Extreme and Rogue Waves in Directional Wave Fields

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Abstract: It is well established that modulational instability enhances the probability of occurrence for rogue waves if the wave field is long crested, narrow banded and sufficiently steep. As a result, a substantial deviation from commonly used second order theory-based distributions can be expected. However the spreading of the wave energy over a number of directional components can notably reduce the effect of modulational instability. In order to achieve a better understanding on the influence of wave directionality and its implication for design work, numerical simulations based on the truncated potential Euler equations were used. Results show the existence of a transition region between strongly and weakly non-Gaussian statistics as short crestedness increases.

Keywords: Deep water waves, extreme waves, modulational instability, wave statistics.

1. INTRODUCTION

Extraordinary (abnormal) waves with a large amplitude and/or a very steep profile, often known as rogue or freak waves, are blamed to have caused a number of accidents to marine structures [1,2] The notion of rogue or freak waves is generally applied for single waves that are extremely unlikely according to the Rayleigh distribution of wave heights [3,4]. Generally, an extreme wave is considered as rogue if its crest-to-trough height exceeds the significant wave height by a factor of 2-2.2 and/or its crest exceeds the significant wave height by a factor of 1.2-1.3 [5,6]; there is no large consensus on the value of these factors, though. Whereas the existence of these extreme waves themselves has generally not been questioned, neither circumstances under which these waves occur nor their physical nature are well understood. Although the knowledge of rogue waves has considerably advanced in the last decade [1], several important questions especially regarding their probability of occurrence in realistic oceanic conditions still remain unanswered. In this respect, the present study discusses the statistical properties of the surface elevation and in particular the probability of occurrence of extreme waves, including rogue waves, in directional wave fields.

It is nowadays established that extreme and rogue waves can arise as a result of several mechanisms such as the wave--current interaction [7-10], linear Fourier superposition and nonlinear wave--wave interaction (see [1] for a review). In the open ocean, in the absence of strong currents, the statistical properties of water waves can be conveniently described by modelling the surface elevation with a second order expansion in the wave steepness ε of the water wave equations, i.e. second order wave theory [11]. Based on this approach a number of probability distributions have been proposed by many authors to describe the statistical properties of the surface elevation [12-15].

Despite the fact that the second order theory agrees with field measurements reasonably well [14,16], deviations from second order based statistical distributions are still documented [17,18]. In this respect, the second order approximation only includes effects related to bound waves while nonlinear dynamics of free waves are neglected. At the third order in wave steepness, however, wave trains tend to be unstable due to small perturbations which cause a local exponential growth in the wave amplitude within a time frame of a few tens of wave periods [4]. The mechanism involved is basically a generalization of the Benjamin--Feir instability [19] or modulational instability [20] and can be explained by the nonlinear Schrödinger equation [20], which is derived from the Euler equations describing a potential flow of free--surface fluid under the hypothesis that waves are weakly nonlinear (i.e. $\varepsilon = ka << 1$, where k is the wavenumber and a is the wave amplitude), and the bandwidth is narrow ($\Delta k / k << 1$, where Δk is a modulation wave vector).

Because numerical models based on the nonlinear Schrödinger equation are not computationally intensive, they have been used extensively to investigate the statistical properties of deep water waves [21-24]. Results have shown that the instability of wave packets and the consequent growth of large amplitude waves can modify substantially the form of the probability density function of the surface elevation. However, strong deviations from Gaussian and second order statistics have only been observed for narrow spectral conditions where most of the wave energy is confined within a narrow range of frequencies and directions [21,23]. For more realistic directional wave fields, the effect of nonlinear dynamics is gradually suppressed at the increase

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of the directional spreading. If the directional spreading is sufficiently broad, in particular, the statistical properties of the surface elevation matches second order predictions and hence only weakly deviates from Gaussian statistics [22-24].

Due to the narrow banded constraint, we cannot rely *a priori* on numerical predictions based on nonlinear Schrödinger equation for the statistical properties of directional wave fields. To eliminate this issue, the evolution of the surface elevation can be simulated with the Zakharov equation, which describes the evolution of weakly nonlinear waves of any bandwidth [25,26]. The surface elevation may also be simulated directly from the potential Euler equations. In this respect, a number of methods is available in the literature [27-30]. Unfortunately, the computational burden for the fully nonlinear Euler equations is rather intense and discourages any attempts to investigate the statistical properties of directional wave fields, which require the simulations of many random realizations of the sea surface. However, if a truncation of the potential Euler equations is considered, simulations of the random sea surface can be conveniently performed by using the Higher Order Spectral Method (HOSM), which was derived independently by Dommermuth and Yue [31] and West *et al.* [32]. This method has been used successfully by many authors [16,33-38] to investigate the random sea surface.

Because a proper understanding of the implications of wave directionality on wave statistics is important for design work [14,39], here we make use of the HOSM to carry out a comprehensive study on the probability of occurrence of extreme waves in directional sea states. In particular, attention is given to relatively broad directional conditions, which are more likely to occur in oceanic wave fields. We mention that for the present study we considered a truncation at the third order so that effects related to the modulational instability have been included.

The paper is organized as follows. In the next section, the numerical model and its initial conditions are briefly introduced. In section 3 and 4, the statistical properties of the surface elevation are presented as a function of the directional spreading. Emphasis is given to the shape of the tail of the probability density function of the surface elevation and wave amplitudes (wave troughs, crests, and heights) and its deviation from the Gaussian surface elevation. As the wave field propagates, the waves spectrum undergoes a number of changes due to the nonlinear wave interaction. A brief description of spectral changes and their effects on wave statistics is presented in section 5. In the last section, results are discussed in perspective to the current design and operational criteria of marine structures.

