The Reliability Evaluation of Electromagnetic Valve of EMUs Based on Two-Parameter Exponential Distribution

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Abstract: In order to solve reliability evaluation of life of electromagnetic valve of EMUs, this paper evaluated the life of electromagnetic valve under small sample size based on zero-failure data. Firstly, prior distribution of the failure probability was taken into consideration and then the posteriori distribution was obtained by using the Bayes method so that the Bayes estimation could be received under the square loss. Finally, according to the p_i received, the reliability parameters of two-parameter exponential distribution were estimated based on weighted least square method. In addition, this paper applied the reliability theory to the reliability life evaluation of electromagnetic valve of EMUs which shows that this method can solve the reliability assessment problem providing certain theoretical basis for the reliability of electromagnetic valve.

1. INTRODUCTION

With the speed increasing by more than 200km/h in operation of EMUs for large area, China Railway has entered the high-speed development stage which is one of the main transportation means in our country. The safety of the system and high-efficiency operation should be ensured when the vehicle is running which are important safeguards of vehicle transportation safety. Prior to estimating the reliability, breakdown should be avoided to guarantee safe and normal operation of the system.

Although the EMUs component has the advantages of high reliability and long life, there can be serious consequences once a failure occurs. So it is necessary to evaluate its reliability life so as to effectively avoid accidents to a certain extent. In reference [1], Zhu Li introduced EMUs reliability index and conducts statistical calculation for breakdown of CRH2 traction system to analyze the reliability; Zhu Dexin used incomplete hybrid Beta distribution as the prior distribution of the failure probability, and solved reliability mathematical model of high-speed train bearing under small sample combining with multi-Bayes evaluation method as shown in reference [2]. In addition, the method of decreasing and increasing function has been put forward by Han Ming to structure prior distribution which provides

a basis for prior distribution [3]; meanwhile, Han Ming makes modified maximum likelihood function and least square estimate for zero-failure data under two-parameter exponential distributions to get the reliability estimation [4]; in reference [5], Zhao Haibing selected $(1-p_2)^2$ and $(1-p_2)^s$ as the core of p_2 for the evaluation of the reliability by making use of exponential distribution with no memory; for bearing life which obeys Weibull distribution, Zhu Dexin summarized the relationship of p_i and made an estimation under small sample size to carry out reliability life assessment based on Bayes [6].

In this paper, Bayes method was applied to the reliability evaluation of EMUs electromagnetic valve on the basis of previous studies. It reasonably analyzed the relationship between failure probabilities and considered prior distribution of the failure probability, and then combined priori information and test information to receive posterior distribution so that Bayes estimation of p_i can be obtained under the quadratic loss function. Thus, the reliability parameters can be estimated on the basis of weighted least square method in order to achieve the purpose of the reliability analysis of electromagnetic valve.

2. THE RELIABILITY MODEL OF EMUS ELECTROMAGNETIC VALVE

This paper established the reliability model of EMUs electromagnetic valve based on the two-parameter exponential distribution where reliability of the model can be evaluated.

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2.1. EMUs Electromagnetic Valve

Rail transport has become an important component of the transportation system in China as a mode of transportation in the long distance with convenience and large capacity. The braking system is one of the important aspects which influences EMUs safety operation, therefore, its life reliability is extremely important.

Electromagnetic valve is automated and is the basic component which belongs to the actuator. It can match different circuits to achieve the desired control. Electromagnetic valve is composed of exhaust valve department and solenoid valve department, which includes two types: ON and OFF, and it makes the movable core move to open or close the exhaust valve by the excitation and demagnetization of solenoid valve coil.

EMUs electromagnetic valve includes emergency braking magnetic valve, EP value and antispin solenoid valve, etc. Sometimes electromagnetic valve indicates some fault, such as: it does not work after getting powered, it does not shut down, internal and external leakage occurs and so on. According to the related statistics, failure rate of the electromagnetic valve in EMUs is higher, and the whole system can be damaged if the electromagnetic valve is not reliable, so it is necessary to evaluate its reliability life.

2.2. Mathematical Model of System Reliability

In the reliability theory, one-parameter exponential distribution is widely used, especially for electronic products. Two-parameter exponential distribution is used to represent the fatigue life of the metal products including vehicles and hydraulic equipment [7].

Its life is assumed to obey two-parameter exponential distribution and the distribution function is given as follows [4]:

$$F(t) = 1 - \exp(-\frac{t-\mu}{\sigma}), \quad t \geq \mu \tag{1}$$

where, $F(t)$ is the failure probability, t is the life of electromagnetic valve which is random variable, μ and σ are parameters of two-parameter exponential distribution.

