Prediction on Deflection of V-core Sandwich Panels in Weak Direction

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Abstract: A V-core sandwich panel consists of two facing plates and a core of V-shaped stiffener. Sandwich panels of this type have high bending and shear-stiffness characteristics in the direction of the core placement. In the transverse direction, however, stiffness, especially the shear stiffness, is relatively weak. In this paper, the shear stiffness in the weak direction for V-core sandwich beams is analyzed. Due to structural geometrical compatibility, a theoretical expression for calculating the deflection in its weaker direction of the V-core sandwich beam is derived. The presented equation takes into account the contact interaction between the facing plates and the flanges of the V-core stiffener. A detailed comparison between the theoretical and numerical results is conducted. Good agreement is obtained between the two results when geometrical parameters are in a valid range, and suitable geometrical range is finally presented based on error analysis.

Keywords: V-core sandwich beam, deflection, weak direction, theoretical equation, finite element analysis.

INTRODUCTION

Sandwich panel structures (SPS), whose high flexural-to-stiffness and strength-to-weight ratios make them an attractive alternative for traditional ship structures, now are more and more applied in various industrial branches, such as building and aircraft industry. A sandwich panel is consisted of two facing sheets and core stiffeners, which are connected to form a composite structure. The core stiffeners can be designed with different materials, such as foam or steel. In a steel core sandwich panel, the type of cores varies in many kinds of forms, among which are C-cores, I-cores, Z-cores, X-cores, and box-cores etc.

Considering that the steel core of the sandwich panel is placed in only one direction, the entire sandwich panel has a strong direction and a weak direction respectively. In the direction of the core stiffener placement, all the rigidities including the shear rigidity, the flexural rigidity and the torsion rigidity are higher, and this direction is called the strong direction. In the direction perpendicular to the direction of the core stiffener placement, however, the shear stiffness is much smaller due to the fact that the stiffeners are not continuous in this direction. Therefore, this direction is called the weak direction. The connection between the core stiffeners and the facing sheets can be achieved by mechanical means such as spot welds, rivets or self-tapping screws. The core stiffeners can transmit shear forces as well as help the facing plates resist bending and local buckling. Hence, it is necessary to know the stiffness to assess the behavior of a steel core sandwich panel.

In the literature, many researchers have conducted numerous works in this field. Using finite-element method, the facing plates and the core stiffeners are modelled as an assemblage of thin plates, and the connectors are modelled as short beams. This requires high computational effort. To simplify the idealization, Tan *et al.* (1993) use the simplified thick plate method to analyze the structural properties of the all-steel sandwich panels [1]. Kang *et al.* (2004) calculated the elastic and shear module of a core in a sandwich beam with a Kagome truss structure theoretically [2]. Hazizan *et al.* (2003) estimated the elastic and shear module of a core in a sandwich beam with an aluminum honeycomb using low-velocity impact tests and deflection theory with the sandwich beam without considering the core stiffness [3]. Fung *et al.* (1994, 1996, 1998) proposed a method for calculating the shear stiffness of C-core and Z-core sandwich panels based on compatibility and recurrence conditions between adjacent segments [4-6].

Some other methods can be found in references [7-9]. In this paper, a simple method is presented to calculate the deflection of a V-core sandwich panel under flexure in weak direction.

THEORETICAL ANALYSIS OF DEFLECTION FOR A V-CORE SANDWICH PANEL

Typical Segment of a V-core Sandwich Panel

A V-core sandwich panel, as shown in Fig. (**1**), is consisted of two facing plates and a series of V-shaped core stiffeners. Fig. (**2**) provides a procedure of laser weld, which is used to connect the facing plates and the stiffeners together.

To describe a V-core sandwich panel more clearly, some definitions are given in Fig. (**3a**). The direction of the V-core placement is in x-axis, which is called the strong direction. Y-axis is called the weak direction because shear stiffness in

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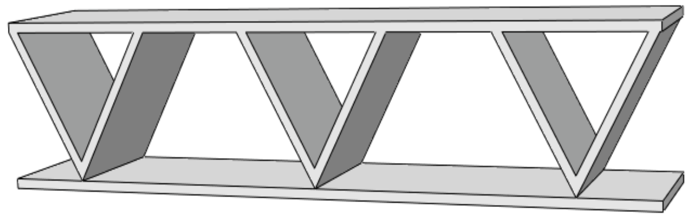


Fig. (1). A V-core sandwich panel.

