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Some Aspects of the Shock Wave in Pair Plasma

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Abstract: This article presents an analytic investigation to calculate the width of the shock in pair plasma. A solitary electromagnetic wave is propagated obliquely in the plasma where the wave direction is characterized by a propagation angle from direction of a background (constant) magnetic field. Based on Sagdeev method, the shock wave is determined by oscillation of pseudo--particle in pseudo--potential well. It is shown that for any fixed value of the propagation angle, the width of the shock becomes an explicit function of *Alfe'vn Mach number* of plasma. Graphs of the width of the shock versus Mach number represent a rapid montonous decreasing functions.

Keywords: Pair plasma, Sagdeev potential, shock wave, solitary wave.

1. INTRODUCTION

Ordinary plasmas consist of electrons and positive ions, and the mass difference between negative- and positive-charged particles essentially causes temporal and spatial varieties of collective plasma phenomena. Pair plasmas, i.e. plasmas consisting of equal mass and absolute charge ions of opposite charge polarity, have been investigated experimentally [1-3] and theoretically [4]. Also, they have recently attracted considerable interest among plasma researchers. Such plasmas exist for instance, in the form of electron--positron (e-p) or electron--positron-ion (e--p--i) plasmas, in pulsar magnetospheres, e. g. [5], in active galactic nuclei (AGN) [6, 7] and other mediums. The physics of pair plasmas was turned into an even more exciting field of investigation when it descended from its astrophysical heights to the terrestrial laboratory. For example, pair plasmas are also of relevance in inertial confinement fusion schemes using ultra-intense lasers [3, 8].

On the other hand, research in plasma physics devotes much attention to nonlinear phenomena. There are two different approaches to study of nonlinear phenomena. The first, which is one of the known approaches and widely employed to investigate the asymptotic behavior of nonlinear excitations, is the so-called reductive perturbation technique [9-13]. The second is Sagdeev (pseudo)-potential approach which is extremely suitable for studying the large amplitude solitary waves in plasmas. One can derive all the one soliton results of perturbation methods and can compare it with the exact results obtained by the pseudo--potential method [14]. Indeed, Sagdeev potential is one particular notion that has become immensely important in soliton and shock researches.

In this work, Sagdeev potential method is applied as well for study of some aspects of the shock wave in pair plasma whose background is a constant magnetic field with a propagating electromagnetic field. The electromagnetic field is considered in the case of oblique propagation (of large amplitude) with respect to the constant magnetic field. The pair plasma is described by the Sagdeev potential and electromagnetic wave has the only dynamical component which (after normalization) plays role of a pseudo--particle. In the case of solitary waves, the pseudo--particle moves (a reciprocating motion) in the potential well and because of energy conservation, produces a soliton or shock. If there is a (little) energy dissipation, the pseudo--particle will be trapped in the well and it oscillates about a minimum point of the well. If, we define Width of the Shock (**WS)** as the length of the one perfect oscillation, then it can be computed by analogy with Harmonic oscillator motion. We will present this analytic computation here, and we will see the WS can be expressed as a function *Alfe'vn Mach number* M.

It must be noted that this possibility (i.e. expressing WS as continues function) is because of the analytical form of the Sagdeev potential. There is not such possibility when the analytic form is not available. But, we can still compute numerically WS in each point (a point-like diagram and not as a continues curve) which has been presented in our previous work [15][1]. The manuscript is organized as follows: Basic equations will be introduced in the next section. The main goal of the work (WS as continues function of Mach number) is achieved in section 3. The remarks and results are summarized in concluding section.

2. BASIC EQUATIONS AND PSEUDO--POTENTIAL

Consider a cold pair (electron-positron) plasma which includes a background constant magnetic field[2]. It is assumed

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[1] In this reference, WS is called as *shock wavelength*.

