High-Energy Scattering by Extreme Dilaton Black Holes

Takuya Maki[*,1] and Kiyoshi Shiraishi[2]

[1]*Japan Women's College of Physical Education, Setagaya, Tokyo 157–8565, Japan*

[2]*Faculty of Science, Yamaguchi University, Yamaguchi-shi, Yamaguchi 753–8512, Japan*

Abstract: We investigate the high and low energy scattering of charged scalar waves from an extreme dilaton black hole (DBH). The analyses here correspond to forward scattering processes of two charged scalar particles with dilaton coupling under a certain extreme condition. We calculate the scattering amplitude and compare it with the known results obtained by other methods. We also discuss the case of incoming wave with a general charge and dilaton coupling.

PACS Number(s): 04.40. Nr, 04.70.Bw, 11.27.+d

1. INTRODUCTION

The study of high-energy scattering process including gravity is very important to explore a possible road to quantum gravity. Since the seminal papers [1, 2] appeared, many authors have studied the scattering of two particles at high energy (for an example, see [3]). The effect of the Coulomb [4-6] and dilatonic forces [7, 8] has been also considered for scattering of charged particles in the leading order in momentum transfer correction.

The simplest object with mass and charges is a black hole. Exact solutions for black holes in string-inspired field theory have been found [9-11]. The interaction of extremely charged dilatonic black holes has been studied in the low-velocity limit by one of the present authors [12, 13]. In particular, the black hole scattering at a low velocity was eagerly investigated there.

In the present paper, we investigate the high-energy scattering of a scalar wave from charged DBHs and reveal the properties by using the Coulomb wave approximation. The result is approximately equivalent to the high-energy scattering of two extremely charged DBHs. Although our analysis relies on a method of pedagogical fashion, the results exhibit the interesting result at both low and high energy.

The structure of the present paper is as follows. In §2, we present a system to study by describing the background fields and wave equations on the back- ground. In §3, we extract the long-range part from the spherical potential and the scattering of scalar wave by the extreme DBH is studied. The case with arbitrary charges and dilaton couplings for the incoming scalar is discussed in §4. The last section (§5) is devoted to summary.

2. THE DBH BACKGROUND AND A CHARGED SCALAR FIELD COUPLED TO DILATON

We suppose an Einstein-Maxwell-dilaton system described by the following action:

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi}\left[R - 2(\nabla\phi)^2 - e^{-2a\phi}F^2 \right] + \left(\text{ surface terms } \right). \quad (1)$$

where R is the scalar curvature, ϕ a dilaton, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the abelian gauge field strength. Here the dilaton coupling a is considered to take a general value ($a = 1$ for effective field theory of string theory and $a = \sqrt{3}$ for reduced theory from the Kaluza-Klein compactification).

Let us consider the high-energy scattering of a charged scalar wave from an extreme DBH in the system. This process corresponds to the scattering of a charged massive object from an extreme DBH. In this section, we describe the background geometry as the target DBH and the wave equation for the incoming particle.

We assume the metric for an extreme DBH as [10, 11][1]

$$ds^2 = -\frac{1}{V^{2/(1+a^2)}} dt^2 + V^{2/(1+a^2)} d\,\mathrm{x}^2 \quad (2)$$

with

$$V = 1 + \frac{(1+a^2)M}{r}, \quad (3)$$

where M is the mass of the DBH. Note that we set the Newton constant G to unity. The dilaton field configuration and electric potential are given as

---

*Address correspondence to this author at the Japan Women's College of Physical Education, Setagaya, Tokyo 157–8565, Japan;
Tel: +81-839335681; Fax: +81-839335681;
E-mail: maki@jwcpe.ac.jp

[1]Strictly speaking, there is a singularity at $r = 0$. Nevertheless, we use the term "black hole" because the extreme case may still have generic properties of black holes in terms of their dynamics as a limiting case. As far as we consider semi-classical scattering cross sections, the phenomena near the horizon (of which property is sensitive on the mass-charge relation) can be neglected.

$$e^{-2a\phi} = V^{2a^2/(1+a^2)},$$ (4)

and

$$A_t = \frac{1}{\sqrt{1+a^2}} \frac{V-1}{V}.$$ (5)

The electric charge Q and the dilatonic charge Σ of the DBH are considered as

$$Q = \sqrt{1+a^2}\, M$$ (6)

and

$$\Sigma = aM,$$ (7)

respectively. These relations are called as an extreme condition among the mass, charge, and dilatonic charge here.