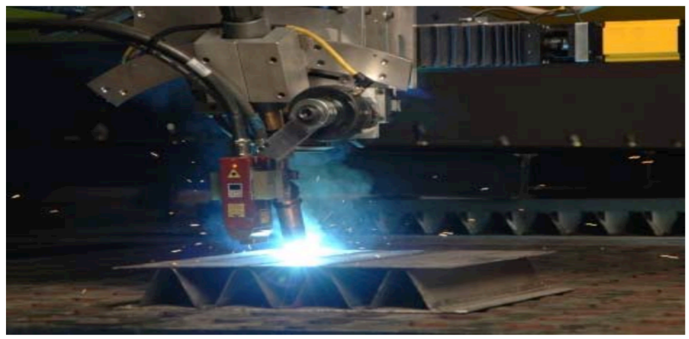


Fig. (2). Laser weld of a V-core sandwich panel.

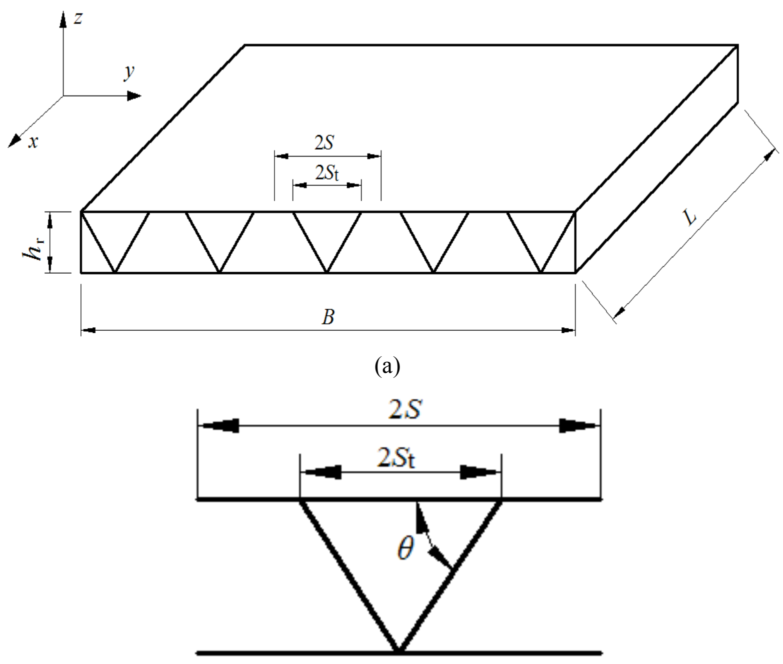


Fig. (3). Definitions and a typical segment of V-core sandwich panel

this direction is weaker due to the hollow web between any two V-core stiffeners. The distance between two adjacent V-cores is denoted by $2(S\text{-}S_t)$, while the distance between the openings of V-cores is $2S_t$. The height between the mid-plane of the top and the bottom facing plates is denoted by h_f. The length and the width of the sandwich panel are expressed as L and B respectively. Accordingly, the deflection of the sandwich panel is assumed to only exist in z-axis. The thickness of the top and the bottom facing places are assumed to be identical, and they can be denoted by t_p, and the web thickness is denoted by t_w.

Recurrence condition is assumed in present study. If the V-core sandwich panel is subjected to uniform loading, the behavior of a typical segment is very similar to the one adjacent to it. In other words, the same segment can be placed next to itself and both the equilibrium and compatibility conditions are still satisfied.

A typical segment of a V-core sandwich panel, extracting from the entire structure in Fig. (**3a**), includes two facing plates and a V-core stiffener, as shown in Fig. (**3b**). Obviously, the widths of the typical segment and the V-core stiffeners are $2S$ and $2S_t$ respectively.

Theoretical Calculation of Deflection

If a V-core sandwich panel is under flexure about x-axis, the total deflection is consisted of two parts: deflection due