The density function and reliability function are as follows:

$$f(t) = \frac{1}{\sigma}\exp(-\frac{t-\mu}{\sigma}), \quad t \geq \mu \tag{2}$$

$$R(t) = 1 - F(t) = \exp(-\frac{t-\mu}{\sigma}), \quad t \geq \mu \tag{3}$$

The failure rate function is as follows:

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{1}{\sigma} \tag{4}$$

Under the condition of limited funds and resources, this paper adopted time truncation censoring test to assume EMUs component with long design cycle, expensive cost and high reliability. Time truncation censoring test refers to setting a time in advance, and the life test is conducted for a number of products and stops once the deadline is reached. In time truncation censoring test, the test data received when the failure does not occur is called zero-failure data for products with high reliability and long life. Components reliability index can be estimated using information offered by zero-failure data [8, 9].

It assumes the reliability model: time truncation censoring test for k times is carried out; truncation time meets $t_1 < t_2 < \cdots < t_k$; the number of test samples in the i-th test is n_i which is zero-failure data. The data can be obtained as follows:

$$(t_i, s_i) \qquad i = 1, 2, \cdots, k$$

where, $s_i = n_i + n_{i+1} + \cdots + n_k$, and s_i shows the number of samples which have not been malfunctioned at time t_i.

3. THE METHOD OF BAYES RELIABILITY EVALUATION

Compared with the classic methods, the Bayes method makes full use of priori information, especially for expensive and small sample products; it can effectively shorten the test cycle, greatly reducing the test cost and improving the test efficiency, having an extensive application prospect. In addition, how to select a priori distribution is the key to Bayes method which can be seen from Bayes formula. And now the failure probability and the parameter of two-parameter exponential distribution are estimated [10, 11].

3.1. The Estimation of the Failure Probability p_i

This paper assumed that failure probability at time t_i obeys the binomial distribution, and the likelihood function is as follows:

$$L(r_i \mid p_i) = C_{s_i}^{r_i} p_i^{r_i} (1-p_i)^{s_i - r_i} \tag{5}$$

where, $C_{s_i}^{r_i}$ means the combination where there is r_i sample which is disabled among s_i samples.

And it can be obtained as follows under zero-failure data:

$$L(0 \mid p_i) = (1-p_i)^{s_i} \tag{6}$$

3.1.1. The Property of the Failure Probability

On the basis of concavity and convexity of distribution function, Zhang Zhihua in the reference [12] obtained the relationship of failure probability, and then Bayes estimation of failure probability and population parameters was calculated in the case of the exponential distribution and normal distribution. Using the method of decreasing function, ore of prior distributions of p_i for two-parameter exponential distribution was chosen [4] on the basis of which this research was carried out. The distribution function of two-parameter exponential distribution is convex function of t [4, 12], so $p_0 = 0$ can be received when i=1, $p_{i-1} < p_i$ when $t_{i-1} < t_i$, and the following can be obtained:

$$\frac{p_{i-1}}{t_{i-1}} > \frac{p_i}{t_i} \quad , \quad i = 2,3,\cdots,m \tag{7}$$

Then

$$p_{i-1} < p_i < \frac{p_{i-1}t_i}{t_{i-1}} \quad , \quad i = 2,3,\cdots,m \tag{8}$$

In this paper $c_i = \frac{p_{i-1}t_i}{t_{i-1}}$ is assumed at the same time, p_{i-1} is replaced by the estimation $\hat{pi-1}$. Then the property of failure probability can be obtained as follows:

$$\hat{pi-1} < p_i < c_i \quad , \quad i = 2,3,\cdots,m \tag{9}$$

3.1.2. The Bayes Estimation of p_1

Through reference [12] and [5], it is obvious that the difference between p_i and $\hat{pi}$ is minor when s_i is major. So $\hat{p}_1 = \frac{0.5}{s_1 + 1}$ is selected as the estimation of p_1 in this paper which is reasonable.