The classical action for a spinless particle, which has an electric charge q and is coupled to the dilaton field with a coupling constant a', is

$$S_p = -\int ds \left[m e^{a'\phi} + qA_\mu \frac{dx^\mu}{ds} \right],$$ (8)

where m and q are the mass and charge of the particle respectively. Quantizing the charged scalar particle coupled to the dilaton, we obtain the scalar wave equation as

$$(D^\mu D_\mu - e^{2a'\phi} m^2)\psi = 0,$$ (9)

where ψ is the wave function and the covariant derivative is defined as

$$D_\mu = \partial_\mu + iqA_\mu.$$ (10)

For the background fields of the extreme DBH (2), (3), (4), (5), the wave equation (9) can be written as

$$V^{\frac{-2}{1+a^2}} \nabla^2 \psi + V^{\frac{2}{1+a^2}} \{ \omega - q(1+a^2)^{-1/2}(1-V^{-1}) \}^2 \psi - V^{\frac{-2aa'}{1+a^2}} m^2 \psi = 0, \quad (11)$$

where we assume $\psi \propto e^{-i\omega t}$.

There are particularly simple cases, when the dilaton coupling constants takes certain values.

If we assume $a'=a$ and the extreme condition for the probing wave, $q = \sqrt{1+a^2}\, m$, the wave equation becomes

$$\nabla^2 \psi + V^{\frac{4}{1+a^2}} \left[(\omega-m)^2 + \frac{2m(\omega-m)}{V} \right] \psi = 0.$$ (12)

Some particular cases are written as follows: When for $a^2 = 3$, the equation reads

$$\nabla^2 \psi + \left[(\omega-m)^2 V + 2m(\omega-m) \right] \psi = 0,$$ (13)

while for $a^2 = 1$, the equation reads

$$\nabla^2 \psi + \left[(\omega-m)^2 V^2 + 2m(\omega-m)V \right] \psi = 0.$$ (14)

In the next section, we solve the wave equation to analyze the scattering of the wave by the extreme DBH.

3. LONG-RANGE EFFECTIVE POTENTIAL AND APPROXIMATION

In this section, we evaluate the cross section for the wave scattering in the lowest-order approximation. Here we expand the potential in terms of inverse of the distance r. Expanding the potential in M/r and dropping terms of higher orders than $O(r^{-2})$, we get the wave equation

$$\left[\nabla^2 + k^2 + 2\alpha \left(\frac{M}{r} \right) + \beta \left(\frac{M}{r} \right)^2 \right] \psi = 0,$$ (15)

where α and β are given by

$$\alpha = (\omega-m)\{2\omega + (1-a^2)m\} = \frac{\omega-m}{M}\{s - qQ - (M-m)^2\}$$
$$= 2k^2 - (1+a^2)m(\sqrt{k^2+m^2} - m),$$ (16)
$$\beta = 2(3-a^2)(\omega-m)(\omega-ma^2)$$
$$= 2(3-a^2)[k^2 - (1+a^2)m(\sqrt{k^2+m^2} - m)].$$

Here, we use $k = \sqrt{\omega^2 - m^2}$. The Mandelstam variables s and t are defined by:

$$s = 2M\omega + M^2 + m^2,$$ (17)

$$t = -|\mathbf{k} - \mathbf{k}'|^2 = -(2k\sin(\theta/2))^2,$$ (18)

where θ is the scattering angle.