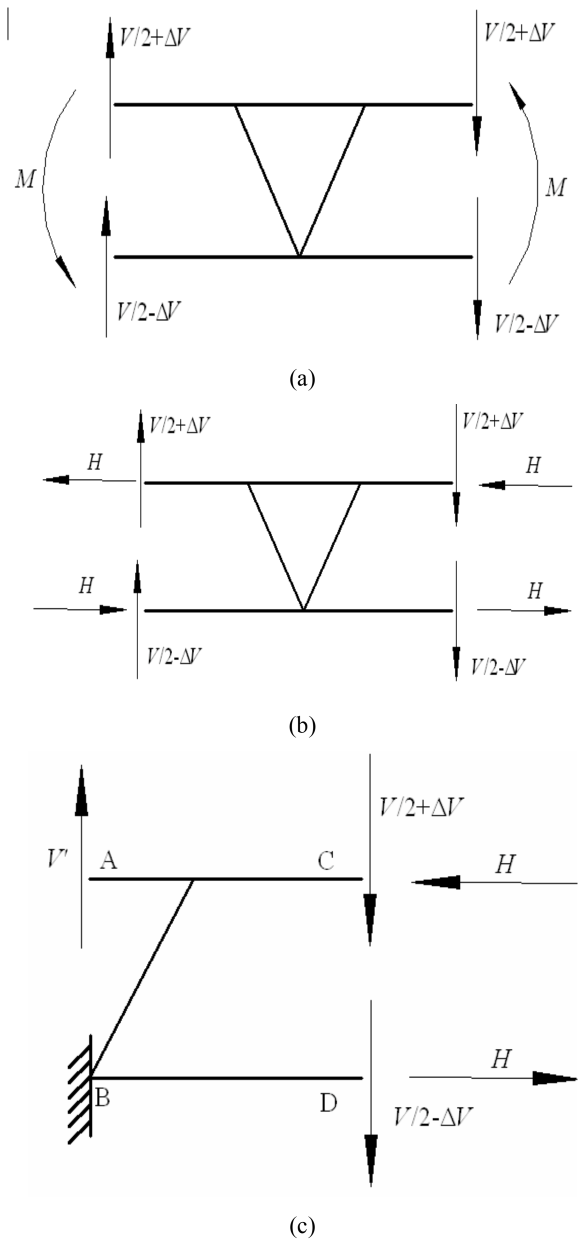


Fig. (4). Internal forces of the typical segment.

to shear w_1 and deflection due to bending w_2. The deflection due to flexural action can be obtained easily from fundamentals of material mechanics or structural mechanics.

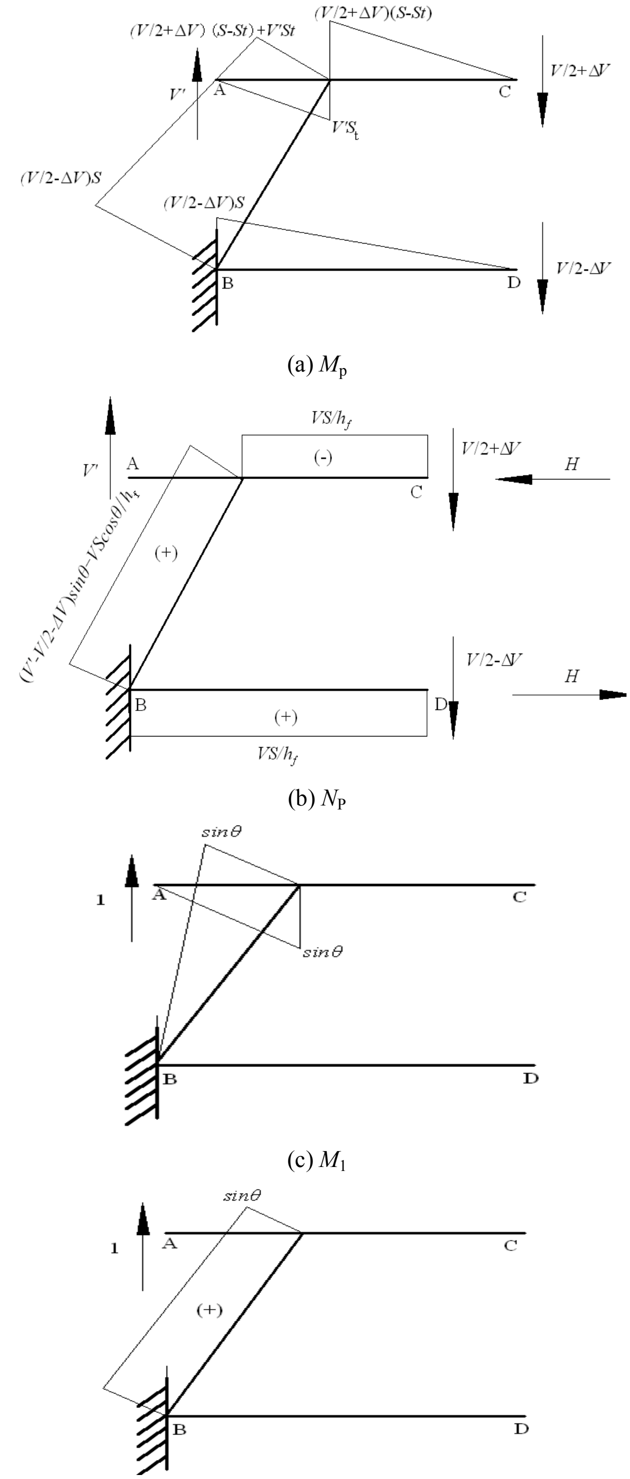
In calculating the deflection w_1 due to shear action, it is assumed that the deformation of each typical segment is identical. For the typical segment, it is assumed that the two ends of the top and bottom facing plates in Fig. (**3b**) are the positions of inflection-points in the V-core sandwich panel.