2. NUMERICAL EXPERIMENTS

2.1. The Model

To model the dynamics of the free surface elevation, we assume an irrotational, inviscid and incompressible fluid flow. In this case there exists a velocity potential $\varphi(x,y,z,t)$ which satisfies the Laplace's equation everywhere in the fluid. We restrict ourselves to the case of domains with constant water depth. At the bottom ($z=-\infty$ for this study) the boundary condition is such that the vertical velocity is zero ($\varphi_z|_{\infty}=0$). At the free surface ($z=\eta(x,y,t)$), the kinematic and dynamic boundary conditions are satisfied for the free surface elevation and the velocity potential at the free surface ($\psi(x,y,t)=\varphi(x,y,\eta(x,y,t),t)$). Using the free surface variables these boundary conditions are as follows [20]:

$$\eta_t+\psi_x\eta_x+\psi_y\eta_y-W\left(1+\eta_x^2+\eta_y^2\right)=0, \tag{1}$$

$$\psi_t+g\eta+\frac{1}{2}\left(\psi_x^2+\psi_y^2\right)-\frac{1}{2}W^2\left(1+\eta_x^2+\eta_y^2\right)=0, \tag{2}$$

where the subscripts denote partial derivatives, and $W(x,y,t)=\varphi_z|_\eta$ represents the vertical velocity evaluated at the free surface.

The time evolution of the surface elevation can be calculated directly from equations (1) and (2) using a higher order spectral method. For the present study, we use the method proposed by West *et al.* [32] as it is more consistent than the one proposed by Dommermuth and Yue [31] (see [40] for a review).

HOSM is a pseudo--spectral method, which uses a series expansion in the wave steepness ε of the velocity potential of the form:

$$\varphi(x,y,z,t)=\sum_{m=1}^{M}\varphi^{(m)}(x,y,z,t), \tag{3}$$

where each $\varphi^{(m)}$ is a quantity of order $O(\varepsilon^m)$. In the above expansion M is the order of approximation in nonlinearity. A Taylor expansion around $z=0$. is then performed for each $\varphi^{(m)}$ term and combined with the above expansion for the potential. After collecting all terms at each order in wave steepness we obtain a system of the form [32]:

$$\varphi^{(1)}(x,y,z=0,t)=\psi(x,y,t);$$

$$\varphi^{(m)}(x,y,z=0,t)=-\sum_{k=1}^{m-1}\frac{\eta^k}{k!}\frac{\partial^k}{\partial z^k}\varphi^{(m-k)}(x,y,z=0,t) \tag{4}$$

$(for\ m=2,3,\ldots,M)$.

Following [32], the vertical velocity at the free surface $W(x,y,t)$ is similarly expanded in series of terms of $O(\varepsilon^m)$:

$$W(x,y,t)=\sum_{m=1}^{M}W^{(m)}(x,y,t), \tag{5}$$

where the terms $W^{(m)}$ are computed from the $\varphi^{(m)}$ terms:

$$W^{(m)}(x,y,t)=\sum_{k=0}^{m-1}\frac{\eta^k}{k!}\frac{\partial^{k+1}}{\partial z^{k+1}}\varphi^{(m-k)}(x,y,z=0,t). \tag{6}$$

For the case of a rectangular domain in space with dimensions L_x and L_y in x and y, assuming periodicity in both directions for the wave field, we can use the following expression based on a double Fourier series for each $\varphi^{(m)}$ term:

$$\varphi^{(m)}(x,y,z,t)=\sum_{k,l}c_{k,l}^{(m)}cos\left(\omega t-\vec{k}_{k,l}\cdot\vec{x}\right), \tag{7}$$

with wavenumbers $k_{k,l} = |\vec{k}_{k,l}|$ and $\vec{k}_{k,l} = (k_x, k_y) = (2\pi k / L_x, 2\pi l / L_y)$; $\omega = \sqrt{g\,|\vec{k}_{k,l}|}$ is the angular frequency and $c_{k,l}^{(m)}$ is the frequency-dependent amplitude of the potential.

Here, we considered a third-order expansion (i.e. $M = 3$) so that three and four waves interaction is included [41]. After evaluating the vertical velocity at the free surface at order M, the free surface velocity potential $\psi(x, y, t)$ and the surface elevation $\eta(x, y, t)$ can be integrated in time from equations (1) and (2). The time integration is then performed by means of a fourth-order Runge--Kutta method with a constant time step. All aliasing errors generated in the nonlinear terms are removed [32].

2.2. Initial Conditions and Numerical Simulations

The numerical experiment consists in simulating the temporal evolution of many random realizations of an initial wave field. In order to prepare the initial conditions, it is necessary to generate a directional, frequency spectrum $E(\omega,\vartheta) = S(\omega)\, D(\omega,\vartheta)$, where $S(\omega)$ is the angular frequency spectrum and $D(\omega,\vartheta)$ is the directional function, and then to transform it into the associated wavenumber spectrum $E(|\vec{k}_{k,l}|)$. As it is frequently used for many practical application, the JONSWAP spectrum was used to express $S(\omega)$, while a frequency-independent $cos^N(\vartheta)$ function was then applied to model the wave directional spreading. The spectrum in wavenumber coordinates can be written as follows:

$$E(|\vec{k}_{k,l}|) = \frac{\alpha}{|\vec{k}_{k,l}|^3} \frac{1}{2\,|\vec{k}_{k,l}|} exp\left[-\frac{3}{2} \left(\frac{k_p}{|\vec{k}_{k,l}|} \right)^2 \right]. \tag{8}$$

$$\gamma^{\exp\left[-\left(\sqrt{|\vec{k}_{k,l}|} - \sqrt{k_p} \right)^2 / \left(2\sigma^2 k_p^2 \right) \right]} cos^N(\vartheta),$$