3.1.3. The Bayes Estimation of p_i

The method of decreasing function has been adopted [12] and $(1-p_2)^2$ is selected as the core of prior distribution of failure probability p_2 on the basis of reference [4] in this paper where $f(\hat{p}_1,1)$ is regarded as the prior distribution of p_2 in the interval $(\hat{p}_1,c_2)$ as follows:

$$f(p_2|c_2) = f(\hat{p}_1,c_2) = \frac{3(1-p_2)^2}{(1-p_1)^3 - (1-c_2)^3} \tag{10}$$

Besides hyper-parameter, c_2 obeys uniform distribution from a_2 to b_2, and the density function is $f(c_2) = \frac{1}{a_2 - b_2}(a_2 < c_2 < b_2)$, where, $a_2 = \hat{p}_1$, $b_2 = \frac{2\hat{p}_1 t_2}{t_1}$ and $\hat{p}_1 < p_2$ [4]. Then multilayer prior distribution function of p_2 is obtained as follows:

$$f(p_2) = \int_0^1 f(p_2|c_2)dc_2$$
$$= \begin{cases} A[g(b_2) - g(p_2)](1-p_2)^2 & \hat{p}_1 < p_2 < b_2 \\ 0 & otherwise \end{cases} \tag{11}$$

where, $A = \frac{3}{b_2 - a_2}$, $g(x) = \int_0^x \frac{dx}{1-(1-x)^3}$

In accordance with the Bayes theory, the posteriori distribution of failure probability p_2 is obtained as follows:

$$\pi(p_2) = \frac{f(p_2)L(0|p_2)}{\int_{\hat{p}_1}^1 f(p_2)L(0|p_2)dp_2}$$
$$= \begin{cases} [q(a_2,b_2,s_2)]^{-1}[g(b_2) - g(p_2)](1-p_2)^{s_2+2} & \hat{p}_1 < p_2 < b_2 \\ 0 & otherwise \end{cases} \tag{12}$$

where, $q(a_2,b_2,s_2) = \int_{a_2}^{b_2}[g(b_2(-g(a_2)](1-p_2)^{s_2+2}dp_2$,

$g(x) = \int_0^x \frac{dx}{1-(1-x)^3}$

Combined with the likelihood function and posterior distribution of the failure probability, multi-Bayes estimation of failure probability p_2 is given under the quadratic loss function as follows:

$$\hat{p}_2 = \int_0^1 p_2\pi(p_2)dp_2 = [q(a_2,b_2,s_2)]^{-1}\int_{\hat{p}_1}^{b_2}[g(b_2) - g(a_2)]p_2(1-p_2)^{s_2+2}dp_2$$
$$= [q(a_2,b_2,s_2)]^{-1}[q(a_2,b_2,s_2) - q(a_2,b_2,s_2+1)]$$
$$= 1 - \frac{q(a_2,b_2,s_2+1)}{q(a_2,b_2,s_2)} \tag{13}$$

Similarly, the hyper-parameter selects uniform distribution from a_i to b_i, where, $a_i = \hat{p}_{i-1}$, $b_i = \frac{2\hat{p}_{i-1}t_i}{t_{i-1}}$, $\hat{p}_{i-1} < p_i$, and $f(\hat{p}_{i-1},1)$ is written for prior distribution of p_i in the interval $(\hat{p}_{i-1},b_i)$ as follows:

$$f(p_i|c_i) = f(\hat{p}_{i-1},c_i) = \frac{3(1-p_i)^2}{(1-p_{i-1})^3 - (1-c_i)^3} \tag{14}$$

The multilayer prior distribution function is obtained as follows:

$$f(p_i) = \int_0^1 f(p_i|c_i)dc_i = \begin{cases} A[g(b_i) - g(p_i)](1-p_i)^2 & \hat{p}_{i-1} < p_i < b_i \\ 0 & otherwise \end{cases} \tag{15}$$

where, $A = \frac{3}{b_i - a_i}$, $g(t) = \int_0^t \frac{dt}{1-(1-t)^3}$

Then the posteriori distribution of p_i is achieved:

$$\pi(p_i) = \frac{f(p_i)L(0|p_i)}{\int_{\hat{p}_i}^1 f(p_i)L(0|p_i)dp_i}$$
$$= \begin{cases} [q(a_i,b_i,s_i)]^{-1}[g(b_i) - g(p_i)](1-p_i)^{s_i+2} & \hat{p}_{i-1} < p_i < b_i \\ 0 & otherwise \end{cases} \tag{16}$$