If a V-core sandwich panel is subjected to flexure about y-axis, the deflection caused by flexural action is dominant. Shear produces very smaller deflection, and thus its effect can be ignored. In this case, beam theory in materials of mechanics can be used to calculate such deflection.

Based on the above reasons, only deflection of a V-core sandwich panel under flexure in its weak direction is studied here.

Compatibility Conditions

For equilibrium at the interfaces of the typical segment, the shear forces and the bending moments at the two ends of the facing plates must be the same. The internal forces in general are shown in Fig. (**4a**). V and M are the shear force and the bending moment per unit width acting at the two ends of the segment respectively. $V/2+\Delta V$ denotes the shear force in the top plate, and $V/2-\Delta V$ denotes the shear force in the bottom plate. The total shear forces at the ends of the segment are V per unit width and the whole segment is in external equilibrium. For brevity in calculation, the bending moment M is denoted as a couple of forces H, as shown in Fig. (**4b**). From equilibrium equation, H can be determined as VS/h_f. From Fig. (**4b**), the forces of the typical segment are anti-symmetric and half segment is isolated from the typical segment, which is shown in Fig. (**4c**).



(a) M_p

(b) N_P

(c) M_1

(d) N_1

Fig. (5). Bending moment diagram and axial force diagram.

It is shown that there are two unknowns $V^{'}$ and ΔV in Fig. (**4c**). Two equations are necessary to solve out the two unknowns. The compatibility conditions are listed as follows:

1. No relative vertical displacement at point A

$$w_A = 0 \tag{1}$$

2. Equal relative vertical displacement at the two ends C and D

$$w_C = w_D \tag{2}$$

Deflection Caused by Shear Force

The bending moment *(M_p, M_1)* and the axial force (N_P, N_1) diagrams are drawn and shown in Fig. (**5**). Using unit load method and the assumption 1, w_A can be calculated and expressed as follow:

$$w_A = \Delta_{MM_1} + \Delta_{NN_1}$$
$$= \sum \int \frac{MM_1}{EI} ds + \sum \int \frac{NN_1}{EA} ds = 0 \tag{3}$$

Eq. (3) is finalized as follow:

$$\left(\frac{2S_t^3}{EI_f} + \frac{2h_f S_t^2}{EI_c \sin\theta} + \frac{6\sin^2\theta}{EA_c}\right)V' + \left[\frac{h_f S_t}{EI_c \sin\theta}\left(3S - 2S_t\right) - \frac{6\sin^2\theta}{EA_c}\right]\Delta V$$
$$= \left[\frac{h_f S_t}{EI_c \sin\theta}\left(S_t - \frac{S}{2}\right) + \frac{3\left(\sin^2\theta + \sin 2\theta \times \frac{S}{h_f}\right)}{EA_c}\right]V \tag{4}$$

Using similar method and basing on assumption 2, w_c - w_D can also be calculated. Its final expression is listed as follow:

$$\left[\frac{\left(3S - 2S_t\right)h_f S_t}{2EI_c \sin\theta} - \frac{3\sin^2\theta}{EA_c}\right]V'$$
$$+ \left\{\frac{\left(S - S_t\right)^3 + S^3}{EI_f} + \frac{\left(3S^2 - 3SS_t + S_t^2\right)h_f}{EI_c \sin\theta} + \frac{3\sin^2\theta}{EA_c}\right\}\Delta V \tag{5}$$
$$= \left\{\frac{S^3 - \left(S - S_t\right)^3}{EI_f} - \frac{\left(S_t - 2S\right)S_t h_f}{2EI_c \sin\theta} - \frac{3\sin\ \theta}{2EA_c}\left(\sin^2\theta + \frac{S}{h_f}\sin 2\theta\right)\right\}V$$