where $k_p = 2\pi / L_p$ is the peak wavenumber and $\vartheta = arctan(k_y / k_x)$; the parameter σ is equal to 0.07 if $\omega < \omega_p$ and 0.09 if $\omega < \omega_p$. For the present study, for convenience, we described the wave field with a dominant wavelength $\lambda_p = 156$ m, which corresponds to a peak period $T_p = 10$ s. In a first instance, we selected Phillips parameter $\alpha = 0.014$ and peak enhancement factor $\gamma = 3$. This configuration corresponds to a significant wave height $H_s = 6.36$ m and hence to a wave steepness $k_p a = 0.13$, where a is half the significant wave height. We then considered a second condition characterized by a steeper and more narrow banded (in the frequency domain) spectrum with $\alpha = 0.016$ and $\gamma = 6$, corresponding to $H_s = 8$ m and $k_p a = 0.16$. Note that the selected values of steepness are not unusual as similar values were observed during ship accidents reported as being due to bad weather conditions [2].

We mention that for uni-directional propagation the Benjamin--Feir Index (BFI), a measure of the relative importance of nonlinearity and dispersion [4,21], provides some indications on the effect of modulational instability on wave statistics. An estimate of this index can be calculated as the ratio of the wave steepness $k_p a$ to the spectral bandwidth $\Delta k / k_p$, where Δk is a measure of the width of the spectrum estimated as the half--width at the half--maximum (see [42] for details). For the above spectral configurations $BFI = 0.7$ and 1.1 respectively. Note that the non-resonant interaction between free modes gives rise to a much larger deviation from the Gaussian statistics than bound waves if $BFI = O(1)$ [4].

In order to consider different degrees of the directional spreading, different values of the spreading coefficient N were used, ranging from fairly long crested (large N) to fairly short crested (small N) waves. The following values were selected: $N = 840$, 500, 200, 90, 48, 24 and 12.

From the wavenumber spectrum, $E(|\vec{k}_{k,l}|)$, an initial two--dimensional surface $\eta(x, y, t = 0)$ was computed using the inverse Fourier transform with the random amplitude and phase approximation. The random phases were assumed to be uniformly distributed over the interval [0, 2π], while the random amplitudes were Rayleigh distributed (the initial wave field is therefore Gaussian). The velocity potential $\psi(x, y, t = 0)$ was obtained from the input surface using linear theory (see equation (7)). The wave field was contained in a square domain of 1404 m with spatial mesh of 256 × 256 nodes. With this resolution, the spatial domain contained about 9 dominant waves; each wave was thus discretized with about 28 grid points.

Theoretical and numerical studies [4,23] have shown that deviations from Gaussian statistics due to third-order nonlinearity occur on a short timescale, typically on the order of 30 peak periods. In the present study, the total duration of the simulation was set equal to 60 T_p. A small time step, $\Delta t = T_p / 200 = 0.05$ s, was used to minimize the energy leakage; we mention that the selected time step was much smaller than the period of the shortest waves considered in this study. The accuracy of the computation was checked by monitoring the variation of the total energy [41]. Despite the fact that the energy content showed a decreasing trend throughout the temporal evolution, its variation was negligible as the relative error in total energy did not exceed 0.4% over the simulation time [37].

The model output consisted in the surface elevation, $\eta(x, y, t)$, which was archived every six peak periods. Because it is not yet clear how to derive wave amplitudes from a two-dimensional surface, we also collected time series from five different grid points far enough to ensure that the collected time series are independent realizations, i.e., the cross--correlation of the time series collected at two arbitrary grid points can be regarded as negligible. Time series were then used for a zero-crossing analysis of wave heights, crests and troughs. Records of the time series started after about 20 peak periods so that the effect of modulational

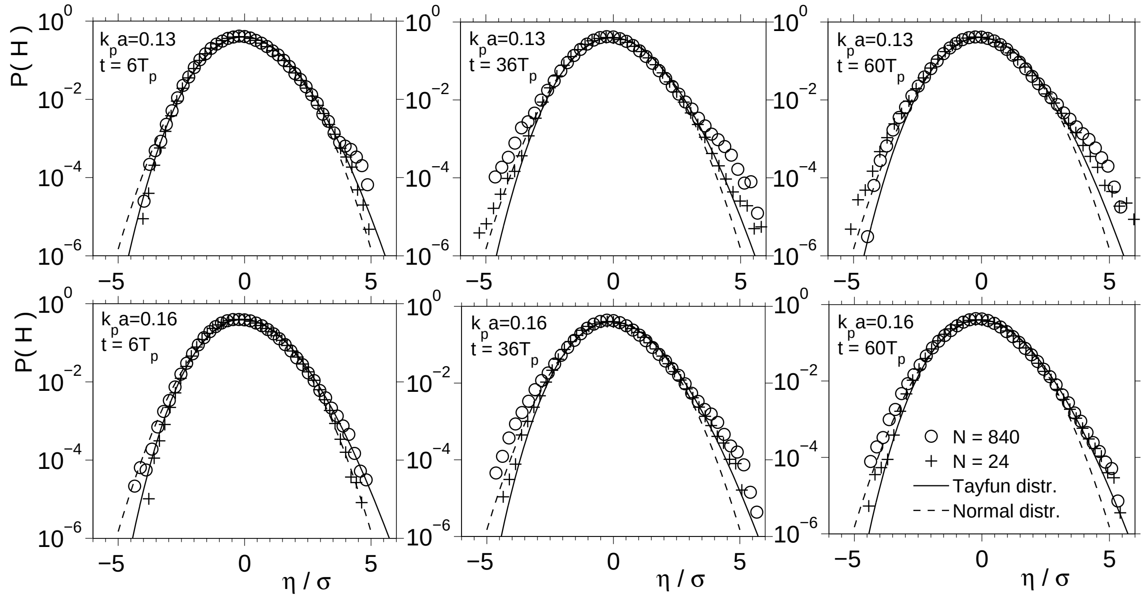

Fig. (1). Probability density function of the surface elevation.

instability is captured. For each of the simulated conditions, we performed 150 repetitions with the same input spectrum and different random amplitudes and random phases in order to have enough samples to achieve statistically significant results. The stability of the statistical moments is discussed in [37].