Under the quadratic loss function, multi-Bayes estimation of the failure probability is as follows:

$$\hat{p}_i = \int_0^1 p_i\pi(p_i)dp_i = [q(a_i,b_i,s_i)]^{-1}\int_{p_{i-1}}^{b_i}[g(b_i) - g(a_i)]p_i(1-p_i)^{s_i+2}dp_i$$
$$= [q(a_i,b_i,s_i)]^{-1}[q(a_i,b_i,s_i) - q(a_i,b_i,s_i+1)]$$
$$= 1 - \frac{q(a_i,b_i,s_i+1)}{q(a_i,b_i,s_i)} \tag{17}$$

where, $q(a_i,b_i,s_i) = \int_{a_i}^{b_i}[g(b_i) - g(a_i)](1-p_i)^{s_i+2}dp_i$

3.2. Weighted Least Square Method of Two-Parameter Exponential Distribution

The common methods of point estimation include moment estimation, order statistics, least square method, etc. Based on zero-failure data, this paper, after the estimation and obtaining p_i according to the Bayes method, estimates the parameters of two-parameter exponential distribution using weighted least square method [7, 13].

According to $F(t) = 1 - \exp(-\frac{t_i - \mu}{\sigma})$, $\ln(1\text{-}F(t)) = -(\frac{1}{\sigma}t_i - \frac{\mu}{\sigma})$ can be obtained. And it is supposed that $\alpha = \frac{\mu}{\sigma}$, $\beta = -\frac{1}{\sigma}$, $x_i = t_i$ and $y_i = \ln(1 - \hat{p}_i)$, then p_i is replaced by $\hat{p}_i$ which introduces error. So now weighted least square method is used to solve the parameters α and β, besides it is supposed as follows:

$$Q = \sum_{i=1}^{k} \omega_i [y_i - \ln(1 - F(t))]^2 = \sum_{i=1}^{k} \omega_i [y_i - \alpha - \beta x_i]^2 \qquad (18)$$

The estimation received in reliability test is related to sample number and testing time. So this paper chooses $\omega_i = n_i t_i / \sum_{i=1}^{k} n_i t_i$ $(i = 1,2\cdots k)$ as weight coefficient, and then:

$$\begin{cases} \dfrac{\partial Q}{\partial \alpha} = -2\sum_{i=1}^{k} \omega_i (y_i - \alpha - \beta x_i) = 0 \\ \dfrac{\partial Q}{\partial \beta} = -2\sum_{i=1}^{k} \omega_i x_i (y_i - \alpha - \beta x_i) = 0 \end{cases} \qquad (19)$$

It is obtained as follows:

$$\begin{cases} \overset{\wedge}{\alpha} = \dfrac{AD - BC}{A^2 - CE} \\ \overset{\wedge}{\beta} = \dfrac{AB - DE}{A^2 - CE} \end{cases} \qquad (20)$$

where, $A = \sum_{i=1}^{k} \omega_i x_i$, $B = \sum_{i=1}^{k} \omega_i y_i$, $C = \sum_{i=1}^{k} \omega_i x_i^2$, $D = \sum_{i=1}^{k} \omega_i x_i y_i$, $E = \sum_{i=1}^{k} \omega_i$.

On the above basis, the estimations of μ, σ are as follows:

$$\begin{cases} \overset{\wedge}{\mu} = \dfrac{AD - BC}{DE - AB} \\ \overset{\wedge}{\sigma} = \dfrac{A^2 - CE}{DE - AB} \end{cases} \qquad (21)$$

The reliability and failure rate at any time can be obtained through bringing the calculated value of $\overset{\wedge}{\mu}, \overset{\wedge}{\sigma}$ into Eq. (3) and (4) as follows:

$$R(t) = \exp(-\frac{t - \overset{\wedge}{\mu}}{\overset{\wedge}{\sigma}}) \qquad (22)$$

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{1}{\overset{\wedge}{\sigma}} \qquad (23)$$

4. RELIABILITY ANALYSIS OF EMUs ELECTROMAGNETIC VALVE

Electromagnetic valves belong to products with high reliability, so this paper adopted zero-failure data in order to avoid a large number of testing expenses. It assumed that the life of electromagnetic valve obeys two-parameter exponential distribution, and now aiming at the reliability life problem of electromagnetic valve, it randomly selected n=20 electromagnetic valves to conduct 8 time truncation censoring tests without zero-failure data in the test results [14, 15]. The specific zero-failure data is shown in Table **1** for the time unit in seconds.