[2] Such plasmas has been studied in a number of works, e.g. [16] and our conventions and frame with the basic equations of this section are extracted from the reference.

that waves propagate along the x-axis of a reference frame and the static magnetic field is in the x-z plane, i.e. $\mathbf{B}_0 = B_{x0}\mathbf{e}_x + B_{z0}\mathbf{e}_z$ with $B_{x0} \geq 0$ and $B_{z0} \geq 0$ and also the propagation angle θ is in the interval $0 \leq \theta \leq 90^0$. The nonlinear dynamics is governed by the following equations:

$$\frac{\partial n_j}{\partial t} + \frac{\partial (n_j v_{jx})}{\partial x} = 0, \tag{1}$$

$$\frac{\partial \mathbf{v}_j}{\partial t} + v_{jx}\frac{\partial \mathbf{v}_j}{\partial x} = \pm\frac{e}{m}(\mathbf{E} + \mathbf{v}_j \times \mathbf{B}), \tag{2}$$

where n_j refers to the number densities of the positrons $(j = i)$ and of the electrons $(\mathrm{j = e})$. The vectorial quantity $\mathbf{v}_j$ is the respective fluid velocities and $\mathbf{E}$ and $\mathbf{B}$ are the total electric and magnetic fields which satisfy the Maxwell's equations

$$\mathbf{e}_x \times \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{B}}{\partial t} = 0\,, \tag{3}$$

$$\mathbf{e}_x \times \frac{\partial \mathbf{B}}{\partial x} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \mu_0 e(n_i \mathbf{v}_i - n_e \mathbf{v}_e) \tag{4}$$

$$\epsilon_0 \frac{\partial E_x}{\partial x} = e(n_i - n_e). \tag{5}$$

It is a common choice to consider the total electric and magnetic fields as $\mathbf{E} = (0, E_y, 0)$ and $\mathbf{B} = (B_{x0}, 0, B_z)$ thus, the magnetic component B_z can be considered as the only variable component in description of the plasma dynamics.

The nonlinear stationary structures allow the change of variable, namely $\xi = x - Mt$ where M is the Mach number. In that context, the motion equation (resulted from the above equations) for the normalized magnetic field $b = B_z / B_0$ is

$$\frac{d^2 b}{d\xi^2} = -\frac{d\psi(b)}{db}, \tag{6}$$

where $\psi(b)$ is the Sagdeev(pseudo)-potential and b defined as pseudo-particle. By the first integration of (6) we will have the first energy integral

$$\frac{1}{2}\Big(\frac{db}{d\xi}\Big)^2 + \psi(b) = 0, \tag{7}$$

where the Sagdeev potential is given by

$$\psi(b) = \frac{M^2(b-\sin\theta)^2\big((b+\sin\theta)^2 - 4(M^2 - \cos^2\theta)\big)}{2(2M^2 + \sin^2\theta - b^2)^2}. \tag{8}$$

To find out the WS later, we must calculate two following derivatives by (8), that is

$$\begin{aligned}\frac{\partial \psi}{\partial b} = \frac{4M^2}{(2M^2 + \sin^2\theta - b^2)^3}\big(b^3\cos^2\theta + 3b^2\\ (M^2 - \cos^2\theta)\sin\theta\\ -b\big(2M^4 + 4M^2\sin^2\theta - 2M^2\cos^2\theta - 3\cos^2\theta\sin^2\theta\big)\\ +(M^2 - \cos^2\theta)(2M^2 + \sin^2\theta)\sin\theta\big)\end{aligned} \tag{9}$$