Note that the wave spectrum changes as the wave field evolves in time due to nonlinear wave-wave interactions. This implies that the spectral peaks is slightly shifted towards lower wavenumbers (spectral downshift) and the directional distribution becomes slightly broader than in the initial condition especially at high wave numbers. A brief description of the spectral changes is presented in section 5.

3. STATISTICAL PROPERTIES OF THE SURFACE ELEVATION

When the initial linear wave field begins to evolve, bound waves are immediately generated [41]. Thus, the effect of second order interaction becomes already visible after a few peak periods (see, for example, [38]). This generates a weak departure from the Gaussian statistics as wave crests become higher and troughs become shallower than linear theory would predict [12]. In this respect, Fig. (**1**) shows that the surface elevation fits a second order based distribution (i.e. Tayfun distribution [12]) reasonably well already after 6 peak periods. As the wave field keeps propagating, however, the instability of free wave packets (i.e. modulational instability) forces some individual waves to grow and reach extremely large amplitudes at the expense of the surrounding waves. Provided waves are long crested, narrow banded in the frequency domain and sufficiently steep, this instability can generate extreme waves far more often than in Gaussian and second order based statistics, which leads to a consequent strong deviation of the tails of the probability density function of the surface elevation [42] (see Fig. **1**). A general measure of the probability of occurrence of extreme values can be conveniently provided by the fourth order moment of the probability density function, i.e. the kurtosis. Note, however, that the latter does not provide specific information on the probability of occurrence of wave exceeding a certain threshold such as rogue waves. We mention that the kurtosis assumes a value of 3 for Gaussian random processes, while it can reach values close to 3.15 in second order wave fields with steepness similar to the ones used for this study.

In Fig. (**2**), the temporal evolution of the kurtosis is presented for selected sea states. Considering the large number of random repetitions, the estimate of the kurtosis is rather accurate. The 95% confidence intervals calculated with bootstrap methods indicates a variability of about ± 0.03. When the waves are long crested, e.g. $N = 840$, the kurtosis rapidly grows as a result of modulational instability (cf. [42]). Its maximum value, which is reached after a time of about 30 peak periods, is 3.5 for an initial spectrum with $k_p a = 0.13$ (or $BFI = 0.7$) and 3.7 for a spectrum with $k_p a = 0.16$ (or $BFI = 1.1$). It is also interesting to see that this large kurtosis is the result of both extremely high crests and deep trough. In particular, whereas positive elevations (upper tail of the probability density function) become significantly higher than second order predictions, negative elevations (lower tail) are observed to be deeper than in Gaussian sea states.

When the wave energy spreads over a wider range of directions, however, the effect of modulational instability is gradually suppressed (modulational instability can still be present though). As a result, the temporal variation of the kurtosis becomes less prominent; this is already notable for $N = 200$, which still remains a rather long crested wave field. A summary of the dependence of the kurtosis on the spreading coefficient is presented in Fig. (**3**). It is evident that the effect of modulational instability on the kurtosis becomes negligible for spreading coefficient $N < 50$. For sufficiently broad directional spectra, in fact, the kurtosis does not overcome substantially second order predictions. As shown in Fig. (**1**), in this respect, the probability density function of simulated sea states with relatively broad directional spreading matches second order based distribution reasonably well even though the negative

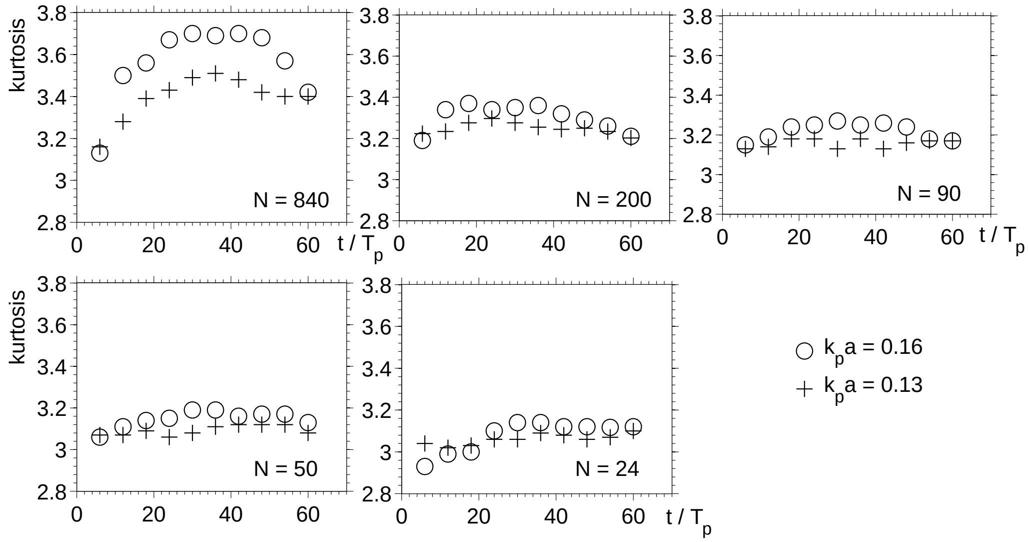

Fig. (2). Temporal evolution of the kurtosis.