Based on zero-failure data of Fig. (**1**) and Eq. (13) and (17), the failure probability at each truncated moment can be obtained as shown in Fig. (**2**). And $a_1 = 1$, $b_1 = \frac{1}{s_1 + 1}$ are selected when $i = 1$.
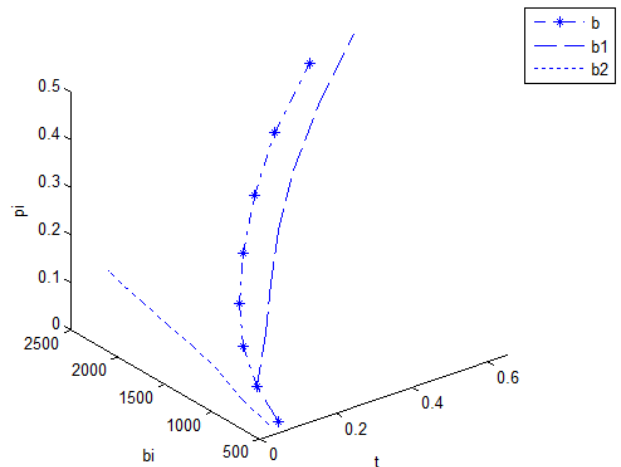

Fig. (1). The comparison of 3D stereogram of failure probability in the three cases.

According to reference [13], this paper chose two other kinds of methods to select hyper-parameter b to compare with the method adopted in this research. It assumed

$$b_{11} = \frac{0.5}{s_1 + 1}, \quad b_{12} = \hat{p}_1 \frac{t_2}{t_1},$$

$$b_{1i} = \hat{p}_{i-1} \frac{(t_i + t_{i-1})/2}{t_{i-2}}, \quad i = 3, \cdots, k,$$

and then the estimation of failure probability p_{1i} at each truncated moment was obtained as shown in Table **3**.

Similarly, it assumed $b_{21} = \frac{1}{s_1 + 1}$, $b_{22} = 2\hat{p}_1 \frac{t_2}{t_1}$,

$$b_{2i} = \hat{p}_{i-1} \frac{(t_i + t_{i-1})}{t_{i-2}}, \quad i = 3, \cdots, k, \quad \text{and the estimation of}$$

Table 1.   The zero-failure data of electromagnetic valve.

i	1	2	3	4	5	6	7	8
t_i	500	850	1200	1500	1800	2100	2350	2500
n_i	2	2	2	2	3	3	3	3
s_i	20	18	16	14	12	9	6	3

Table 2.   The estimation of failure probability p_i at each truncated moment.

i	1	2	3	4	5	6	7	8
a_i	0	0.0238	0.0468	0.0780	0.1170	0.1647	0.2236	0.2965
b_i	0.0476	0.0809	0.1321	0.1950	0.2808	0.3844	0.5005	0.6308
p_i	0.0238	0.0468	0.0780	0.1170	0.1647	0.2236	0.2965	0.3909

**Table 3.   The estimation of failure probability p_{1i} at each truncated moment.**

i	1	2	3	4	5	6	7	8
a_{1i}	0	0.0238	0.0468	0.0780	0.1170	0.1647	0.2236	0.2965
b_{1i}	0.0238	0.0405	0.0648	0.0739	0.0813	0.0904	0.0983	0.1023
p_{1i}	0.0238	0.0316	0.0465	0.0591	0.0695	0.0795	0.0886	0.0954

Table 4.   The failure probability estimation p_{2i} at each truncated moment.

i	1	2	3	4	5	6	7	8
a_{2i}	0	0.0238	0.0468	0.0780	0.1170	0.1647	0.2236	0.2965
b_{2i}	0.0476	0.0810	0.1912	0.2802	0.3726	0.4880	0.6158	0.7469
p_{2i}	0.0238	0.0468	0.0882	0.1355	0.1877	0.2491	0.3234	0.419

failure probability p_{2i} at each truncated moment was obtained as shown in Table **4**.