and

$$\begin{aligned}\frac{\partial y}{\partial b} = \frac{4M^2}{(2M^2 + sin^2\theta - b^2)^3}(b^3 cos^2\theta + 3b^2(M^2 - cos^2\theta)sin\theta\\ -b(2M^4 + 4M^2 sin^2\theta - 2M^2 cos^2\theta - 3cos^2\theta sin^2\theta)\\ +(M^2 - cos^2\theta)(2M^2 + sin^2\theta)sin\theta)\\ \frac{\partial^2 y}{\partial b^2} = \frac{4M^2}{(2M^2 + sin^2\theta - b^2)^4}(3b^4 cos^2\theta + 4b^3\\ (M^2 - cos^2\theta)sinq + b^2(3cos^2\theta(2M^2 + sin^2\theta) -\\ 5(2M^4 + 4M^2 sin^2\theta - 2M^2 cos^2\theta - 3sin^2 q cos^2\theta))\\ +12b(M^2 - cos^2\theta)(2M^2 + sin^2\theta)sin\theta - (2M^2 + sin^2\theta)\\ (2M^4 + 4M^2 sin^2\theta - 2M^2 cos^2\theta - 3cos^2\theta sin^2\theta)).\end{aligned} \tag{10}$$

2.1. Solitary Waves Solution and Mach Number Allowed Values

It is easily to show that the values assigned by the Mach number should be confined to the interval where the positive (negative) sign corresponds to the positive (negative) soliton[3].

Inequality (11) ensures the existence of the positive soliton, if a negative soliton exists, but the inverse is not true. Indeed, there is a critical angle $\theta_c = 30^\circ$ so that for $\theta < \theta_c$, we will have both of type solitons, but for $\theta > \theta_c$ there is only positive soliton.

$$1 < M^2 \leq 2(1 \pm \sin\theta). \tag{11}$$

3. WS AS FUNCTION OF *M*

Energy integral (7) can be interpreted as energy condition for a virtual particle of unit mass in a conservative field. In fact, in the case of solitary waves pseudo-particle motion is restricted to the potential well which is an allowed part of the potential curve. The potential curves corresponding to Sagdeev potential (8), are shown in Figs. (**1-3**) (right) for some spicific values of M^2 and θ. A quick view on these figures reveals that pseudo--particle enters the well from the left and the it goes to right side of the well. Due to the energy conservation, in a first point at which potential vanishes, it must be stopped and then returned to the entrance point. Thus, it makes single transition between two points and this is nothing but a soliton or shock wave, which is

[3] The inequality (11) comes from the condition for existence of the (positive and negative) solitons.

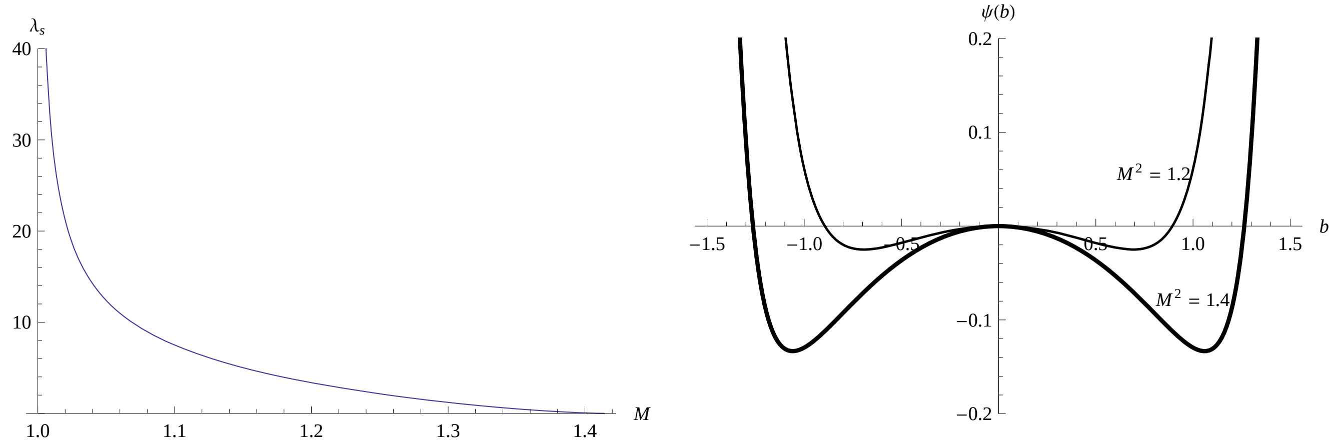

Fig. (1). Left: Plot of the λ_s versus M −Right: (pseudo) potential curves versus b for M^2=1.2, 1.4 as $\theta = 0^\circ$.