For brevity, Eq. (4) and Eq. (5) can be expressed as follows:

$$C_1 V' + C_2 \Delta V = C_3 V \tag{6}$$

$$D_1 V' + D_2 \Delta V = D_3 V \tag{7}$$

Then V' and ΔV can be obtained from Eq. (6) and Eq. (7), and they are expressed as follows:

$$V' = \frac{C_3 D_2 - C_2 D_3}{C_1 D_2 - C_2 D_1} V \tag{8}$$

$$\Delta V = \frac{C_3 D_1 - C_1 D_3}{C_1 D_2 - C_2 D_1} V \tag{9}$$

where

$$C_1 = \frac{2S_t^3}{EI_f} + \frac{2h_f S_t^2}{EI_c \sin\theta} + \frac{6\sin^2\theta}{EA_c} \tag{10a}$$

$$C_2 = \frac{\left(3S - 2S_t\right)S_t h_f}{EI_c \sin\theta} - \frac{6\sin^2\theta}{EA_c} \tag{10b}$$

$$C_3 = \frac{\left(2S_t - S\right)S_t h_f}{2EI_c \sin\theta} + \frac{3\left(\sin^2\theta + \frac{S}{h_f}\sin 2\theta\right)}{EA_c} \tag{10c}$$

$$D_1 = \frac{\left(3S - 2S_t\right)S_t h_f}{2EI_c \sin\theta} - \frac{3\sin^2\theta}{EA_c} \tag{10d}$$

$$D_2 = \frac{\left(S - S_t\right)^3 + S^3}{EI_f} + \frac{\left(3S^2 - 3SS_t + S_t^2\right)h_f}{EI_c \sin\theta} + \frac{3\sin^2\theta}{EA_c} \tag{10e}$$

$$D_3 = \frac{S^3 - \left(S - S_t\right)^3}{2EI_f} - \frac{\left(S_t - 2S\right)S_t h_f}{2EI_c \sin\theta} - \frac{3\left(\sin^2\theta + \frac{S}{h_f}\sin 2\theta\right)}{2EA_c} \tag{10f}$$

The deflection of each typical segment caused by the shear force can be calculated by the following equations:

$$w_0 = w_C + w_D$$
$$= \left[\frac{\left(3S - 2S_t\right)h_f S_t}{6EI_C \sin\theta} - \frac{\sin^2\theta}{EA_c}\right]V'$$
$$+ \left[\frac{\left(S - S_t\right)^3 - S^3}{3EI_f} + \frac{h_f}{6EI_C \sin\theta}\left(3S^2 - 3SS_t + S_t^2\right) + \frac{\sin^2\theta}{EA_c}\right]\Delta V \tag{11}$$
$$+ \left[\frac{\left(S - S_t\right)^3 + S^3}{6EI_f} + \frac{\left(S_t - 2S\right)h_f S_t}{6EI_C \sin\theta} + \frac{\sin^2\theta + \frac{S}{h_f}\sin 2\theta}{2EA_c}\right]V$$

Deflection Caused by Bending

The deflection of a V-core sandwich panel caused by bending can be simply calculated from classical beam theory, i.e., for a cantilever beam, the deflection can be calculated from the following equation:

$$w' = \frac{VL^3}{3EI_2} \tag{12}$$

Overall Deflection

Combing shear deflection and flexural deflection, the overall deflection of a V-core sandwich beam is calculated from superposition of Eq. (11) and Eq. (12) with the consideration of number of segment. Considering that beam is a problem of plane stress while plate is in plane strain condition, elastic modulus E in the equations should be replaced by $E/(1-v^2)$.

CASE STUDY

A typical V-core sandwich cantilever beam, as shown in Fig. (**6**), is selected to for analysis. The sandwich beam has 9 V-cores and its total length is ($16S$+ $2S_t$). The facing plates and the V-cores are made of same materials with elastic modulus E=206GPa and Poisson's ratio v=0.3. A vertical line load with a value of F=-1N/m is located at beam end. Finite element software ABAQUS is used to carry out the numerical analyses. As is known that fixed end and loading place have much influence on the deformation of a sandwich panel due to stress concentration, the fixed end and the last unit of the cantilever plate are ignored when the presented equation is used. Considering the deflection of the remaining 7 units, the displacement can be calculated theoretically.

