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Non-Akhiezer-Polovin Model on Plasma Electrostatic Wave and Electron Beam

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Abstract: We study, beyond the well-known Akhiezer-Polovin model, plasma electrostatic wave and unmagnetized electron beam.Our investigation is based on a stricter theory in which a long-lasting misconception about zero-temperature fluid motion equation is removed. Our theory explains some authors' puzzle about why *a narrowly focused* (charged particles) *beam is preserved in the presence of strong space charge forces*. The interaction of such a unmagnetized charged particles beam with a plasma electrostatic wave is studied in details and some universal results are revealed. These exact information are crucial to accurate estimation of the quality of a plasma wakefield and hence its performance in acceleration.

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1. INTRODUCTION

Among numerous theoretical investigations on plasma-based particle acceleration [1-16], Akhiezer-Polovin (A-P) model is a popular description on zero-temperature plasma electrostatic (ES) wave [3-5, 9, 15]. According to this model, there is a convective term appearing in the fluid motion equation in the zero-temperature limit.

$$\partial_t p(u) + u \cdot \nabla_r p(u) = -[E + u \times B] \quad (1)$$

where u is fluid velocity, $p(u) = \frac{u}{\sqrt{1-u^2}}$ and $u \cdot \nabla_r p(u)$ is the so-called convective term. Because the fluid motion equation in the zero-temperature limit and 4 Maxwell equations (Meqs) form a basis for studying macroscopic property of charged particles, it is very crucial to ensure these equations being correct or strictly derived from microscopic Vlasov-Maxwell (V-M) theory, which is also the theoretical basis of beam physics [17-27].

The derivation of the A-P model is completely a standard textbook procedure. By deriving a finite-temperature fluid motion equation [28] from Vlasov equation (VE) and then putting this equation in the zero-temperature limit, we can discard the thermal-pressure term and finally obtain Eq. (1). This procedure seems to be perfect. Eq. (1) can also be derived from other way [see later]. Agreement between different derivation methods further confirms Eq. (1) to be strictly correct.

However, we cannot ignore a deep inconsistency behind the above-described standard procedure of deriving the Eq. (1). That is, when putting the finite-temperature fluid motion equation in the zero-temperature limit, we actually have subconsciously believed that zero-temperature type distribution is a strict solution of the VE. This belief might be the fundamental reason for a long-lasting misconception popular in plasma and beam physics. We have pointed out this inconsistency elsewhere [31]. For convenience of readers, we also introduce them in details in this work.

This forces us, beyond the A-P model, to re-consider ES wave, electron beam and various aperiodic ES structures in charged particles. (here, wave is a periodic ``structure''. Because studied physical quantities are not bound to be periodic, we introduce a more general term "structure"). This will be done in Sec.II. The interaction between periodic structure and aperiodic one is the content of the third subsection of Sec.II. The 4-th subsection is for phase space coherent structure. Section. III is a brief summary.

2. THEORY

2.1. Starting Model Equations

We start from well-known V-M equation set [28, 29]

$$\left[\partial_t + v \cdot \nabla - [E + v \times B] \cdot \partial_{p(v)}\right] f = 0, \quad (2)$$

$$\partial_t E = nu + \nabla \times B; \quad (3)$$

$$\nabla \cdot E = -n + ZN_i; \quad (4)$$

$$\nabla \times E = -\partial_t B; \quad (5)$$

$$\nabla \cdot B = 0. \quad (6)$$

where $p(v) = m_e \frac{v}{\sqrt{1-v \cdot v}}$, $n = \int f d^3 v$, $u = \frac{\int v f d^3 v}{\int f d^3 v}$ and m_e is the static mass of an electron. Moreover, electron charge e

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has been absorbed into E and B. Moreover, all equations are expressed in term of Euler variables (r,t), and v is independent of r and t.