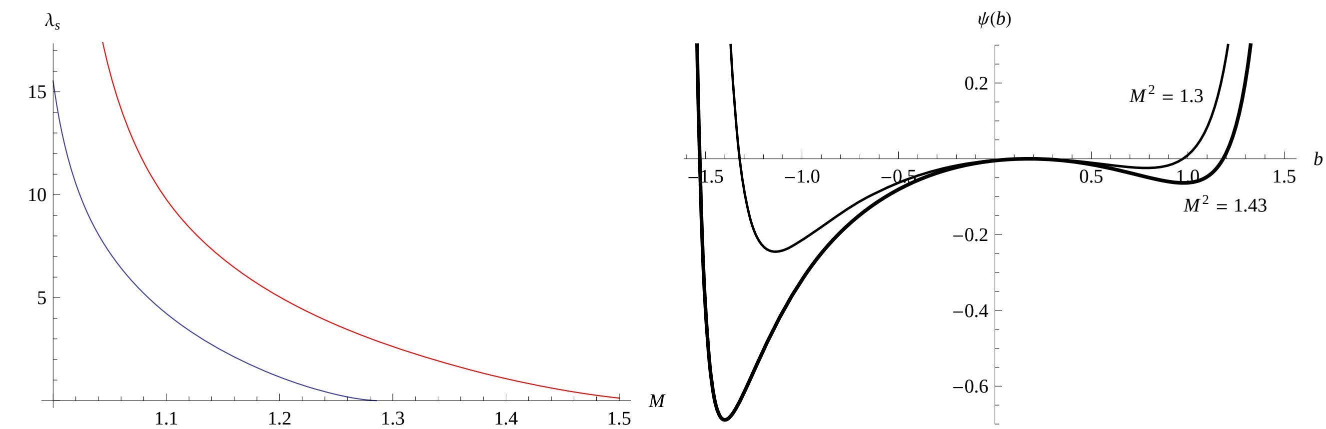

Fig. (2). Left: Plots of the λ_s versus M corresponding to the $\pm$ soliton, + (red) and - (blue). Right: Potential curves versus b for M^2=1.3, 1.43 as $\theta = 10^\circ$.

propagated in the medium as a potential disturbance. But in the case of small dissipation of energy (17), the pseudo--particle will never return to the entrance point and is trapped inside the potential well and will oscillate about a minimum point b_m. Each oscillation produces a shock wave, and hence we have a shock train. This situation is analogous to Harmonic oscillation of a real particle which can also be deduced by comparison between Eq. (6) and the familiar motion equation

$$m\frac{d^2x}{dt^2} = -\frac{dV(x)}{dx}.$$

$$x \to b, \quad V \to \psi, \quad t \to \xi \qquad and \qquad m \to 1$$

Furthermore, for a real particle in a potential well, the angular frequency of periodic motion (around a minimum point x_m) is given by the relation,

$$\omega^2 = \frac{1}{m}\frac{d^2V(x)}{dx^2}\Big|_{x=x_m}. \tag{12}$$

A little dimensional analysis clears that in our plasma system the analogous quantity to the angular frequency ω, is the wave number k_s whose inverse is in proportion to WS which after here is denoted by λ_s whit $k_s = 2\pi/\lambda_s$. Therefore, analogous formula (to Eq. 12) also holds for the shock wave number which by above correspondences can be written as

$$k_s^2 = \frac{d^2\psi(b)}{db^2}\Big|_{b=b_m}. \tag{13}$$

Now, we are prepared to calculate λ_s by the latter equation with the Sagdeev potential (8). But, first of all, we must obtain minimum point b_m and this is done by vanishing the first derivative (9), leading to the following cubic equation in b