We take the lowest order approximation, where the term of $O(M^2/r^2)$ is omitted. Then, the wave equation reduces to

$$\left(\nabla^2 + k^2 + \frac{2\alpha M}{r} \right) \psi = 0.$$ (19)

The asymptotic form of the solution for scattering is [3, 6]

$$\psi \approx e^{i\left[kz - \frac{\alpha M}{k} \ln k(r-z) \right]} + \frac{e^{i\left(kr + \frac{\alpha M}{k} \ln 2kr \right)}}{r} f(\theta),$$ (20)

and then the scattering amplitude $f(\theta)$ is given as [3, 6]

$$f(\theta) = \frac{\alpha M}{2k^2 \sin^2(\theta/2)} \exp\left[i\frac{\alpha M}{k} \ln(\sin^2\theta/2) + 2i\delta_0 \right]$$
$$= \frac{1}{2ik} \frac{\Gamma(1-i\alpha M/k)}{\Gamma(i\alpha M/k)} \left(\frac{4k^2}{-t} \right)^{1-i\alpha M/k},$$ (21)

where the phase shift is obtained as

$$\delta_0 = \arg\Gamma(1 - i\alpha M/k).$$ (22)

The scattering cross section of the lowest-order, in other words, in the Coulomb wave approximation is

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

$$= \frac{1}{4k^2 \sin^4(\theta/2)} \left| \frac{\Gamma(1 - i\alpha M / k)}{\Gamma(i\alpha M / k)} \right|^2 \tag{23}$$

$$= \frac{\alpha^2 M^2}{4k^4 \sin^4(\theta/2)}.$$

For the case that $k \ll m$, we find

$$\frac{d\sigma}{d\Omega} = \frac{(3 - a^2)^2 M^2}{16 \sin^4(\theta/2)}. \tag{24}$$

This result agrees with the previous result (of one of the present authors) obtained by the classical analysis of the moduli space of slowly-moving extreme DBHs [13]. For $a = 1$, the expression (24) coincides with Eq. (3.14) in Ref. [13]. For $a^2 = 3$, almost no scattering occurs at the low energy, since there is no force between two extremal DBHs in the slowly-moving limit.

On the other hand, in the high-energy limit, $\alpha M / k \approx s - Qq - (M - m)^2$. Thus

$$\frac{d\sigma}{d\Omega} = \frac{[s - Qq - (M - m)^2]^2}{4k^2 \sin^4(\theta/2)}. \tag{25}$$

Then the result (25) agrees with one of 't Hooft [1] when M-m is neglected. The gravitational interaction is certainly dominant in the Planckian energy scale. It should be noted that no effect of dilaton couplings and charges can be seen either in the lowest order approximation in high-energy scattering.

The validity of the approximation is guaranteed by assuming the large impact parameter, because only the lowest order in M/r is expected to be dominant. This is equivalent to the case with small momentum transfer, or nearly forward scattering.

As further exploration on the validity of the approximation, we will estimate the feasible value for the impact parameter. If we expand the wave function as the angular momentum eigenvalue l, we find the centrifugal potential as $l(l+1)/r^2$. Thus, the $O(M^2/r^2)$ term of potential can be neglected if

$$l(l+1) \gg |\beta| M^2. \tag{26}$$

Since the impact parameter is estimated semiclassically as $b \approx \sqrt{l(l+1)}/k$,

$$b \gg \sqrt{|\beta|} M / k. \tag{27}$$

For the case that $k \ll m$, the approximation holds for

$$b \gg \sqrt{|(3 - a^2)(1 - a^2)|} M, \tag{28}$$

while the approximation holds for the case that $k \gg m$

$$b \gg \sqrt{2 |(3 - a^2)|} M. \tag{29}$$

4. THE CASE WITH ARBITRARY CHARGES AND DILATON COUPLINGS FOR THE INCIDENT WAVE

The wave equation with an arbitrary coupling and charge (11) is approximated to be in the form of Eq. (19). Then the coefficient α is

$$\alpha M = M[(2\omega - \sqrt{1 + a^2} q)\omega + (aa' - 1)m^2]$$
$$= (2\omega^2 - m^2)M - Qq\omega + \Sigma\Sigma' m, \tag{30}$$

where $\Sigma' = a' m$.

