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Reliability Analysis on Gear Contact Fatigue Strength Considering the Effect of Tolerance

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Abstract: Tolerance allocation influences product performance especially for miniature precise assembly device. The purpose of this paper is to explore an approach to optimize manufacturing tolerances by combining the reliability of product performance indicators under actual working condition. The paper describes the principle and ways of tolerance handling in the finite element control equations for the displacement and stress, and then introduces the implementation of Monte-Carlo Finite Element Analysis method. We illustrate this method on a gear-tooth of port-cleaning-equipment gear pump and carry out the reliability analysis on gear contact fatigue strength considering the effect of manufacturing tolerance, and a sensitivity analysis is also performed to identify the key tolerances and improve them in order to attain the expected reliability.

Keywords: Finite element analysis, gear contact fatigue strength, manufacturing tolerance allocation, port-cleaning-equipment, reliability analysis.

1. INTRODUCTION

Tolerance design can impact product quality as well as manufacturing costs. To evaluate the impact of tolerance on product quality, much research has been conducted over the past years: Bruyere and his partners proposed an approach to analyze the tolerances based on Tooth Contact Analysis and Monte Carlo simulation [1]; Govindarajalu and Karuppan studied the optimal tolerance design for an assembly simultaneously considering manufacturing cost, quality loss and deformation due to inertia effects [2]. Pierre L and his partners described how thermomechanical strains are integrated into tolerance analysis and how they influence the controlling clearance between the tips of the high pressure turbine blades and the stator [3, 4]. Xu Dong performed performance analysis of the reheat-stop-valve mechanism under dimensional tolerance, misalignment and thermal impact [5, 6]; Manarvi and Juster used finite element method to simulate the effect of tolerance on part deformation [7]; Serban and his partner studied the effect of manufacturing tolerances on the low-noise amplifier performance by means of sensitivity analyses [8]. Zhang established the finite element model of gear of microminiaturize clock mechanism to find the main tolerance that impact the performance of the mechanism [9].

However, present studies of tolerance analysis are more concerned with assembly quality and the motion characteristics of specific mechanism, the reliability of miniature precise assembly device simultaneously considering working condition and manufacturing tolerance is seldom studied. Actually, both manufacturing tolerance and actual working condition usually exercises a combined influence on product working performance simultaneously, even the part deformation caused by actual working condition also makes the original tolerance value meaningless. Therefore, the isolated calculation of them in traditional design procedure should be abandoned, to ensure product reliability in harsh operating conditions, the preferred method is to take an overall consideration the impact of manufacturing tolerances, material characteristics and loading conditions on product reliability.

In this paper, the manufacturing tolerance as a random variable was considered during the Finite Element Analysis (FEA) on gear contact fatigue strength of gear pump, firstly the treatment method of tolerance calculation in the finite element control equations for the displacement and stress was presented; Secondly the calculation method of reliability and sensitivity using Monte Carlo-based stochastic Finite Element Methods(FEM) was put forward to enhance the reliability of gear contact fatigue strength through adjusting tolerance value.

2. CALCULATING METHOD OF TOLERANCE IN FINITE ELEMENT CONTROL EQUATIONS

In mechanical engineering, engineer needs predications of stiffness and elastic deformation of components. The deformation results from forces and moments under operating conditions as well as from residual stress of press fits as a superposition of both [10]. Displacements in the contact area shown in Fig. (**1a**) have their cause in tolerance specifications and other factors in Fig. (**1b**). FEA model is always based on various assumptions and idealized conditions despite the fact that material properties, manufacturing tolerances, boundary conditions and working conditions are uncertain, which results in a greater difference between the calculation results and the actual condition [11].