Influence of Thickness Ratio t_c/t_p,

Considering the influence of the thickness ratio t_c/t_p, overall 12 models have been chosen for analysis, in which the thickness of the facing plates for each model is defined

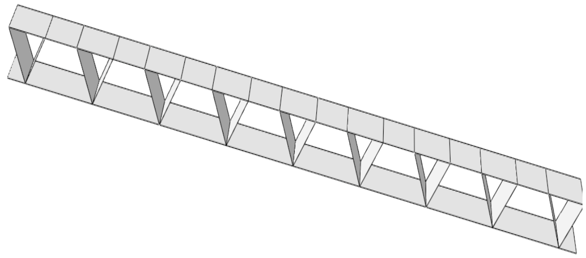


Fig. (6). A V-core sandwich beam

Table 1. Dimensions of Sandwich Beams

Model	t_p mm	t_c mm	S_t mm	S mm	L mm	h_f mm	Θ degree
A_1	2.0	1.0	34.65	51.975	900.9	60	60
B_1	2.0	1.5	34.65	51.975	900.9	60	60
C_1	2.0	2.0	34.65	51.975	900.9	60	60
D_1	2.0	2.5	34.65	51.975	900.9	60	60
E_1	2.0	3.0	34.65	51.975	900.9	60	60
F_1	2.0	3.5	34.65	51.975	900.9	60	60
G_1	2.0	4.0	34.65	51.975	900.9	60	60
H_1	2.0	5.0	34.65	51.975	900.9	60	60
I_1	2.0	6.0	34.65	51.975	900.9	60	60
J_1	2.0	7.0	34.65	51.975	900.9	60	60
K_1	2.0	10.0	34.65	51.975	900.9	60	60
L_1	2.0	15.0	34.65	51.975	900.9	60	60

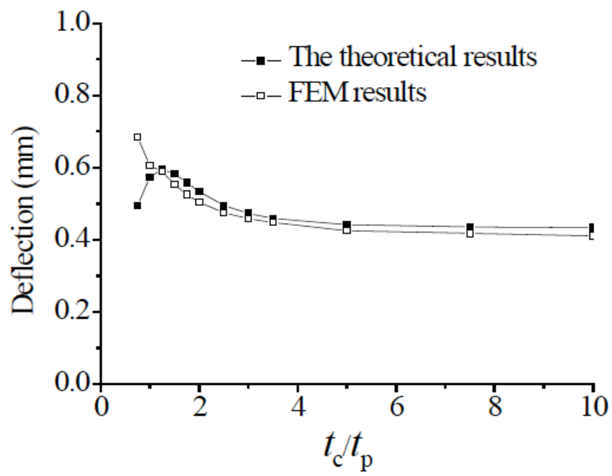
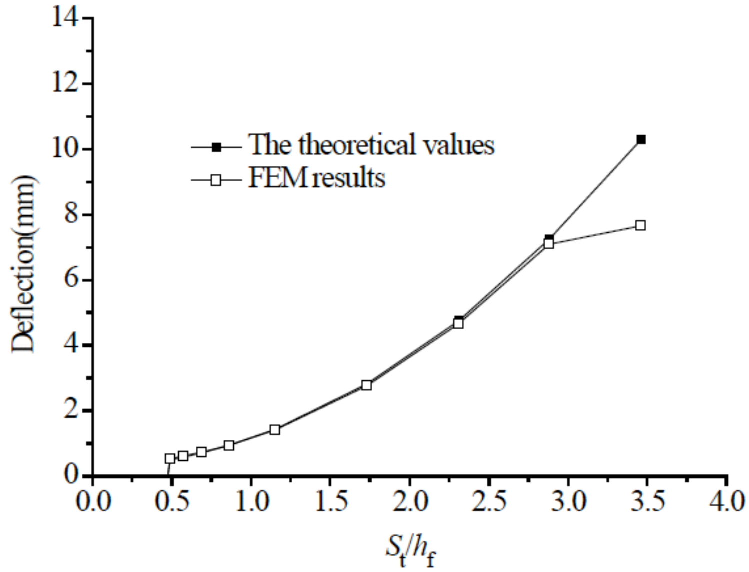