$$\begin{aligned}&(\cos^2\theta_0)b^3 + ((3(M^2-\cos^2\theta_0)\sin\theta_0)b^2 - \\ &(2M^4+4M^2\sin^2\theta_0-2M^2\cos^2\theta_0-3\cos^2\theta_0\sin^2\theta_0)b + \\ &(M^2-\cos^2\theta_0)(2M^2+\sin^2\theta_0)\sin\theta_0 = 0,\end{aligned} \tag{14}$$

where θ_0 is a fixed value of the propagation angle θ. In general, Eq. (14) can't be solved exactly, but fortunately, we know from theory of cubic equations, if one of cubic roots is known, other roots of the cubic equation can be obtained. Then, knowing the one extremum (maximum) of $\psi(b)$ at the point $b = \sin\theta_0$ [4] Eq. (14) is reduced to the following quadratic equation

$$Ab^2 + Bb + C = 0, \tag{15}$$

with

$$A = \cos^2\theta_0,$$

$$B = (3M^2 - 2\cos^2\theta_0)\sin\theta_0$$

and

$$C = \cos^2\theta_0 \sin^2\theta_0 - M^2\sin^2\theta_0 + 2M^2\cos^2\theta_0 - 2M^4.$$

Thus, roots of Eq. (15) are

$$b_m = b_m(M) = \frac{-(3M^2 - 2\cos^2\theta_0)\sin\theta_0 \pm M\sqrt{9M^2 - (8+M^2)\cos^2\theta_0}}{2\cos^2\theta_0}, \tag{16}$$

where one must note that the expressions under the radical sign in the latter equation is always positive and hence, Eq. (15) has always two real roots which are minimum points corresponding to the $\pm$ solitons. To obtain the wave number (13), we substitute (16) into the second derivative (10) and thus we will have, not a very simple relation equation:

$$\begin{aligned}
k_s^2 = {} & M^2\Big[-16M^6 + M^2(16-10M^2)\Big(3M^2\sin\theta_0 \pm \\
& M\sqrt{-8\cos^2\theta_0 + M^2(8+\sin^2\theta_0)}\Big)^2 \sec^4\theta_0 \\
& + \frac{3}{4}\Big(3M^2\sin\theta_0 \pm M\sqrt{-8\cos^2\theta_0 + M^2(8+\sin^2\theta_0)}\Big)^4 \\
& \sec^6\theta_0 + 16M^4\sin^2\theta_0 + \\
& M^2\sin^4\theta_0 + \cos^2\theta_0(16M^4 + \sin^4\theta_0) + M^2\sin^2 2\theta_0 \\
& + \Big(3M^2\sin\theta_0 \pm M\sqrt{-8\cos^2\theta_0 + M^2(8+\sin^2\theta_0)}\Big) \\
& (\sin^3\theta_0 - M^2(16+8M^2)\sec\theta_0\tan\theta_0) + \Big(3M^2\sin\theta_0 \\
& \pm M\sqrt{-8\cos^2\theta_0 + M^2(8+\sin^2\theta_0)}\Big)^3(\sec^3\theta_0\tan\theta_0 - 6M^2\sec^5\theta_0\tan\theta_0)\Big] \\
& \Big[2M^2 - \frac{1}{4}\Big(3M^2\sin\theta_0 \pm M\sqrt{-8\cos^2\theta_0 + M^2(8+\sin^2\theta_0)}\Big)^2 \\
& \sec^4\theta_0 + (3M^2\sin\theta_0 \pm M\sqrt{-8\cos^2\theta_0 + M^2(8+\sin^2\theta_0)})\sec\theta_0\tan\theta_0\Big]^{-4}.
\end{aligned} \tag{17}$$

Having the above wave number, we can obtain λ_s as function of Mach number by

$$\lambda_s(M) = \frac{2\pi}{k_s(M)}, \tag{18}$$

where we consider Eqs. (17) and (18) for the following fixed values $\theta_0 = 0^\circ, 10^\circ,\ 90^\circ$.

