + \frac{9}{2} - \frac{1}{2n} \right) \cos^2 \theta$$

$$R = \frac{l^3}{EI} \left(\frac{5}{9}n^3 + 8n^2 - \frac{307}{72}n + \frac{7}{6} - \frac{1}{8n} \right) - \frac{l}{EA} \left(6n^2 + 6n - \frac{9}{2} + \frac{3}{2n} \right) + \frac{6l}{5GA} \left(\frac{2}{3}n^3 + 10n^2 - \frac{17}{3}n + \frac{9}{2} - \frac{1}{2n} \right)$$

Obviously, the relationships of $\begin{cases} S^2 > 0 \\ Q > 0 \\ R > 0 \end{cases}$ always hold, and

$$k'(\theta) = \frac{3 \left[Q(1 + \sin^2 \theta) 2 \cot^3 \theta + R \sin 2\theta \right]}{S^2} > 0 \quad \text{for}$$

$0 \leq \theta < \frac{\pi}{2}$. Therefore, function $k(\theta)$ increases with the increasing of θ . Letting $k'(\theta) = 0$, one can find that $\theta = \frac{\pi}{2}$.

So

$$\left[k(\theta) \right]_{\max} = k\left(\frac{\pi}{2}\right) = \frac{3EA}{l \left(6n^2 + \frac{3}{2}n + \frac{5}{2} - \frac{1}{n} \right)} \quad (35)$$

Suppose there are four sets of parameters of the foldable tower shown in Table 1. Next one can examine the effects of the parameters on the stiffness of the foldable tower.

Table 1. Primary Parameters of the Foldable Tower

Set Series	$E(Pa)$	$G(Pa)$	$I(m^4)$	$A(m^2)$	$l(m)$	n
a	2.06×10^{11}	7.9×10^{10}	8.33×10^{-6}	10^{-2}	1	20
b	2.06×10^{11}	7.9×10^{10}	8.33×10^{-6}	10^{-2}	1	10
c	2.06×10^{11}	7.9×10^{10}	8.33×10^{-6}	10^{-2}	1	6
d	2.06×10^{11}	7.9×10^{10}	5.21×10^{-7}	2.5×10^{-3}	1	6

With the parameters in Table 1 and equation (32), one can gain the curves of the stiffness of the foldable tower with respect to θ . Fig. (17) shows the different effects of the parameters of the foldable tower.