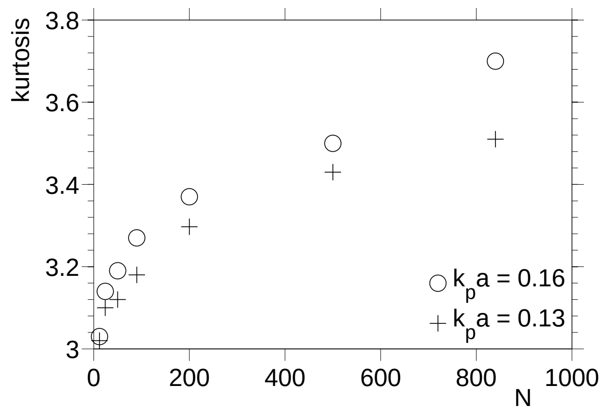

Fig. (3). Kurtosis as a function of the directional spreading coefficient.

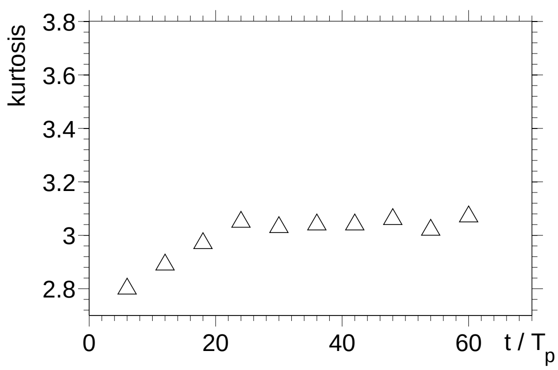

Fig. (4). Temporal evolution of the kurtosis for a sea state with steepness $k_p a = 0.21$ and $N = 24$.

elevations appears to be slightly deeper. This result is consistent with recent laboratory experiments on random waves in a directional wave tank [43]. It is important to note that the broadest directional wave field considered in this study (i.e. $N = 12$) still represents a narrow directional spectrum if compared with commonly observed directional wave spectra of wind generated waves. Nonetheless, narrow spectra ($N \geq 12$) are not uncommon, especially during tropical storms [44].

It is reasonable to expect that an increase of the steepness would increase the kurtosis also in directional wave field as observed for long crested waves. However, a few tests performed with even higher steepness (see an example in Fig. (**4**) for a steepness $k_p a = 0.21$, $\gamma = 6$, and $N = 24$), even though unrealistic, show that the kurtosis does not overcome the second order prediction if the initial spectrum is sufficiently broad in the directional domain.

It is important to remark that we performed the present simulations with a frequency-independent directional function. However, the lateral shape of the directional spectrum can be described by a number of different spreading functions (see, for example, [45] for a review). Nonetheless, because unstable modes are mainly distributed around the spectral peak [46], the form of the directional spreading function is not expected to have a significant effect on the results, provided the initial spreading is kept constant at the spectral peak. To support this conjecture, we have repeated the simulations of directional wave fields with initial steepness $k_p a = 0.16$ with a $cos^{2s(\omega)}(\vartheta)$ function, where the spreading coefficient $s(\cdot)$ is a function of the angular frequency ω (see [47] and references therein). This function has the same directional spreading at the spectral peak (provided $N = 2s(\omega_p)$), but becomes broader at higher frequency. In Fig. (**5**), we show the maximum kurtosis as a function of the directional spreading coefficient at the spectral peak (i.e. N and $2s(\omega_p)$). Results confirm that the percentage of extreme waves does not depend upon the shape of the directional function. Substantial variations of the kurtosis are only observed as a consequence of different directional spreading at the spectral peak (see also [48]).

4. CREST, TROUGH AND HEIGHT DISTRIBUTION

It is now instructive to show the statistical distributions of wave crests, troughs and heights as they are often used for practical applications. Because it is not clear how to extract individual waves from a two-dimensional surface, here we

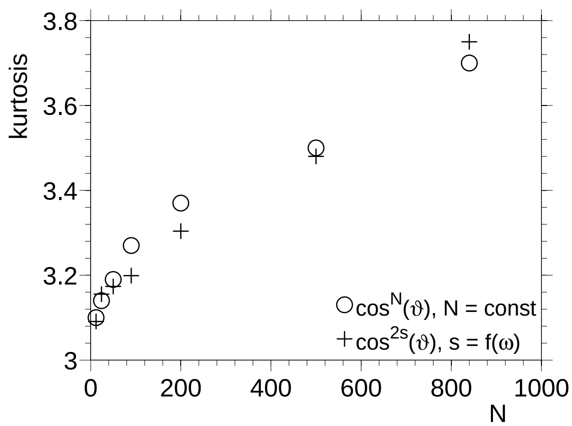

Fig. (5). Maximum kurtosis as a function of the directional spreading coefficient N computed with two different directional distributions; initial $k_p a = 0.16$.

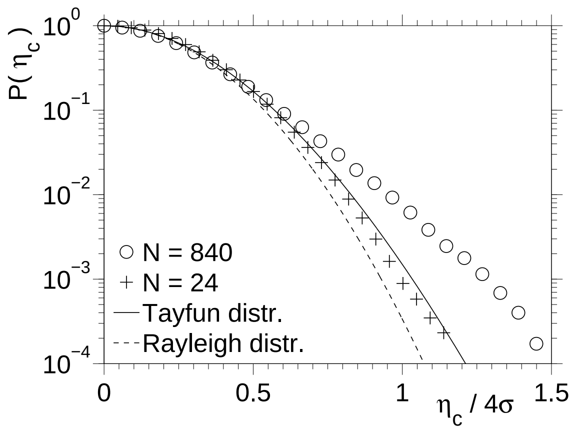

Fig. (6). Wave crest distribution for a sea state with $k_p a = 0.13$ (or $BFI = 0.7$).