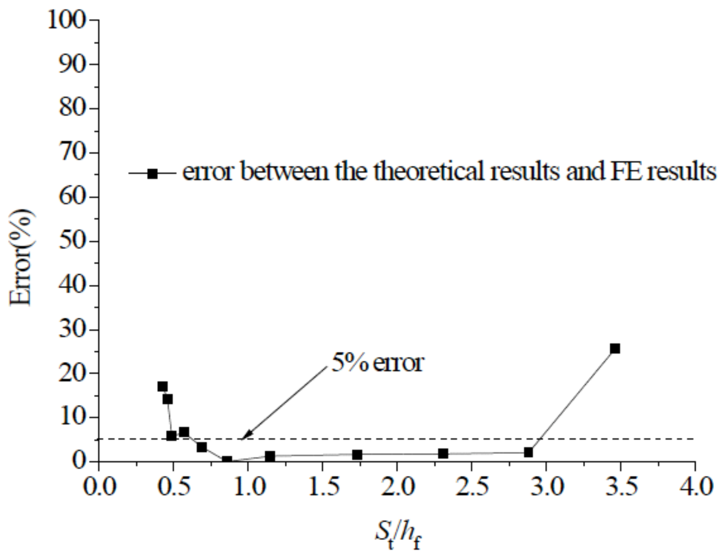
Fig. (7). Comparison of deflection.

as t_p, the thickness of the V-core steel web is t_c, and the length of each typical V-core is $2S$, as is tabulated in Table **1**.

The finite element results and the theoretical predictions of the deflection for 7 V-cores are plotted together in Fig. (**7**). From the comparison, it can be concluded that the presented theoretical equation is reasonably accurate for predicting the deflection of a V-core sandwich beam when the t_c/t_p ratio is not less than 1.0. When this ratio is much smaller than 1.0, the theoretical value is not yet accurate. This may be due to the fact that the recurrence conditions are not satisfied in this case. In overall, the presented equation is reliable when such thickness ratio is not too small.



(a) Deflection of the theoretical and FE results



(b) Error between the theoretical and FE results

Fig. (8). Comparison of the deflection and errors of the V-core sandwich beam.

Influence of Ratio S_t/h_f

In order to study the influence of the ratio S_t/h_f, 11 models have been analyzed, and the dimensions of all the models are tabulated in Table **2**. The comparison between theoretical values and FE results is plotted in Fig. (**8**).

From Fig. (**8**), it is found that the deflection grows rapidly with the ratio S_t/h_f increasing regularly. When the ratio S_t/h_f ranges from 0.5 to 3.0, good agreement is obtained between the theoretical predictions and the FEM results, in which the error is acceptable within a small percentage less than 5%. On the contrary, when the S_t/h_f ratio is greater than 3.0 or less than 0.5, the difference between the theoretical values and the FEM results becomes larger, resulting in inaccurate theoretical prediction. Therefore, $0.5< S_t/h_f<3.0$ is regarded as the reasonable validity range of the derived equation.

Influence of Ratio S_2/h_f

Similarly, the effect on the deflection of the V-core sandwich beam is observed by changing the value of S_2, which is shown in Fig. (**9**), and other parameters are kept as

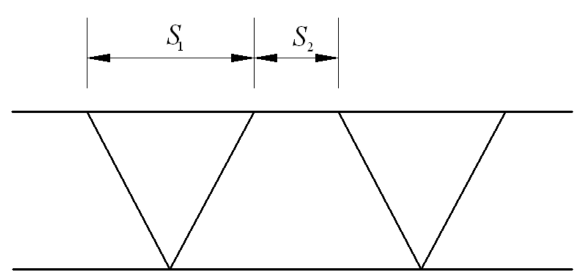


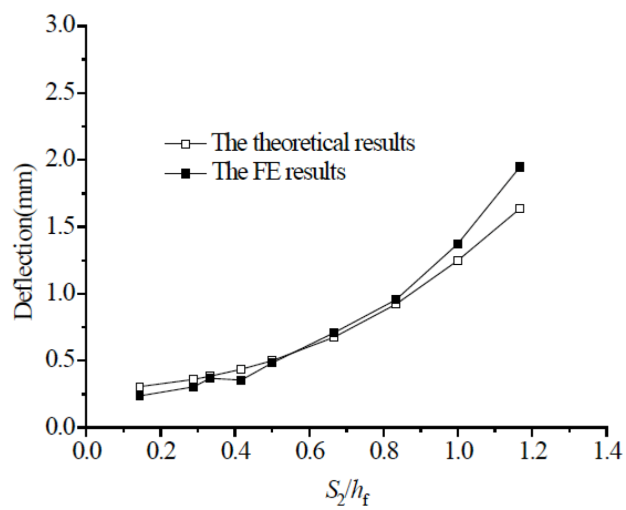
Fig. (9). The dimension of the V-core.