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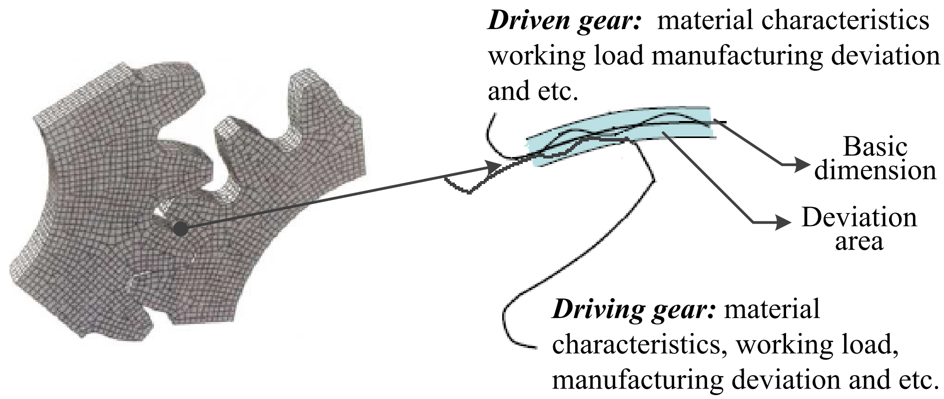

(a) Gear model meshing **(b) Influence factors in gear contact area**

Fig. (1). Multi-factor influence on finite element analysis.

Therefore, the influence of irregularities of manufactured part surfaces to the residual stress of fitting parts should be taken into the FE calculation method.

According to the basic theory of Stochastic Finite Element [12, 13], the displacement and stress finite element equations can be derived as follows.

According to the principle of virtual displacements and stress-strain relations, the nodal equilibrium equation can be expressed by displacement:

$$\left[\mathbf{K}\right]\left[\mathbf{U}\right]=\left[\mathbf{F}\right] \tag{1}$$

where K is the known stiffness matrix of the system, U is the vector of displacements or rotations and F is the vector of external forces or moments.

The system of equations can be transformed in the following way:

$$\left[\mathbf{U}\right]=\left[K\right]^{-1}=\left[F\right] \tag{2}$$

In terms of stress- strength interference theory, stress - strain relations can be expressed in the following equation:

$$[\boldsymbol{\sigma}]=[D][\varepsilon]=[\mathbf{D}][\mathbf{B}][\mathbf{U}] \tag{3}$$

where σ is the vector of stress, D is material elastic matrix, ε is strain vector.

With the aid of (2) and (3), σ and ε can be calculated. At the assumption of K, F and D are the function of random variable $X=\left(X_1,X_2,\cdots,X_{\mathrm{m}}\right)$, the number of variable is m, then U and σ can be expressed with X as follows:

$$U\left(X\right)=U\left(X_1,X_2,\cdots,X_m\right) \tag{4}$$

$$\sigma\left(X\right)=\sigma\left(X_1,X_2,\cdots,X_m\right) \tag{5}$$

where random variables X include various loads, part geometric parameters and physical parameters (Poisson's ratio, elastic modulus E, thermal expansion coefficient, etc.).

Based on Monte-Carlo method, a groups of random variables that were randomly selected from m mutual independence variables ($X_1,X_2,\cdots,X_{\mathrm{m}}$) are listed as: $X_1=\left(X_{11},X_{21},\cdots,X_{\mathrm{m}1}\right)$, then U and σ can be calculated in the following way after N groups of variables were selected:

$$\begin{cases} U\left(X_1\right)=U\left(X_{11},X_{21},\cdots,X_{m1}\right) \\ U\left(X_2\right)=U\left(X_{12},X_{22},\cdots,X_{m2}\right) \\ \qquad\cdots \\ U\left(X_n\right)=U\left(X_{1n},X_{2n},\cdots,X_{mn}\right) \end{cases} \tag{6}$$

$$\begin{cases} \sigma\left(X_1\right)=\sigma\left(X_{11},X_{21},\cdots,X_{m1}\right) \\ \sigma\left(X_2\right)=\sigma\left(X_{12},X_{22},\cdots,X_{m2}\right) \\ \qquad\cdots \\ \sigma\left(X_n\right)=\sigma\left(X_{1n},X_{2n},\cdots,X_{mn}\right) \end{cases} \tag{7}$$

The statistical characteristic of $\left\{U\right\}$ and $\left\{\sigma\right\}$ can be obtained from the above statistics, when product performance is expressed as a function of U or σ, the reliability of product performance can be calculated based on the function of U or σ.