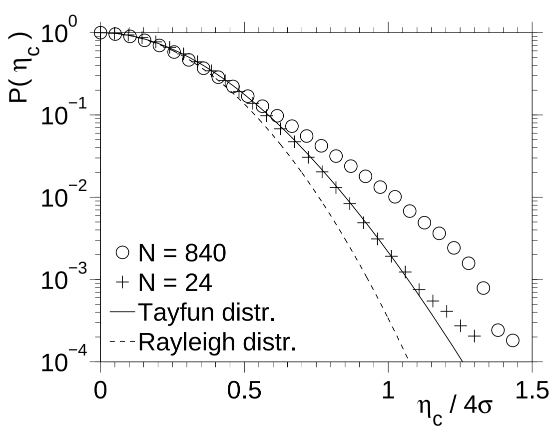

Fig. (7). Wave crest distribution for a sea state with $k_p a = 0.16$ (or $BFI = 1.1$).

calculated the individual amplitudes (i.e. crests, troughs and heights) from the recorded time series. A zero down-crossing analysis was used to this end. A similar approach was applied in [37]. Note that the estimate of the probability distribution of wave amplitudes are rather accurate also at low probability levels. For example, the 95% confidence intervals estimated with bootstrap methods indicate a

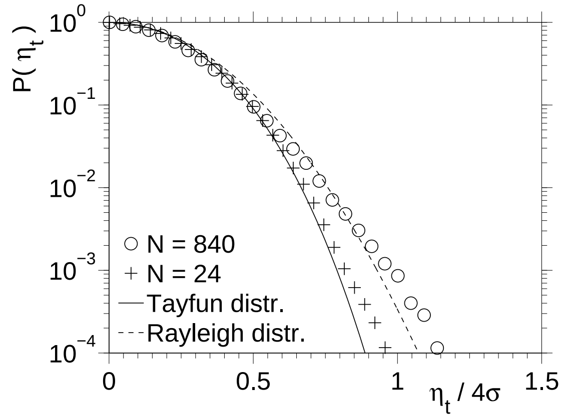

Fig. (8). Wave trough distribution for a sea state with $k_p a = 0.13$ (or $BFI = 0.7$).

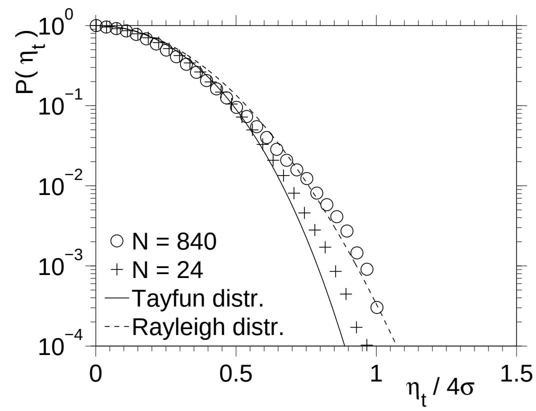

Fig. (9). Wave trough distribution for a sea state with $k_p a = 0.16$ (or $BFI = 1.1$).

variability of the dimensionless amplitude of ±0.05 at probability levels as low as 0.0001.

In Figs. (**6**) and (**7**), the wave crest distribution is shown; the Rayleigh distribution for linear waves and the Tayfun distribution [12] for second order waves are presented as reference. When the energy spectrum is concentrated on a narrow range of directions, modulational instability is responsible for notable increase of the kurtosis (Fig. **2**). As a result, extreme crests occur far more often than in linear and second order theory. The wave crest distribution substantially deviates from both theoretical distributions already at probability level of 0.01. When the energy spreads over a larger number of directions, however, extreme wave crests are no more frequent than in second order theory. For a sea states with steepness $k_p a = 0.16$ (or $BFI = 1.1$), nevertheless, the wave crests weakly deviate from the Tayfun distribution at probability levels as low as 0.005. Note that our simulations are qualitatively consistent with laboratory experiments in [43]. It should also be mentioned that the use of the Generalized Pareto Distribution (GPD) has recently been proposed to describe extreme crests in short term statistics [49]. An attempt to fit the GPD to our simulated wave crests has not been satisfactory though. The best fit has significantly underpredicted (beyond the 95% confidence intervals) long crested wave fields and overpredicted short crested waves.

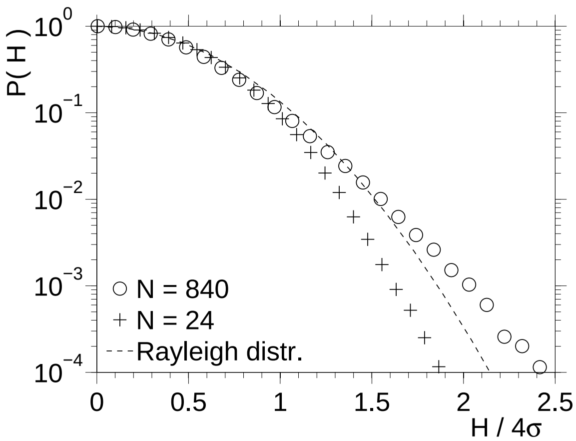

Fig. (10). Wave height distribution for a sea state with $k_p a = 0.13$ (or $BFI = 0.7$).