For $\theta = 0^\circ$, Eq. (17) and inequality (11) become[5]

$$k_s^2 = 2\frac{3M^2 - M^4 - 2}{M^2(2-M^2)^4}, 1 < M^2 \le 2, \tag{19}$$

substituting this equation into Eq. (18), we get

$$\lambda_s = \lambda_s(M) = \frac{2\pi}{\sqrt{2}} \frac{M(2-M^2)^2}{\sqrt{3M^2 - M^4 - 2}} 1 < M^2 \le 2. \tag{20}$$

Fig. (**1**) (left) shows the function (20) versus Mach number, it represents a monotonic decreasing function which could be explained based on potential curves (Fig. **1**-right):

Fig. (**1**) (right) shows depth of the potential well increases as the Mach number increases. On the other hand, the deeper wells get shorter length in the round trip of pseudo--particle and this is nothing but the shorter width λ_s (Fig. **1**-left).

For $\theta = 10^\circ$, Eq. (17) becomes

$$\begin{aligned}
k_s^2 = {} & M^2\Big[0.0008 + 0.117M^2 + \\
& 15.999M^4 - 16M^6 + (0.005 - 2.864M^2 - 1.432M^4) \\
& (0.520M^2 \pm M\sqrt{-7.758 + 8.030M^2}) + \\
& M^2(17.010 - 10.631M^2)(0.520M^2 \pm \\
& M\sqrt{-7.758 + 8.030M^2})^2 \\
& +(0.184 - 1.142M^2)(0.520M^2 \pm \\
& M\sqrt{-7.758 + 8.030M^2})^3 + 0.822\begin{pmatrix} 0.520M^2 \\ \pm M\sqrt{-7.758 + 8.030M^2} \end{pmatrix}^4\Big] \\
& \Big[2M^2 + (0.179 - 0.265(0.520M^2 \pm \\
& M\sqrt{-7.758 + 8.030M^2}))(0.520M^2 \\
& \pm M\sqrt{-7.758 + 8.030M^2})\Big]^{-4},
\end{aligned} \tag{21}$$

which the signs $\pm$ are corresponding to $\pm$ solitons. Similar to the previous case, the functions λ_s become

$$\begin{aligned}
\lambda_s(M) = {} & \frac{2\pi}{M}\Big[2M^2 + (0.179 - 0.265 \\
& (0.520M^2 \pm M\sqrt{-7.758 + 8.030M^2})) \\
& (0.520M^2 \pm M\sqrt{-7.758 + 8.030M^2})\Big]^2 \\
& \Big[0.0008 + 0.117M^2 + 15.999M^4 - 16M^6 \\
& +(0.005 - 2.864M^2 - 1.432M^4) \\
& (0.520M^2 \pm M\sqrt{-7.758 + 8.030M^2}) + \\
& M^2(17.010 - 10.631M^2) \\
& (0.520M^2 \pm M\sqrt{-7.758 + 8.030M^2})^2 + \\
& (0.184 - 1.142M^2)(0.520M^2 \pm M\sqrt{-7.758 + 8.030M^2})^3 + \\
& 0.822\Big(0.520M^2 \pm M\sqrt{-7.758 + 8.030M^2}\Big)^4\Big]^{-1},
\end{aligned} \tag{22}$$

which inequality (11) imposes that

[4] Note that the necessary condition for soliton type solution is $\psi(\sin\theta) = \psi'(\sin\theta) = 0$

[5] In this case, each of both $\pm$ solitons exist, but with the same (squared) wave numbers (19).