However, when geometric dimension of parts are regarded as random variable, it is necessary to establish the constraint relation between dimensions and tolerances, the methods used were as follows:

First of all, parts dimension allowed values can be expressed as $x\pm\Delta x$ where x is basic dimension, $\pm\Delta x$ is the upper and lower tolerance. The dimensional variables generally obey the Gaussian distribution. According to "$3\sigma_{\mathrm{x}}$ principle" in statistics, parts dimension mean μ and standard deviation σ_x can be expressed in the following way [14]:

$$\mu=x\ ,\ \sigma_x\approx\frac{\left(\bar{x}+\Delta x\right)-\left(\bar{x}-\Delta x\right)}{6}=\frac{\Delta x}{3} \tag{8}$$

Secondly, when parts dimension ranges from $x_{\max}$ to $x_{\min}$, then μ and σ_x can be expressed in the following way:

$$\mu = \frac{x_{\min} + x_{\max}}{2}, \sigma_x = \frac{x_{\max} - x_{\min}}{6} \quad (9)$$

In addition, " $3\sigma_x$ principle" is also applied to other random variables, i.e. the standard deviation (σ_x) can be determined by the scope of parameters.

The above calculation and analysis process can be implemented through the combination of FEM and Monte-Carlo method, specific steps are shown in Fig. (**2**):

Step 1: the three-dimensional CAD model is imported into ANSYS environment for FEM analysis.

Step 2: carrying out parametric modeling and finite element discretization on CAD model;

Step 3: setting random variables, variable distribution functions and the target function in ANSYS application environment;

Step 4: the input variables were created in a random order with a known probability distribution based on the Monte Carlo method, then the value of target function can be calculated.

Step 5: we can get the statistical characteristic after computing all samples, based on which the effect of key variables on performance indicators (i.e. the target function) can be calculated, such as product reliability, parameters sensitivity and so on. In this paper we focus on gear contact fatigue strength especially considering the effect of tolerance.

3. RELIABILITY ANALYSIS METHOD CONSIDE-RING THE EFFECT OF TOLERANCE

The basic principle of reliability analysis is to repeatedly substitute the random variables into the finite element control equations and to solve the defined target function, then get the distribution characteristics of the target function result or directly calculate the failure probability, the opposite is the reliability of function index. Actually, ANSYS/PDS module will help complete the reliability analysis of gear strength effectively, the details are as follows:

Setp 1: To establish the finite element equations and the formula of intensity, and determine the number (N) of samples;

Step 2: Defined random variables and their probability distribution type;

Step 3: Generating a group of random numbers;

Step 4: Substitute the random numbers into the finite element control equations to calculate the stress S, and Substitute the random numbers into the formula of intensity to calculate the strength R;

Step 5: Through comparing the magnitude of stress and strength, the product performance is considered as reliable if $R - S > 0$, it counts as 1 and $k = k + 1$; otherwise the product performance is considered as component failure if $R - S \leq 0$, it counts as 0.

Step 6: repeat the above steps 3 and 5 until the total number (N) of samples has been finished.

Step 7: the reliability of target index as can be obtained according to f N time sample function calculation, the reliability is expressed as $R_S = P[(R - S) > 0]$.

The above procedure can be generalized to calculate the reliability of yield strength, the frequency reliability and etc. Based on both sensitivity analysis, the key influencing factors of product reliability can be determined and then optimized.

4. CASE STUDY: PROBLEM DESCRIPTION, SOLUTION AND REALIZED STRATEGY

Micro gear pump is commonly used as the driving component of port cleaning instrument, the changing of actual working condition and the existing manufacturing deviation cause wearing condition of gear tooth flank or loss of gear transmission accuracy in the process of gears meshing, therefore it is important to carry out contact fatigue strength analysis and verification, which is based on the comparison of theoretical value of Hertz contact theory with the calculated value under actual working condition as well as the effect of tolerance.