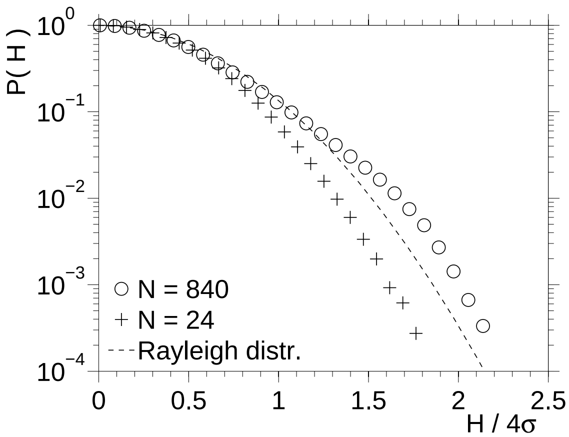

Fig. (11). Wave height distribution for a sea state with $k_p a = 0.16$ (or $BFI = 1.1$).

Substantial deviations from theoretical distributions were also observed for wave troughs. In Figs. (**8**) and (**9**), the wave trough distribution is presented together with the Rayleigh and second order distribution (details for the latter can be found in [50]). As observed for wave crests, strong deviations occur in long crested wave fields. In this respect, the troughs are slightly deeper than linear wave theory would predict. When the energy spreads over a number of directions, however, wave troughs become less deep. The concurrent distribution migrates towards a second order based distribution at the decreasing of the directional spreading coefficient (i.e. spectrum becomes wider). Nonetheless, the simulated trough distribution still weakly departs from second order predictions for probability levels lower than 0.01. It is interesting to mention that this deviation is observed for both initial sea states.

In Figs. (**10**) and (**11**), the wave height distribution is presented. Both trough-to-crest and crest-to-trough heights were used to produce an estimate of the probability distribution. Because second order theory does not have any significant effect on the waves height, it is reasonable to suppose that the wave height distribution would fit the Rayleigh distribution, provided the wave spectrum is narrow banded in the frequency domain. However, when waves are long crested, the modulational instability produces higher waves than in linear theory. Thus, the Rayleigh distribution notably underpredicts the simulated wave height. In these circumstances, a proper description of the wave height distribution can be conveniently provided by a modified Edgeworth-Rayleigh distribution (see [51,52] for details). As the effect of modulational instability is suppressed by the increased of directional spreading, nonetheless, the amplitude of the wave height is reduced. Note that, because the input spectra does not really satisfy the narrow banded hypothesis, under which the Rayleigh distribution is applicable, the Rayleigh distribution results in an overprediction of the simulated wave height; this finding is consistent with results in [53].

5. THE EVOLUTION OF THE DIRECTIONAL WAVE SPECTRUM

The nonlinear interaction between wave components is responsible for an energy transfer across frequencies and directions. In Fig. (**12**), the directional wave spectrum at different times is presented for a wave field with initial $k_p a = 0.16$ and $N = 24$. The spectrum was calculated from the output surface elevation using a Fast Fourier Transform; an ensemble average over the 150 random repetitions was considered (no smoothing was applied).

As the wave field evolves a large fraction of the spectral energy is moved towards lower wavenumbers (or wave frequencies), generating the downshift of the spectral peak. Because the energy remains constant throughout the simulations and the peak period increases due to the downshift, the wave field becomes less steep. Furthermore, a fraction of the energy is also transferred across directions. In particular, the energy is redirected along two main directions forming and angle of $\pm 35.5^\circ$ with the mean direction of propagation (see [47] and references therein for details). The directional redistribution produces a significant broadening of the wave spectrum, which is more accentuated at high wavenumbers. Thus, although the initial directional spreading function is frequency-independent, the wave field develops a frequency-dependent directional spreading.

The reduction of wave steepness and the broadening of the wave spectrum result in an attenuation of the non-Gaussian properties. Therefore, strong deviation from Normality may eventually vanish also for steep and long crested wave fields if the waves can evolve for a long period. In order to verify this conjecture, we traced the evolution of a long ($N = 200$) and a short ($N = 24$) crested wave field for a period of $150T_p$. The temporal evolution of the kurtosis is presented in Fig. (**13**). For long crested wave fields, non-Gaussian properties are evident soon after the wave field starts propagating. In this respect, the kurtosis shows a first overshooting with a maximum at $18T_p$ (cf. Fig. **2**), after which it decreases. Owing to the recurrent nature of modulational instability, the kurtosis also shows subsequent peaks. However, as the wave spectrum gradually changes due to the nonlinear wave interaction, the wave field is not longer able to sustain strong non-Gaussian properties in the long period. Therefore, subsequent peaks reach substantially lower maxima than in the first overshooting.

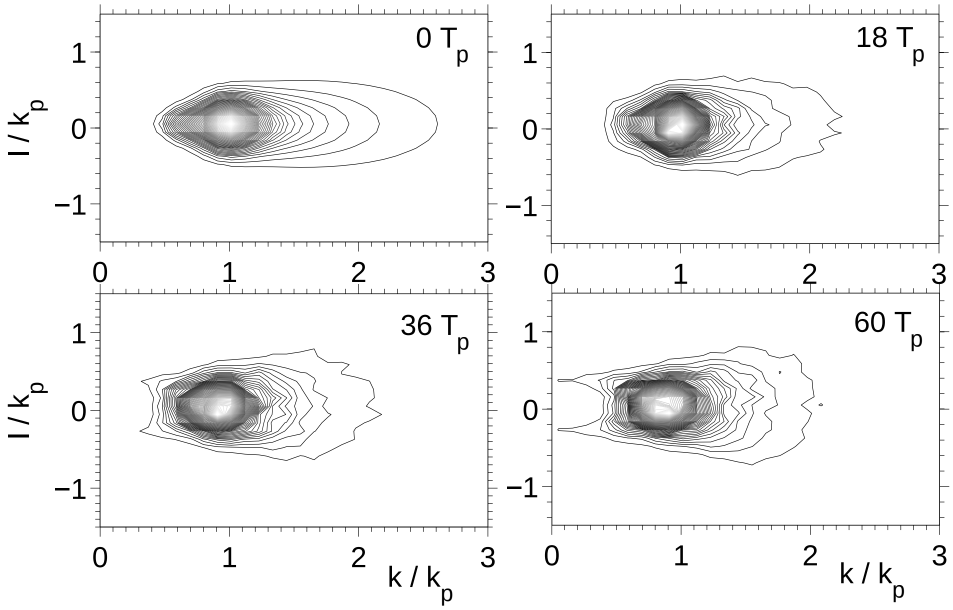

Fig. (12). Evolution of the directional wave spectrum (initial $k_p a = 0.16$ and $N = 24$).