A strict analysis have revealed that if $f(r,v,t)$ is a strict solution of the VE, $f_0 = f\left(r, \frac{\int v f d^3v}{\int f d^3v}, t\right) * \delta\left(v - \frac{\int v f d^3v}{\int f d^3v}\right)$ is not [31, 32]. This result can be found from more general Klimontovich-Dupree (K-D) theory [28]. According to the K-D theory, a particle system can be described by a function $N(r,v,t) = \sum_i \delta(r_t(t) - r)\delta(d_t r_i(t) - v)$ which meets the VE. Obviously, a function with more constraint $N_0(r,v,t) = \sum_i \delta(r_t(t) - r)\delta(d_t r_i(t) - v)\delta(v - u(r,t))$ will correspond to a zero-temperature type distribution function. Even though $N(r,v,t) = \sum_i \delta(r_t(t) - r)\delta(d_t r_i(t) - v)$ always meets the VE, $N_0(r,v,t) = \sum_i \delta(r_t(t) - r)\delta(d_t r_i(t) - v)\delta(v - u(r,t))$ does not. This can be directly verified by substituting N_0 back into the VE.

For any particle system described by a microscopic distribution function f, we can always view it as the summation of two subsystems of a same fluid velocity u, one consists of all particles whose velocities are equal to $u = \frac{\int v f d^3v}{\int f d^3v}$ and the other is described by a "hollow" distribution f_{ho} which meets $f_{ho}(r, v = u, t) = 0$ and $u = \frac{\int v f_{ho} d^3v}{\int f_{ho} d^3v}$. Above results reveal that each subsystem does not have a conserved total particle number and hence exchange particles with the other. This might be the fundamental reason for why the zero-temperature type subsystem, which corresponds to $f_0 = f - f_{ho}$, does not meet the VE.

The microscopic dynamics equation of $f_0 = n_0(r,t)\delta(v - u(r,t))$, where $n_0 = \int f_0 d^3v$, can be derived straightforward [31]. According to strict theory [31, 32], the continuity equation associated with n_0 becomes

$$\partial_t n_0 + u \cdot \nabla_r n_0 = 0, \tag{7}$$

rather than our familiar $\partial_t n_0 + u \cdot \nabla_r n_0 = -n_0 \nabla_r \cdot u$ (i.e. $\partial_t n_0 + \nabla_r \cdot (n_0 u) = 0$). This reflects the subsystem described by f_0 having particle exchange with other. Namely, because E is space-time dependent, a charged particle system cannot be at zero-temperature state in which at any space position, all particles have a same velocity. Space-inhomogeneous E will lead to, in some space positions, the temperature differing from 0 and hence thermal spread in particles' velocities appearing (which means some particles being out of the kernel group described by f_0 and into the hollow group described by $f - f_0$) .

Likewise, a macroscopic fluid motion equation can be derived [31, 32]

$$\partial_t \frac{u}{\sqrt{1-u^2}} + E + u \times B = 0. \tag{8}$$

In contrast, we can, from the VE, derive our familiar fluid motion equation (for arbitrary temperature)

$$\partial_t u + \frac{\int (v-u)\nabla_r[vf]d^3v}{n} + \frac{\int [E(r,t) + v\times B(r,t)]\cdot[\sqrt{1-v^2}]^3 * f d^3v}{n} = 0. \tag{9}$$

Obviously, two strict equations of u suggest a balance relation

$$\frac{\int (v-u)\nabla_r[vf]d^3v}{n} = \left[-\frac{\int [E(r,t) + v\times B(r,t)]\cdot[\sqrt{1-v^2}]^3 * f d^3v}{n} + [E + u\times B]\left(\sqrt{1-u^2}\right)^3\right], \tag{10}$$

which is an implicit equation of the temperature or the thermal spread.

Above discussions have displayed in details, from microscopic theory, why Eq. (1) is inaccurate. On the other hand, Eq. (1) can also be derived completely *via* macroscopic fluid theory [20, 21]. This forces us to analyze what is misunderstood when deriving Eq. (1) from fluid theory.