Nylon MC901 is the material of choice due to its light weight and corrosion resistance properties. Material properties of pump gears are listed in Table **1**. The parameters of gear pair are listed in Table **2**. In addition, gear pump normal pressure is 2.5MPa, Maximum output torque is 1000N.mm.

Table 1. Properties of pump gears.

Material Properties	Unit	Value
Density (23 °C)	kg/m^3	$1.15 \sim 1.17 \times 10^3$
Young's modulus	MPa	3.2×10^3
Poisson's ratio	—	0.4

4.1. Selecting Design Parameters of Gear Pump

The parameters that affect strength reliability are randomized, which mainly include tooth breadth(B), gear

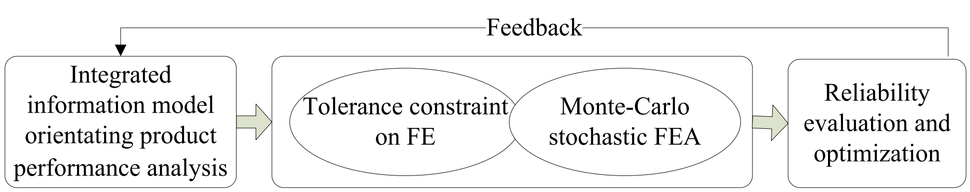

Fig. (2). Product performance reliability evaluation method.

Table 2. Basic parameters of gear pairs.

Module M (mm)	Pressure Angle (°)	Number of Active Tooth z_1	Number of Driven Gear z_2	Tooth Breadth B (mm)
1.5	20	10	10	16

Table 3. Random variables and their distribution.

Random Variables	Distribution	Center Value	Standard Deviation	Tolerance
Tooth breadth (B)/mm	GAUS	15.865	0.045	$16^{0}_{-0.27}$
Gear center distance (AW)/mm	GAUS	15	0.03	15 ± 0.09
Gear pitch diameter (D)/mm	GAUS	14.963	0.007	$\varphi15^{-0.016}_{-0.059}(\varphi15f9)$
Mounting holes diameter (DW)/mm	GAUS	7.976	0.004	$\varphi8^{-0.013}_{-0.035}(\varphi8f8)$
Young modulus (E)/Mpa	GAUS	3.2×10^{3}	128	
ρ/tonne/mm^3	GAUS	1.16×10^{-9}	4.64×10^{-11}	
λ	GAUS	0.4	0.016	

pitch diameter(D), gear shaft diameter (D_W), gear center distance (A_W), Young modulus(E), density, Poisson's ratio. They are listed as Table **3**.

4.2 Hertz Contact Theory for Solving Contact Stress

First of all, the gear contact stress σ_H is calculated based on Hertz contact theory [15]:

$$\sigma_H = \sqrt{\frac{F\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)}{\pi\cdot \mathrm{b}\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}} \tag{10}$$

where σ_H is Hertz Stress, F is normal force, E_1 and E_2 are Young modulus of contacting body, μ_1 and μ_2 are Poisson's ratios, b is contact length, ρ_1 and ρ_2 are the curvature radius at the node of the tooth profile. In this example, $E_1 = E_2 = 3200MPa$, $\mu_1=\mu_2=0.4$, $b=16$mm, $F=48.5\,N$, $\rho_1=\rho_1={}^{d}\!/_{2}=7.5$mm , the above values are substituted into (10) results in σ_H , that is $\sigma_H = 22.14MPa$.

4.3. Contact Stress Analysis of Gear Based on Monte Carlo-FEM

Building the Finite Element Model shown in Fig. (**3**): In order to reduce the computational workload for FEA, only part of parametric model with ignored fillet and other small features has been built based on ANSYS/APDL. The parametric model grid is based on MESH 200 and using ANSYS eight node SOLID 45 isoparametric elements.