Hence, for arbitrary couplings, when the extreme condition is not satisfied, the scattering of very low energy is nothing but the Rutherford scattering with the cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 M^2}{4k^4 \sin^4(\theta/2)}$$

$$= \frac{(Mm - Qq + \Sigma\Sigma')^2 m^2}{4k^4 \sin^4(\theta/2)}, \tag{31}$$

for $Mm - Qq + \Sigma\Sigma' \neq 0$. This result is trivial because of the existence of residual long-range static forces. Of course, at very high energy, the cross section is similar to (25) because of the gravity dominance.

5. SUMMARY AND OUTLOOK

We have investigated the scattering of charged scalar wave from an extreme DBH. The scalar field is supposed to satisfy the extreme condition. This corresponds to the Planckian forward scattering process of two charged scalar particles with the extreme condition. We have evaluated the scattering cross section for the corresponding case of a large impact parameter.

We have found that our simple analyses can reveal the behavior of the eikonal scattering from black holes. Therefore we intend to study the scattering with ex- cited states such as in the Kaluza-Klein and string theory in the similar manner and compare the results with ones obtained from various methods.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

ACKNOWLEDGEMENTS

Declared none.

REFERENCES

[1]   (a) 't Hooft G. Graviton dominance in ultra-high-energy scattering. Phys Lett B 1987; 198: 61-3; (b) On the factorization of universal poles in a theory of gravitating point particles. Nucl Phys B 1988; 304: 867-76.

[2]   (a) Amati D, Ciafaloni M, Veneziano G. Superstring collisions at planckian energies. Phys Lett B 1987; 197: 81-8; (b) Classical and quantum gravity effects from planckian energy superstring collisions. Int J Mod Phys A 1988; 3: 1615-61; (c) Higher-order gravitational deflection and soft bremsstrahlung in planckian energy superstring collisions. Nucl Phys B 1990; 347: 550-80.

[3]   Kabat D, Ortiz M. Eikonal quantum gravity and planckian scattering. Nucl Phys B 1992; 388: 570-92.

[4] Lousto C, Sánchez N. The curved shock wave space-time of ultrarelativistic charged particles and their scattering. Int J Mod Phys A 1990: 5: 915-38.

[5] Jackiw R, Kabat D, Ortiz M. Electromagnetic fields of a massless particle and the eikonal. Phys Lett B 1992; 277: 148-52.

[6] Das S, Majumdar P. Aspects of Planckian scattering beyound the eikonal Pramana 1998; 51: 413-9.

[7] Das S, Majumdar P. Shock wave mixing in Einstein and dilaton gravity. Phys Lett B 1995; 348: 349-54.

[8] Das S, Majumdar P. Eikonal particle scattering and dilaton gravity. Phys Rev D 1997; 55: 2090-8.

[9] Gibbons GW, Maeda K. Black holes and membranes in higher-dimensional theories with dilaton fields. Nucl Phys B 1988; 298: 741-75.

[10] Garfinkle D, Horowitz G, Strominger A. Charged black holes in string the- ory. Phys Rev D 1991; 43: 3140-3; (Erratum) 1992; 45: 3888.

[11] Shiraishi K. Multi-centered solution for maximally-charged dilaton black holes in arbitrary dimensions. J Math Phys 1993; 34: 1480-6.

[12] Shiraishi K. Moduli Space Metric for Maximally Charged Dilaton Black holes. Nucl Phys B 1993; 402; 399-410.

[13] Shiraishi K. Classical and Quantum Scattering of Maximally Charged Dila- ton Black holes. Int J Mod Phys D 1993; 2: 59-77.