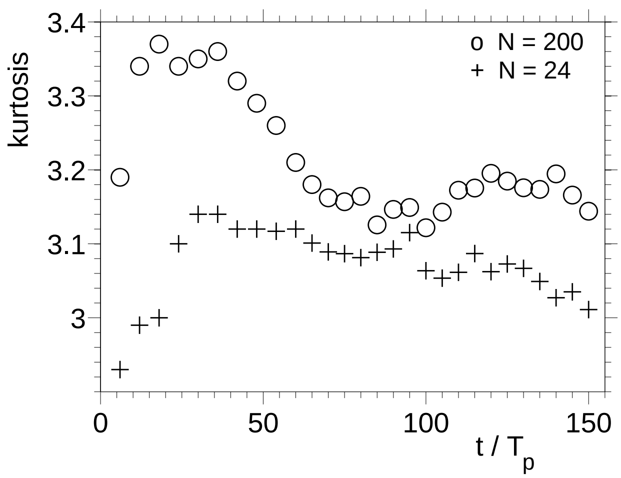

Fig. (13). Long term evolution of wave fields with initial $k_p a = 0.16$.

For short crested conditions, the kurtosis only weakly deviates from the Gaussian values as a result of bound wave contribution (i.e. second order interaction). Modulational instability does not have any significant effect of wave statistics, even for a long evolution time. It is important to mention that the nonlinear dynamics of free waves is still present in short crested wave fields (see spectral changes in Fig. **12**). However, this mechanism does not contribute to wave statistics significantly.

6. DISCUSSION AND CONCLUSIONS

Although enhancing safety at sea through specification of uncertainties related to environmental data and models is today one of the main concerns of the offshore and shipping industry, extreme waves still remain an issue. An extended knowledge about extreme and rogue waves, and in particularly their probability of occurrence and marine structures behaviour in them, is mandatory for evaluation of possible revision of classification societies' rules [54]. Further, a consistent risk-based approach combining new information about extreme and rogue waves in a design perspective needs to be proposed [55]. Although the Norwegian offshore standards (NORSOK Standard [56]) take into account extreme wave conditions by requiring that a 10000-year wave does not endanger the structure integrity (Accidental Limit State, ALS), there is no consensus within the offshore industry on wave models for the prediction of extreme and rogue waves and design scenarios to be included in a possible ALS control procedure.

Statistical characteristics of the sea surface like skewness and kurtosis as well as the wave crest, trough and height distribution will need to be a part of the risk-based approach. Further, understanding of implications of wave directionality on statistical properties of the surface elevation is essential for design work, e.g. for specification of an air gap for offshore installations. Today the second-order theory-based wave statistics like, for example, the Forristall distributions for wave crest [14], is commonly used in design practice (see, e.g., [57]). However, second order models do not include effects related to the nonlinear dynamics of free waves, i.e. modulational instability, which is responsible for the formation of large amplitude waves. Here we used the potential Euler equations to investigate comprehensively the effect of modulational instability in directional wave fields. Our results clearly show that when the wave energy spreads on a wider range of directions the effect of modulational instability is gradually suppressed and the second order wave theory is adequate to describe the statistical behaviour of ocean waves. This result is consistent with tank experiments [43]. Furthermore, field data also suggest that extreme events in broad-banded waves are no more frequent than commonly applied statistical models would predict (e.g. [58]).

Nonetheless, we cannot exclude that the wave spectrum can sometimes become more narrow banded both in the frequency and directional domain (see the discussion in [59]). This would represent a potentially dangerous situation as the occurrence of extreme waves is more likely under these circumstances. In this respect, an investigation of directional wave spectra measured during tropical storms [44] indicates that the spectral peak may become as narrow as our case with $N = 50$. Moreover, owing to their narrow banded nature, swells might develop instability and hence deviate from second order predictions provided they retain sufficient energy. The occurrence of energetic swells is not unlikely along for example the west coast of Africa. Furthermore, under the influence of rising wind speed, a coexisting swell may also grew exponentially at the expense of the wind sea, potentially leading to an energetic narrow banded sea state [60]. Thus, the need for detailed investigations of meteorological and oceanographic conditions in which extreme and rogue waves may occur is vital as already pointed out by several authors at the Rogue Waves 2008 Workshop in Brest. There is also a need for more field data to study extreme waves in the ocean. Attentions should be given to the development of the directional distribution of the wave spectrum during storm conditions. Possibly this should be done considering very short time frame in order to investigate whether local narrowing of the directional spectrum are a realistic and robust feature of the ocean.

It should be mentioned that the present study is limited to deep water and to one generation mechanism for rogue waves, i.e. the modulational instability. The shown solution is not fully nonlinear. An external forcing like wind, wave breaking and current are not included in the numerical solution applied. Further, the existing knowledge about wave directional spreading models may have some deficiencies. These are the limitations which call for further investigations.

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