Contact stress analysis of gear: in this example, specific treatment methods are as follows: the profile surface of driven gear tooth is supposed as the target surface and the profile surface of driving gear tooth is supposed as the contact surface; The contact stiffness factor FKN and the maximum allowable penetration tolerance FIIDN are set at 1.0 and 0.1 respectively. The applied load on each node of driving gear mounting holes surface in the direction of tangential force is as follows:

$$F_t = \frac{T}{r_w\times n} = \frac{1000}{4\times726} = 0.344N$$

where T is the torque of the driving gear, r_w is the radius of the inner gear ring, n is the node number of inner gear ring.

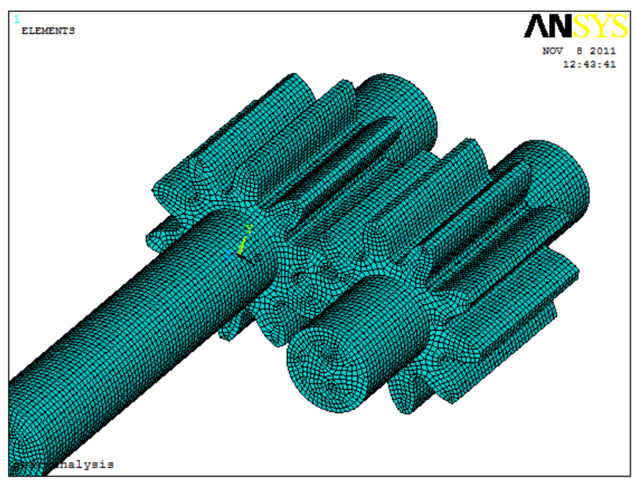

Fig. (3). FEA of gear mesh.

The calculated result shows that the maximum contact stress in meshing process is 16.77MPa, which occur at the gear tooth root shown in Fig. (**4**).

4.4. Reliability-Based Tolerance Optimization for the Key Dimensions of Gear Pump

Based on Latin hypercube sampling in ANSYS/PSD module, the reliability of contact strength under the confidence level of 95% is 72.4%, which shows that the reliability is low and needs to be further optimized.

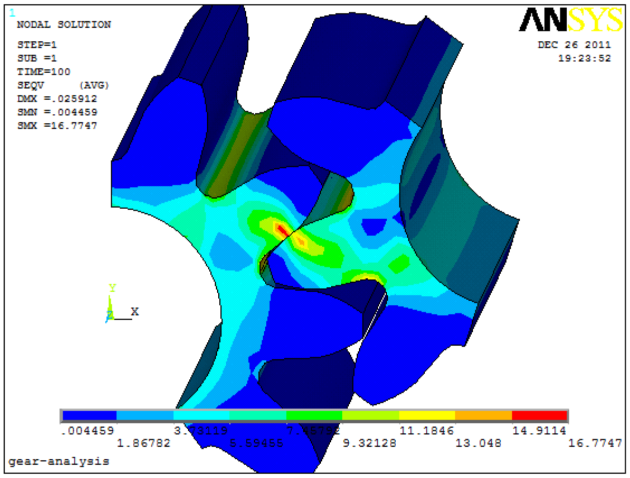

Fig. (4). Gear stress cloud chart.

According to the defined variables in Table **1**, the random parameter sensitivity chart in Fig. (**5a**) and the sensitivity values in Table **4** have been obtained based on Latin hypercube sampling in ANSYS/PSD module. The reliability of contact strength under the confidence level of 95% is 72.4%, which showed that the reliability is low and needs to be further optimized.

As shown in Fig. (**5a**) the target output variable as Z_cont has a positive correlation with D_W, D and MD, i.e. the reliability decrease with the increase of these variables; Meanwhile, Z_cont has a negative correlation with B, A_W, E and λ, i.e. the reliability increase with the increase of these variables; A_W, B and D_W are the main factors that influenced gear contact stress and can be optimized. We carry out PDS cycle program again after readjusting the values of A_W, B and DW according to their sensitivity direction, the optimized reliability and new parameters sensitive graph can be obtained as showed in Fig. (**5b**).