Because in standard fluid theory, if a physical field is expressed by Lagrangian variable $Q(\tilde{r}(t),t)$, its total differential with respect to t will be strictly $d_t Q(\tilde{r}(t),t) = \partial_t Q(\tilde{r}(t),t) + d_t\tilde{r} \cdot \nabla_{\tilde{r}} Q(\tilde{r}(t),t)$. Thus, if Q stands for momentum p, there will be $\partial_t Q(\tilde{r}(t),t) + d_t\tilde{r} \cdot \nabla_{\tilde{r}} Q(\tilde{r}(t),t) = d_t Q(\tilde{r}(t),t) = F(\tilde{r}(t),t)$ (where $F(\tilde{r}(t),t)$ stands for the field of force). For this equation, people often, by the relation $d_t\tilde{r} \equiv u(\tilde{r}(t),t)$ which defines the trajectory of a fluid element, re-write it as a more familiar form $F(\tilde{r}(t),t) = \partial_t p(\tilde{r}(t),t) + u(\tilde{r}(t),t) \cdot \nabla_{\tilde{r}} p(\tilde{r}(t),t) = \partial_t p\left(u(\tilde{r}(t),t)\right) + u(\tilde{r}(t),t) \cdot \nabla_{\tilde{r}} p\left(u(\tilde{r}(t),t)\right)$, where $p(u) = \frac{u}{\sqrt{1-u\cdot u}}$. Then, after transforming this familiar form from an expression in term of Lagrangian variables $(\tilde{r}(t),t)$ to that in term of Euler variable (r,t), we will obtain Eq. (1).

However, if noting that the relation $d_t\tilde{r} \equiv u(\tilde{r}(t),t)$ can lead to 3 equivalent expressions of $d_t\tilde{r} \cdot \nabla_{\tilde{r}} p\left(u(\tilde{r}(t),t)\right)$: 1. $u(\tilde{r}(t),t) \cdot \nabla_{\tilde{r}} p\left(u(\tilde{r}(t),t)\right)$ (merely replacing $d_t\tilde{r}$ left to $\cdot$ operator with $u(\tilde{r}(t),t)$); 2. $d_t\tilde{r} \cdot \nabla_{\tilde{r}} p(d_t\tilde{r})$ (merely replacing $u(\tilde{r}(t),t)$ with $d_t\tilde{r}$); 3. $u(\tilde{r}(t),t) \cdot \nabla_{\tilde{r}} p(d_t\tilde{r})$. Namely, the relation $d_t\tilde{r} \equiv u(\tilde{r}(t),t)$ will lead to $d_t\tilde{r} \cdot \nabla_{\tilde{r}} p\left(u(\tilde{r}(t),t)\right) \equiv u(\tilde{r}(t),t) \cdot \nabla_{\tilde{r}} p\left(u(\tilde{r}(t),t)\right) \equiv d_t\tilde{r} \cdot \nabla_{\tilde{r}} p(d_t\tilde{r}) \equiv u(\tilde{r}(t),t) \cdot \nabla_{\tilde{r}} p(d_t\tilde{r})$. Most essentially, the property $d_t\tilde{r} \cdot \nabla_{\tilde{r}} p(d_t\tilde{r}) = d_t\tilde{r} \cdot \frac{dp(\lambda)}{d\lambda}|_{\lambda = d_t\tilde{r}} \nabla_{\tilde{r}} d_t\tilde{r} \equiv 0$ (because of $\nabla_{\tilde{r}} d_t\tilde{r} \equiv 0$) determines all these 4 expressions to be $\equiv 0$. In other words, the so-called convective term $u(\tilde{r}(t),t) \cdot \nabla_{\tilde{r}} p\left(u(\tilde{r}(t),t)\right)$ is indeed 0. More detailed demonstration are presented in an appendix.