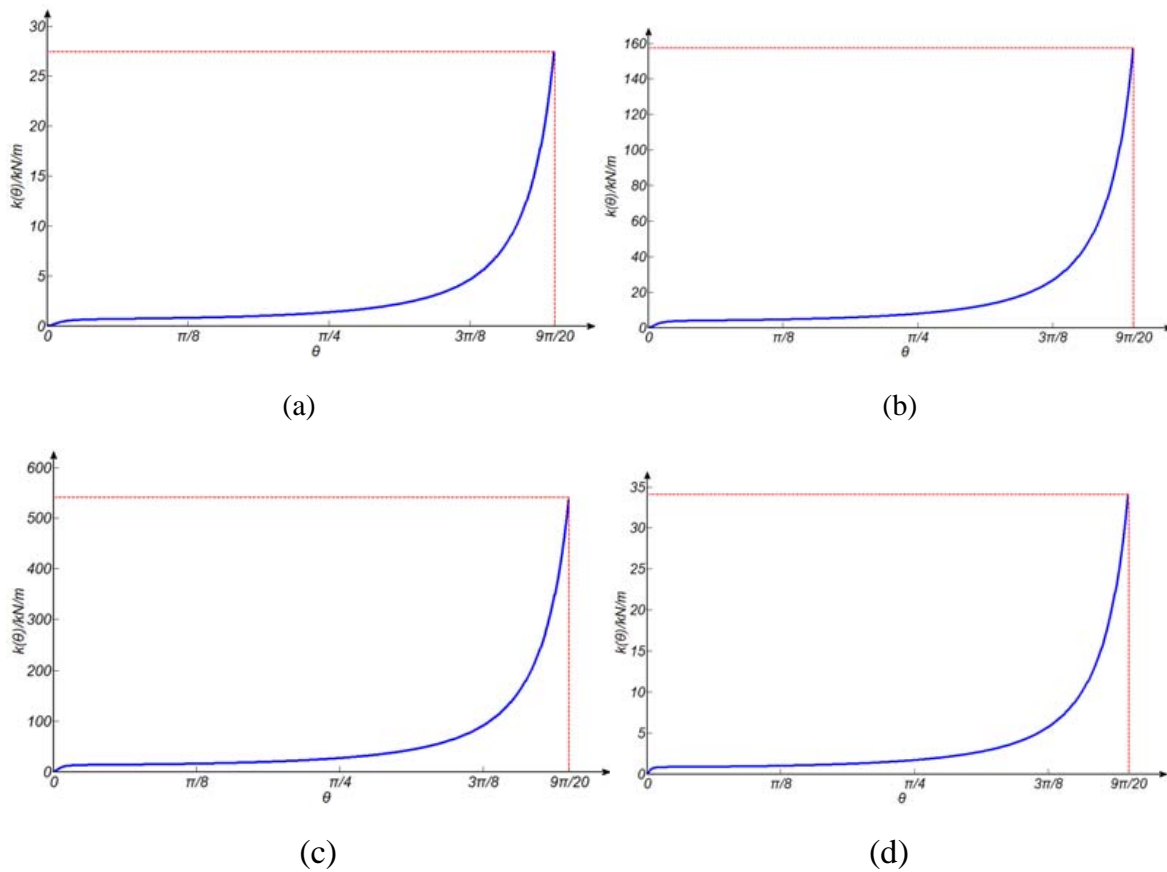


Fig. (17). The stiffness of the foldable tower of different parameters.

Fig. (17) shows that the stiffness of the foldable tower changes with the change of parameters and the deployable angle θ . The result that the stiffness of the foldable tower increases much faster than the increasing of the deployable angle θ indicates the most invaluable possibility in the applications of wind turbine.

CONCLUSIONS

This paper focused on the foldable tower which should be used as the support of wind turbine and investigated the method to establish the equivalent stiffness expression of the whole structure. The advantage of the flexibility to fold and unfold can be utilized for protecting the wind turbine when destructive weather occurs. The stiffness of the foldable tower is especially high when it is deployed completely. This is a particularly suitable structure as the strut of wind turbine and it should have a fascinating future.

CONFLICT OF INTEREST

None declared.

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REFERENCES

- [1] T. Burton, D. Sharp, N. Jenkins, and E. Bossanyi, *Wind Energy*, Handbook. England: John Wiley and Sons, 2001.
- [2] I. Lavassas, G. Nikolaidis, P. Zervas, E. Efthimiou, I. N. Doudoumis, and C. C. Baniotopoulos, "Analysis and design of the prototype of a steel 1-MW wind turbine tower", *Eng. Struct.*, 25, 1097-1106, 2003.
- [3] C. G. Justus, and A. Mikhail, "Height variation of wind speed and wind distributions statistics", *Geophys. Res. Lett.*, 3, 261-264, 1976.
- [4] A. G. Fuhrlander, 2009, FL 2500 Brochure, German.
- [5] L. Fingersh, M. Hand, and A. Laxson, "Wind turbine design cost and scaling model", Technical Report, NREL/TP-500-40566. 2006.
- [6] C. J. Gantes, and E. Konitopoulou, "Geometric design of arbitrarily curved bi-stable deployable arches with discrete joint size", *Int. J. Solids Struct.*, 41, 5517-5540, 2004.
- [7] E. L. Christiansen, J. H. Kerr, H. M. Fuentes, and W. C. Schneider, "Flexible and deployable meteoroid/debris shielding for spacecraft", *Int. J. Impact Eng.*, 23, 125-136, 1999.
- [8] J. F. V. Vincent, "Deployable structures in nature: Potential for biomimicking", *Proc. Inst. Mech. Eng.*, 214, 1-10, 2000.
- [9] X. Z. Wei, Y. A. Yao, Y. B. Tian, and R. Fang, "A new method of creating expandable structure for spatial objects", *Proc. Inst. Mech. Eng. C-J. Mech. Eng. Sci.*, 220, 1813-1818, 2006.
- [10] W. W. Gan, and S. Pellegrino, "Numerical approach to the kinematic analysis of deployable structures forming a closed loop",

Proc. Inst. Mech. Eng., Part C: *Proc. Inst. Mech. Eng. C- J. Mech. Eng. Sci.*, 220, 1045-1056, 2006.

[11]

A. Bedford, and M. Kenneth, *Mech. Mater.*, 2000. NJ: Prentice Hall.

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