So the comparison of 3D stereograms between hyper-parameter b_i at each truncated moment and failure probability in the three cases supposed was obtained as shown in Fig. (**1**), in which the curves of b, b_1, b_2 respectively correspond to the values of Tables **2-4**. The failure probability received according to the first method to select hyper-parameters is more smooth and continuous than another two, which shows that the selection is more reasonable. Meanwhile, the relationship between failure probability and time is shown in Fig. (**2**), and the relationship between the failure probability and the hyper-parameters is shown in Fig. (**2**) in which they basically have a linear correlation. In addition, the estimations of two parameters of two-parameter exponential distribution, reliability and failure rate can be obtained as follows:
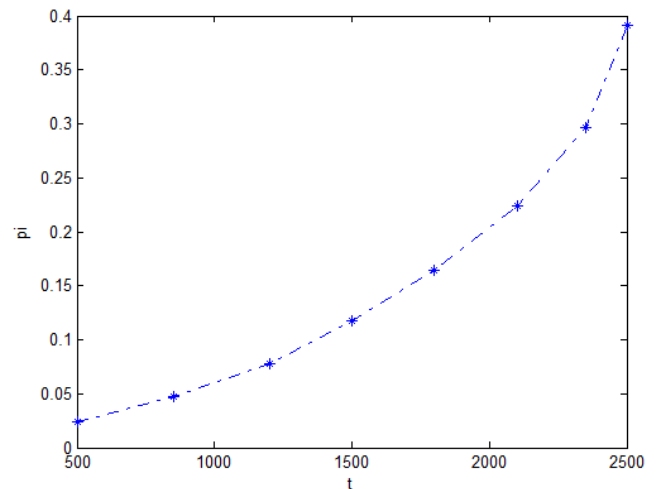
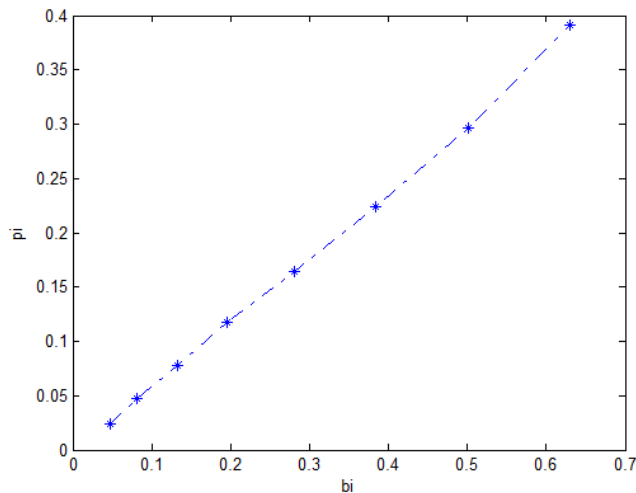**Fig. (2).** The relationship between failure probability and time.

Fig. (3). The relationship between failure probability and hyper-parameter.
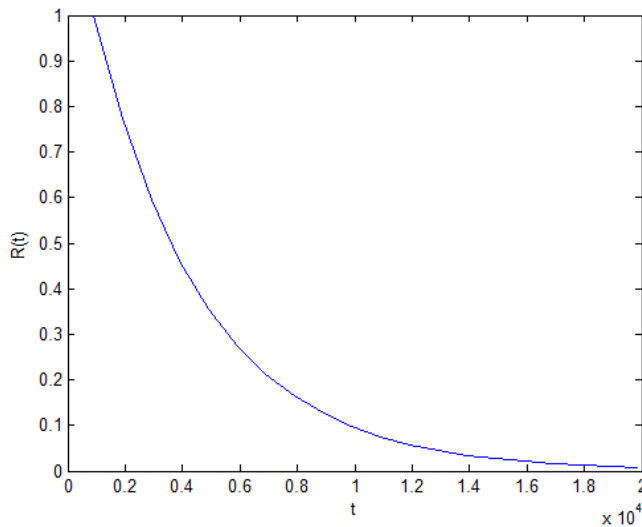

Fig. (4). The reliability curve of EMUs electromagnetic valve.

$$\hat{\mu} = \frac{AD - BC}{DE - AB} = 904.609$$

$$\hat{\sigma} = \frac{A^2 - CE}{DE - AB} = 3869.492$$

$$R(t) = \exp(-\frac{t - \hat{\mu}}{\hat{\sigma}}) = \exp(-\frac{t - 904.609}{3869.492}) \ , \quad t \geq 904.609$$

$$\lambda(t) = \frac{1}{\hat{\sigma}} = 2.584 \times 10^{-4}$$

Thus, the reliability curve of corresponding EMUs electromagnetic valve can be obtained as shown in Fig. (**4**).

CONCLUSION

This paper mainly researched on the parameter estimation of two-parameter exponential distribution, starting from the theoretical analysis, and concretely applying it to reliability evaluation of EMUs electromagnetic valve, as shown in Fig. (**3**), which can obtain estimation which is $R(1000) = 0.9755$ with high reliability.

There is little difference between failure rate of an EMU which is 2.323×10^{-4} and the estimated rate in this paper, which indicates that this method can basically make a precise estimate of failure rate. The application of this method can achieve the purpose of the reliability evaluation of electromagnetic valve, so there is certain theoretical significance for research on two-parameter exponential distribution for electromagnetic valve.

CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

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