Clearly, if merely using the relation $d_t\tilde{r} \equiv u(\tilde{r}(t),t)$ but on purpose ignoring the fact that $u(\tilde{r}(t),t) \cdot \nabla_{\tilde{r}} p\left(u(\tilde{r}(t),t)\right)$ is indeed $\equiv 0$, we can find that $d_t p\left(u(\tilde{r}(t),t)\right) = F(\tilde{r}(t),t)$ will agree with $\partial_t p\left(u(r,t)\right) + u(r,t) \cdot \nabla_{\tilde{r}} p\left(u(r,t)\right) = F(r,t)$. Incomplete/partial utilization of the relation $d_t\tilde{r} \equiv u(\tilde{r}(t),t)$ can yield a different agreement.

In mathematical language, incomplete/partial utilization will lead to extraneous root of the equation set

$$F(\tilde{r}(t),t) = d_t p(d_t \tilde{r}); \tag{11}$$

$$d_t \tilde{r} \equiv u(\tilde{r}(t),t). \tag{12}$$

i.e. the solution of $\partial_t p\big(u(r,t)\big) + u(r,t)\cdot\nabla_{\tilde{r}} p\big(u(r,t)\big) = F(r,t)$ cannot meet Eqs. (11, 12) but the solution of Eqs. (11, 12) can meet $\partial_t p\big(u(r,t)\big) + u(r,t)\cdot\nabla_{\tilde{r}} p\big(u(r,t)\big) = F(r,t)$.

In short, by analyzing on what condition Eqs. (11, 12) have a common solution of $\tilde{r}(t)$, we can find this demanding a relation between two functionals $F(r,t) = \partial_t p\big(u(r,t)\big)$,i.e. Eq. (8).

2.2. Macroscopic ES Structures in Neutral and Non-Neutral Plasmas

For different ES structures, we wish to find related solutions which are static in a moving frame of a constant velocity $\frac{1}{\eta}c$. Therefore, we introduce following definitions

$$\xi = \eta z/c - t; p = \frac{u}{\sqrt{1-[u/c]^2}}; u(z,t,r,\theta) = u(\xi,r,\theta), \tag{13}$$

where r, θ and z are coordinates in the cylindric frame.

2.2.1. ES Wave

Detailed analysis reveals that the ES wave in 3-D case corresponds to an equation (where $\beta = \frac{p_r}{p_z}$ and $\lambda = \frac{p_\theta}{p_z}$, p_z is dimensionless momentum) [30]

$$\left[1 - \frac{\eta(1+\beta^2+\lambda^2)p_z}{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2}}\right]\partial_{\xi\xi}p_z = -ZN_i\frac{p_z}{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2}}, \tag{14}$$

which corresponds to a first integral of general form

$$\left(\partial_\xi p_z\right)^2 + f_0(r,\theta,p_z) = G(r,\theta), \tag{15}$$

where $G(r,\theta)$ is a binary function of r and θ, and f_0 stands for well-known Sagdeev potential.

The relation between p_z and density profile n reads [30]

$$n = \frac{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2}}{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2-\eta(1+\beta^2+\lambda^2)p_z}}ZN_i. \tag{16}$$

Note that the condition $n† \geq 0$ will lead to a constraint on p_z

$$\sqrt{1+(1+\beta^2+\lambda^2)p_z^2} - \eta(1+\beta^2+\lambda^2)p_z > 0, \tag{17}$$

$$\text{or} p_z < \frac{1}{\sqrt{\eta^2(1+\beta^2+\lambda^2)^2-(1+\beta^2+\lambda^2)}} < \frac{1}{\sqrt{\eta^2-1}} \text{if} \eta > 1. \tag{18}$$