Table 2. Dimensions of Sandwich Beams

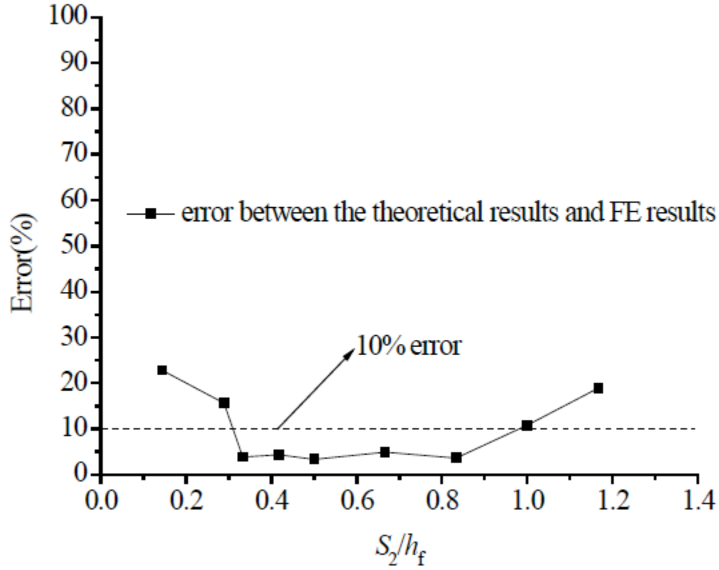
Model	t_p mm	t_c mm	S_t mm	S mm	L mm	h_f mm	Θ degree
A_2	2.0	2.0	34.65	51.975	900.9	10	60
B_2	2.0	2.0	34.65	51.975	900.9	12	60
C_2	2.0	2.0	34.65	51.975	900.9	15	60
D_2	2.0	2.0	34.65	51.975	900.9	20	60
E_2	2.0	2.0	34.65	51.975	900.9	30	60
F_2	2.0	2.0	34.65	51.975	900.9	40	60
G_2	2.0	2.0	34.65	51.975	900.9	50	60
H_2	2.0	2.0	34.65	51.975	900.9	60	60
I_2	2.0	2.0	34.65	51.975	900.9	70	60
J_2	2.0	2.0	34.65	51.975	900.9	75	60
K_2	2.0	2.0	34.65	51.975	900.9	80	60

Table 3. Dimensions of Sandwich Beams

model	t_p mm	t_c mm	S_1 mm	S_2 mm	L mm	h_f mm	è degree
A_3	2.0	2.0	69.3	8.66	693.0	60	60
B_3	2.0	2.0	69.3	17.32	762.3	60	60
C_3	2.0	2.0	69.3	20	783.7	60	60
D_3	2.0	2.0	69.3	25	823.7	60	60
E_3	2.0	2.0	69.3	30	863.7	60	60
F_3	2.0	2.0	69.3	40	943.7	60	60
G_3	2.0	2.0	69.3	50	1023.7	60	60
H_3	2.0	2.0	69.3	60	1103.7	60	60
I_3	2.0	2.0	69.3	70	1183.7	60	60



(a) Deflection of the theoretical and FE results



(b) Error between the theoretical and FE results

Fig. (10). Comparison of the deflection and errors of the V-core sandwich beam.

constants. As is tabulated in Table **3**, 9 models have been chosen for analysis. The comparison between the theoretical values and the FEM results is plotted in Fig. (**10**).

Fig. (**10**) shows that both the two kinds of values increase fast in accordance with the ratio S_2/h_f. The presented theoretical equation is reasonably accurate for calculating the deflection of the V-core sandwich beams when the ratio S_2/h_f ranges from 0.3 to 1.0, while the theoretical values agree not well with the FE results when the S_2/h_f ratio is greater than 1.0 or less than 0.3. Thus, $0.3<S_2/h_f<1.0$ is considered to be the suitable validity range, and in this range the error is not bigger than 10%.

CONCLUSIONS

Theoretical analysis of the deflection of a V-core sandwich beam in weak direction is carried out based on compatibility and recurrence conditions, and a corresponding equation for calculating the deflection is presented. Through comparison with finite element analyses, the accuracy and reliability of this equation is proved. Based on parametric study, the geometrical validity range of the V-core sandwich panel is also specified through error analysis.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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Declared none.

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