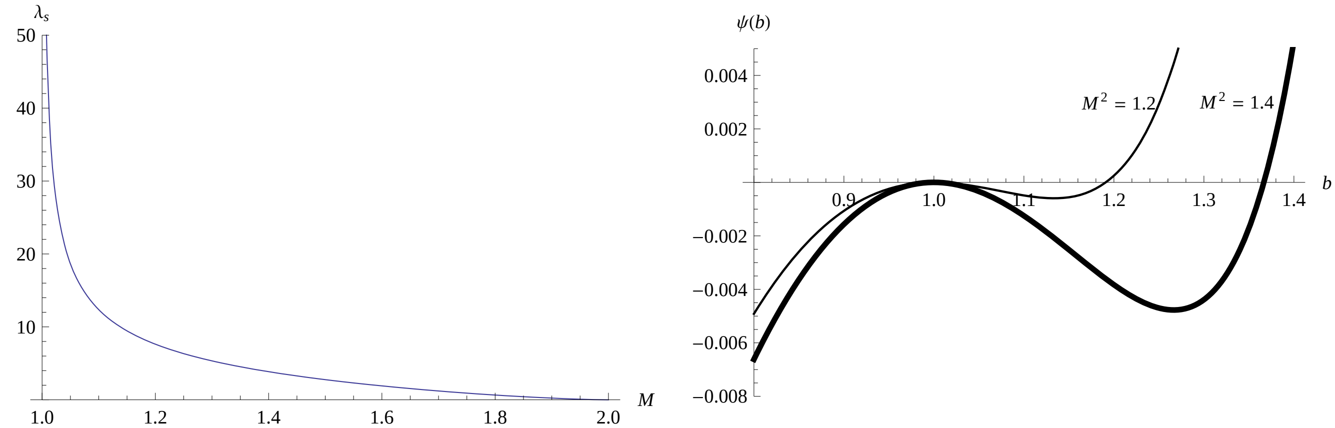

Fig. (3). Left: Plot of the λ_s function *versus* M . Right:(pseudo) potential curves *versus* b for M^2=1.2, 1.4 as $\theta = 90^\circ$.

$$1 < M^2 \le 2.346 \text{ or } 1 < M \le 1.531 \text{ postivesoliton}$$
$$1 < M^2 \le 1.645 \text{ or } 1 < M \le 1.286 \text{ negativesoliton.} \tag{23}$$

The two functions (22) are plotted in Fig. (**2**) (left) which again represent decreasing functions, but there are two main differences between them:

First, in the limit $M \to 1$, the negative soliton (blue) plot tends to a finite value while the positive one (red) is unbounded. The reason for this can be explained in what follows:

Fig. (**2**) (right) shows when the Mach number decreases the well potential (corresponding to the positive soliton) tends to the horizontal axis (b) (contrary to the negative solitons) and this in turn causes the length of the round trip (λ_s) to grow unbounded.

Second, the graph corresponding to negative soliton is damped faster than to the positive one. This is expected according to physics, because according to (Fig. **2**-right), the potential well corresponding to negative soliton is deeper than the positive one and as a result, produced shock by the negative soliton has shorter λ_s .

In the case $\theta = 90^\circ$, Eq. (17) and inequality (11) read[6]

$$k_s^2 = 729 \frac{-M^4 + 5M^2 - 4}{(1+2M^2)^3(4-M^2)^4} M^4, \quad 1 < M^2 \le 4 \tag{24}$$

and therefore,

$$\lambda_s(M) = \frac{2\pi}{27} \frac{(4-M^2)^2(2M^2+1)^{3/2}}{M^2\sqrt{-M^4+5M^2-4}}, \tag{25}$$

which is graphed in Fig. (**3**) (left). The physical reason for decreasing λ_s is the same as two investigated previous cases.

At the end of this section, we note that, the plots with $0 \le \theta \le \theta_c = 30^\circ$ are qualitatively similar to plot for $\theta = 10^\circ$, namely there are two λ_s functions corresponding to both ± solitons. Also, the plot with $\theta > \theta_c$ are qualitatively similar to plot for $\theta = 90^\circ$, namely there is only one λ_s function corresponding to positive solitons. These plots differ with each other quantitatively and this is what will be discussed in below.