$f_0(r,\theta,p_z)$ and $G(r,\theta)$ in Eq. (15) read [30]

$$f_0(r,\theta,p_z) =$$

$$\frac{2}{c}\left\{\begin{matrix} -\sqrt{1+(1+\beta^2+\lambda^2)p_z^2} - \frac{1}{2}\frac{\eta(1+\beta^2+\lambda^2)}{\sqrt{c}}\ln\frac{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2}-\frac{\eta(1+\beta^2+\lambda^2)}{\sqrt{c}}}{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2}+\frac{\eta(1+\beta^2+\lambda^2)}{\sqrt{c}}} \\ -\eta(1+\beta^2+\lambda^2)p_z + \frac{1}{2}\frac{\eta(1+\beta^2+\lambda^2)}{\sqrt{c}}\ln\frac{\sqrt{c}p_z-1}{\sqrt{c}p_z+1} \end{matrix}\right\}ZN_i \tag{19}$$

$$G(r,\theta) = f_0(r,\theta,p_z=0) + \left(\partial_\xi p_z\right)^2|_{p_z=0}, \tag{20}$$

where

$$c = [\eta^2(1+\beta^2+\lambda^2)^2 - (1+\beta^2+\lambda^2)] > 0 if \eta > 1. \tag{21}$$

In the A-P model [5], $\partial_\xi[\eta p - \Gamma] = E$ is used and u is a function of $\eta p - \Gamma$ (where $\Gamma = \sqrt{1+p^2}$, $\xi = z - \frac{1}{\eta}ct$ and η is a constant). A first integral of $\eta p - \Gamma$ (which is denoted by W) is thus derived (where Z is ionic charge and N_i is ionic density).

$$\left(\partial_\xi W\right)^2 + \frac{ZN_i}{(\eta^2-1)}\left[\eta\sqrt{(W^2+\eta^2)-1} \pm W\right] = const \tag{22}$$

Following similar procedure, we use $\partial_\xi \eta p = E$ and derive a first integral of p.

$$\left(\partial_\xi p\right)^2 + \frac{ZN_i}{3\eta}\left[\left(\sqrt{p^2+1}\right)^3 + p^3\right] = const \tag{23}$$

As shown in Fig. (**1**), under same value of $\left[\partial_\xi p|_{\xi=0}, p|_{\xi=0}\right]$, the behaviors of p and E from different models are somewhat different. Luckily, there is no marked difference between the magnitudes of two shapes. Also, two shapes are almost of a same wavelength. Actually, even though some authors have studied 1-D ES wave beyond the A-P model [6], they seem to be not aware that their treatment is indeed rigorous rather than an approximation, and hence underestimate the importance of their works.

2.2.2. Unmagnetized Charged Particles Beam

Same procedure can yield similar formulas about the unmagnetized electron beam

$$\left[1 - \frac{\eta(1+\beta^2+\lambda^2)p_z}{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2}}\right]\partial_{\xi\xi}p_z =$$
$$\left\{\left[\left(\partial_r\beta + \frac{\beta}{r} + \frac{1}{r}\partial_\theta\lambda\right)\partial_\xi p_z\right]\right\}\frac{p_z}{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2}}. \tag{24}$$

which implies

$$\partial_\xi p_z - \left(\partial_r\beta + \frac{\beta}{r} + \frac{1}{r}\partial_\theta\lambda\right)f_0(r,\theta,p_z) = G(r,\theta). \tag{25}$$

Likewise, n reads

$$n = -\left(\partial_r\beta + \frac{\beta}{r} + \frac{1}{r}\partial_\theta\lambda\right)\partial_\xi p_z - \eta(1+\beta^2+\lambda^2)\,\partial_{\xi\xi}p_z$$
$$= -\frac{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2}}{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2} - \eta(1+\beta^2+\lambda^2)p_z}\left(\partial_r\beta + \frac{\beta}{r} + \frac{1}{r}\partial_\theta\lambda\right)\partial_\xi p_z$$
$$= -\frac{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2}}{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2} - \eta(1+\beta^2+\lambda^2)p_z}$$
$$\left[\left(\partial_r\beta + \frac{\beta}{r} + \frac{1}{r}\partial_\theta\lambda\right)^2 f_0(r,\theta,p_z) + \left(\partial_r\beta + \frac{\beta}{r} + \frac{1}{r}\partial_\theta\lambda\right)G(r,\theta)\right]. \tag{26}$$