After tolerance optimization of B, A_W, D, D_W, the reliability of gear contact strength reached 83%, which is higher than before optimization, Table **5** presents optimized tolerance and their distribution, Fig. (**5b**) shows that parts material is also sensitive to the gear strength after tolerance optimization, so the replacing of gear material or adjusting other parameters may contribute to higher reliability of gear strength.

CONCLUSION

A reliability analysis method considering the effect of tolerance has been presented and applied to analyze gear contact fatigue strength. To improve the reliability of gear contact fatigue strength, the tolerance values are to be optimized according to sensitivity analysis method. As a matter in fact, there are still several problems requiring further study:

(1) Comprehensive optimization of different reliability indicators including vibration reliability, reliability of gear contact fatigue strength and etc.

(2) Multi-objective tolerance optimization considering reliability and cost.

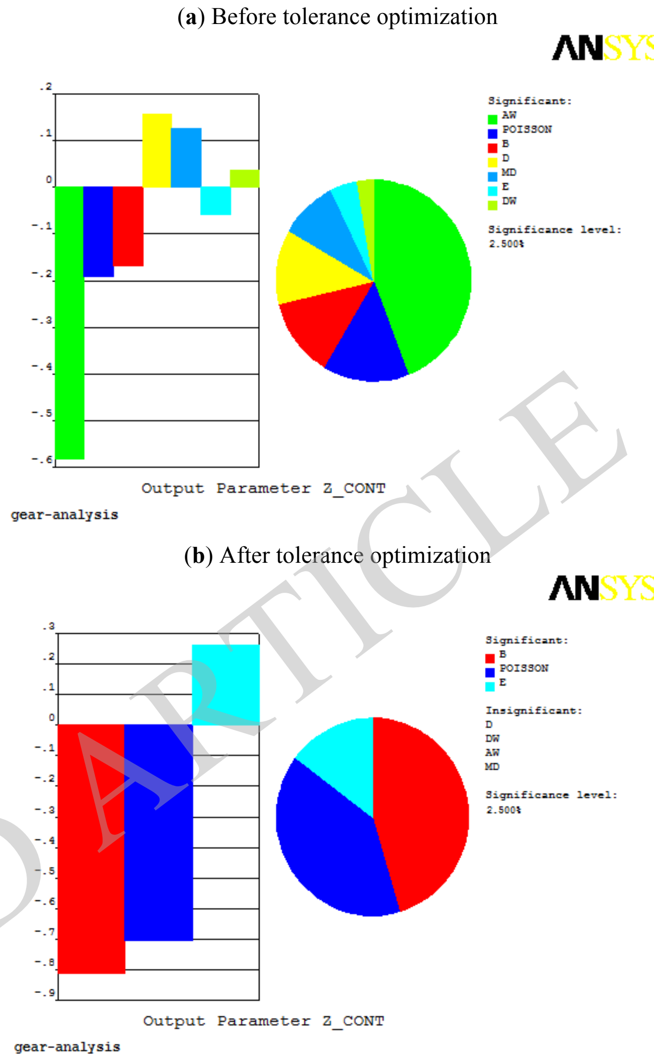

Fig. (5). Random variables sensitive graph.

Table 4. The correlation coefficient of random variables.

Out\Inp	B	D	D_W	A_W	E	M_D	POISSON
Z_CONT	-0.167	0.157	0.037	-0.581	-0.058	0.126	-0.190

Table 5. Optimized tolerance and their distribution.

Random Variable	GAUS	Center Value	Standard Deviation	Tolerance
B/mm	GAUS	16.135	0.045	$16^{+0.27}_{0}$
A_W/mm	GAUS	15.09	0.03	$15^{+0.18}_{0}$
Pitch diameter (D/mm)	GAUS	14.949	0.012	$\varphi 15^{-0.016}_{-0.086}(\varphi 15f10)$
D_W/mm)	GAUS	7.942	0.015	$\varphi 8^{-0.013}_{-0.103}(\varphi 8f11)$

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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