4. PLOTS CHANGES *VERSUS* ANGLE CHANGES

So far, we showed that, for a fixed value of propagation angle θ_0 , the width of the shocks are functions of M. The graphs of functions were monotonic descending ones so that each graph has its own slope (decreasing rate). For a better comparison, it is appropriate to consider the change in graphs versus the angle changes. The comparison between decreasing rates is more viewable when the graphs are presented all in one illustration. This is done in (Fig. **4**) for positive (left) and negative (right) solitons. In the case of positive solitons, the corresponding λ_s decreases more slowly whit increasing angle. Geometrically, it means that the graph with large amounts of propagation angle has larger slope at any point. This situation is reversed in the case of negative solitons as shown in the Fig. (**4**) (right).

CONCLUSION

We focus on width of the shock λ_s in a pair plasma with a background constant magnetic field. An electromagnetic wave is propagated in the pair plasma with propagation angle θ measured from the background field. The large amplitude solitary waves propagate in plasma which is described by Sagdeev (pseudo)-potential. The only dynamical variable is normalized magnetic component of the wave which is considered as pseudo-particle. Shock wave (in the medium) is produced by oscillations of pseudo--particle (in potential well). For a given value of propagation angle θ_0 , width of the shock λ_s is a function of the Mach number M which is bounded below by unity and bounded above by the value depending on the propagation angle. All of the functions

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[6] In this case, there is only a single positive soliton.

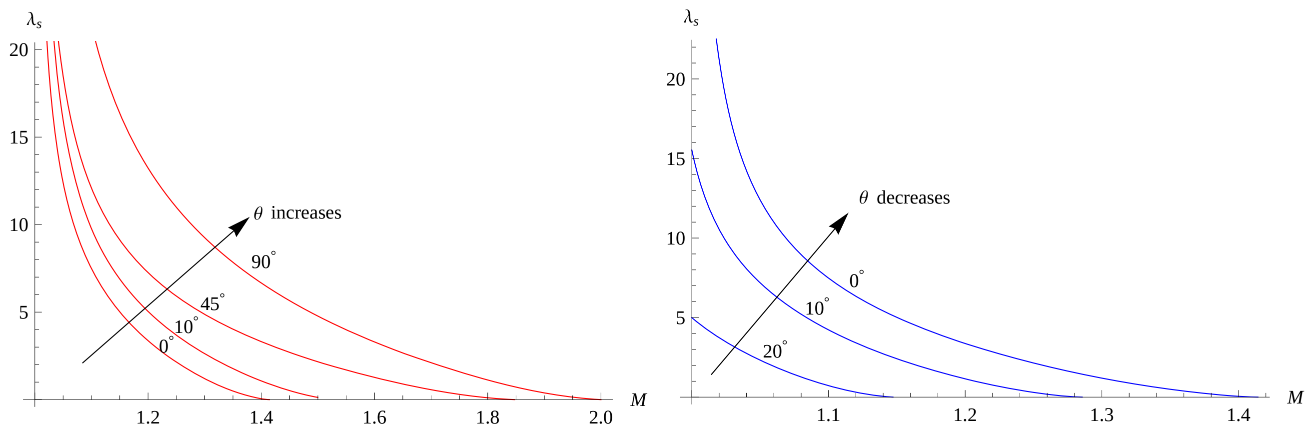

Fig. (4). Comparison between the decreasing rate for λ_s plots as propagation angle changes, positive solitons (left) and negative solitons (right).

plots (λ_s *versus* the Mach number) have a main and common feature which is monotonic rapidly decreasing character. There are also other features about λ_s plots, said in below:

1- For $\theta_0 < \theta_c$ and the limiting case $M \to 1$, width of the shock corresponding to positive soliton is unbounded while the negative one is bounded.

2- Since, width of the shock λ_s is the length of a perfect sweep of pseudo--particle (in the potential well) and potential wells corresponding to the negative solitons are deeper than the positive one, therefore, generated shock by the positive solitons has a greater width.

3- In the case of positive (negative) solitons, width of the shock corresponding to smaller (larger) values of θ_0 , is decreased more rapidly.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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