It is well-known that such a general form

$$f_2(y)y'' + f_1(y) * y' + f_0(y) = 0 \tag{27}$$

which contains a linear term of y', cannot correspond to a first integral unless $f_1(y) = 0$. According to Eq. (24) and

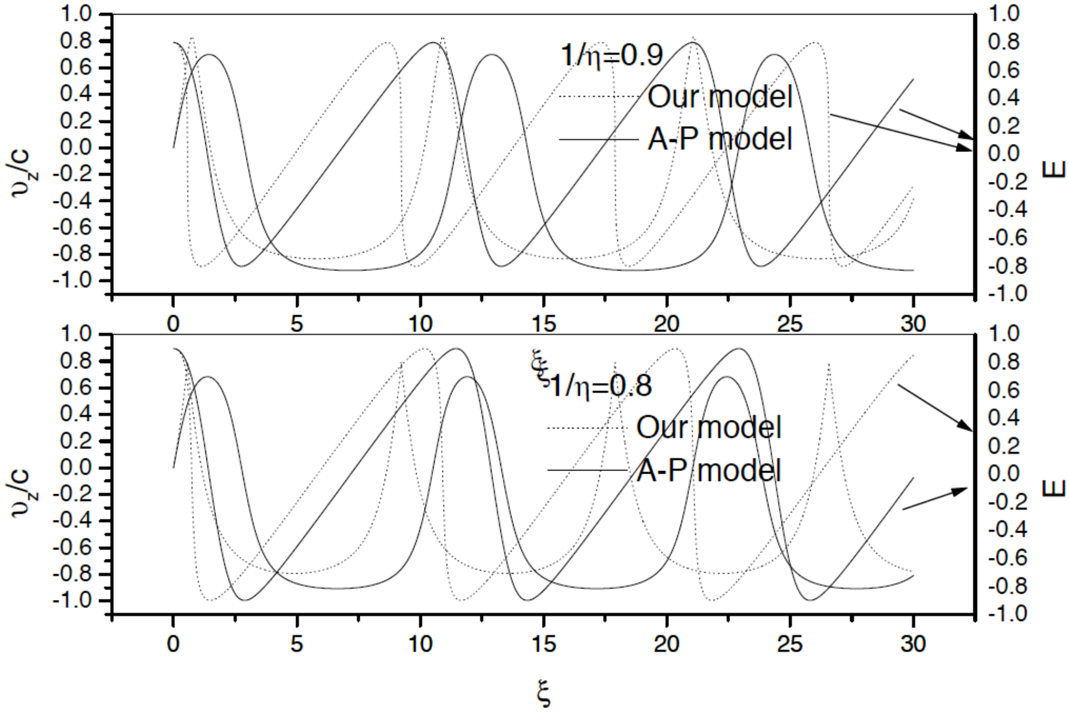

Fig. (1). The behaviors of p and E VS $\xi = z - \frac{c}{\eta}t$, where $\frac{c}{\eta}$ is the phase velocity of the ES wave and $ZN_i = 1$ is choosen.

Eq. (26), an unmagnetized electron beam is impossible to be described by a periodic solution because this will imply $\partial_r\beta + \frac{\beta}{r} + \frac{1}{r}\partial_\theta\lambda = 0$ and $n = 0$ anywhere. Thus, an unmagnetized electron beam must have

$$\partial_r\beta + \frac{\beta}{r} + \frac{1}{r}\partial_\theta\lambda \neq 0 \tag{28}$$

Note that there should be $n \geq 0$ anywhere. This will lead to a constraint on p_z

$$\frac{\left[\left(\partial_r\beta+\frac{\beta}{r}+\frac{1}{r}\partial_\theta\lambda\right)^2 f_0(r,\theta,p_z)+\left(\partial_r\beta+\frac{\beta}{r}+\frac{1}{r}\partial_\theta\lambda\right)G(r,\theta)\right]}{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2}-\eta(1+\beta^2+\lambda^2)p_z} \P \leq 0 \tag{29}$$

$f_0(r,\theta,p_z)$ and $G(r,\theta)$ in Eq. (25) read

$$f_0(r,\theta,p_z) = \frac{1}{c}\left\{\begin{array}{l} -\sqrt{1+(1+\beta^2+\lambda^2)p_z^2} - \frac{1}{2}\frac{\eta(1+\beta^2+\lambda^2)}{\sqrt{c}}\ln\frac{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2}-\frac{\eta(1+\beta^2+\lambda^2)}{\sqrt{c}}}{\sqrt{1+(1+\beta^2+\lambda^2)p_z^2}+\frac{\eta(1+\beta^2+\lambda^2)}{\sqrt{c}}} \\ -\eta(1+\beta^2+\lambda^2)p_z + \frac{1}{2}\frac{\eta(1+\beta^2+\lambda^2)}{\sqrt{c}}\ln\frac{\sqrt{c}p_z-1}{\sqrt{c}p_z+1} \end{array}\right\} \tag{30}$$

$$G(r,\theta) = -\left(\partial_r\beta + \frac{\beta}{r} + \frac{1}{r}\partial_\theta\lambda\right) f_0\left(r,\theta,p_z = \frac{1/\eta}{\sqrt{1-(1/\eta)^2}}\right), \tag{31}$$

where $c = [\eta^2(1+\beta^2+\lambda^2)^2 - (1+\beta^2+\lambda^2)] > 0 if \eta > 1.$(32)

Here, the reason for why $G(r,\theta)$ has above expression is for following consideration: the p_z-value at the position meeting $\partial_t p_z = E = 0$ is equal to $\frac{1/\eta}{\sqrt{1-(1/\eta)^2}}$ and thus $1/\eta$ is a characteristic constant velocity of such an ES structure. Eq. (30) indicates that for $p_z > 0$, $f_0(r,\theta,p_z)$ decreases with p_z increasing. Correspondingly, the constraint Eq. (29) will become

$$\sqrt{1+(1+\beta^2+\lambda^2)p_z^2} - \eta(1+\beta^2+\lambda^2)p_z \P \leq 0 \tag{33}$$

$$\text{or} p_z \dagger \frac{1}{\sqrt{\eta^2(1+\beta^2+\lambda^2)^2-(1+\beta^2+\lambda^2)}} \text{if} \eta > 1. \tag{34}$$

According to Eqs. (26, 29, 30, 31), $p_z > \frac{1}{\sqrt{\eta^2-1}}$ will correspond to $n < 0$. Thus, for a unmagnetized beam, its p_z is confined within a regime

$$0 < \frac{1}{\sqrt{\eta^2(1+\beta^2+\lambda^2)^2-(1+\beta^2+\lambda^2)}} \leq \P p_z \P \leq \frac{1}{\sqrt{\eta^2-1}}. \tag{35}$$

Some examples of p_z-profile and corresponding n-profile are presented in Fig. (**2**). Transverse boundary conditions are dependent on longitudinal one through β and λ. Here, we plot these curves in similar shapes. For example, p_z rises monotonically to $\frac{1}{\sqrt{\eta^2-1}}$ and n drops monotonically to 0. This same trend just reflects these curves are from a common equation with different parameter values. The effect of related parameter values are exhibited by different ranges of coordinate regime. Note that at $r = 0.01$ (see Fig. (**2b**)), p_z rises from 0.5773334 to 0.5773350 over a regime $0 < \xi < 0.025$. In contrast, at $r = 10$, p_z rises from 0.025 to 0.35 over a regime $0 < \xi < 0.030$. Such a difference due to different r-values could also be found from n-curves. At larger r regime, n drops relatively gently from a value, which is far below that at smaller r regime, to 0. From those n-curves at different r-values, one could find that for a given ξ, n is inverse proportional to r.

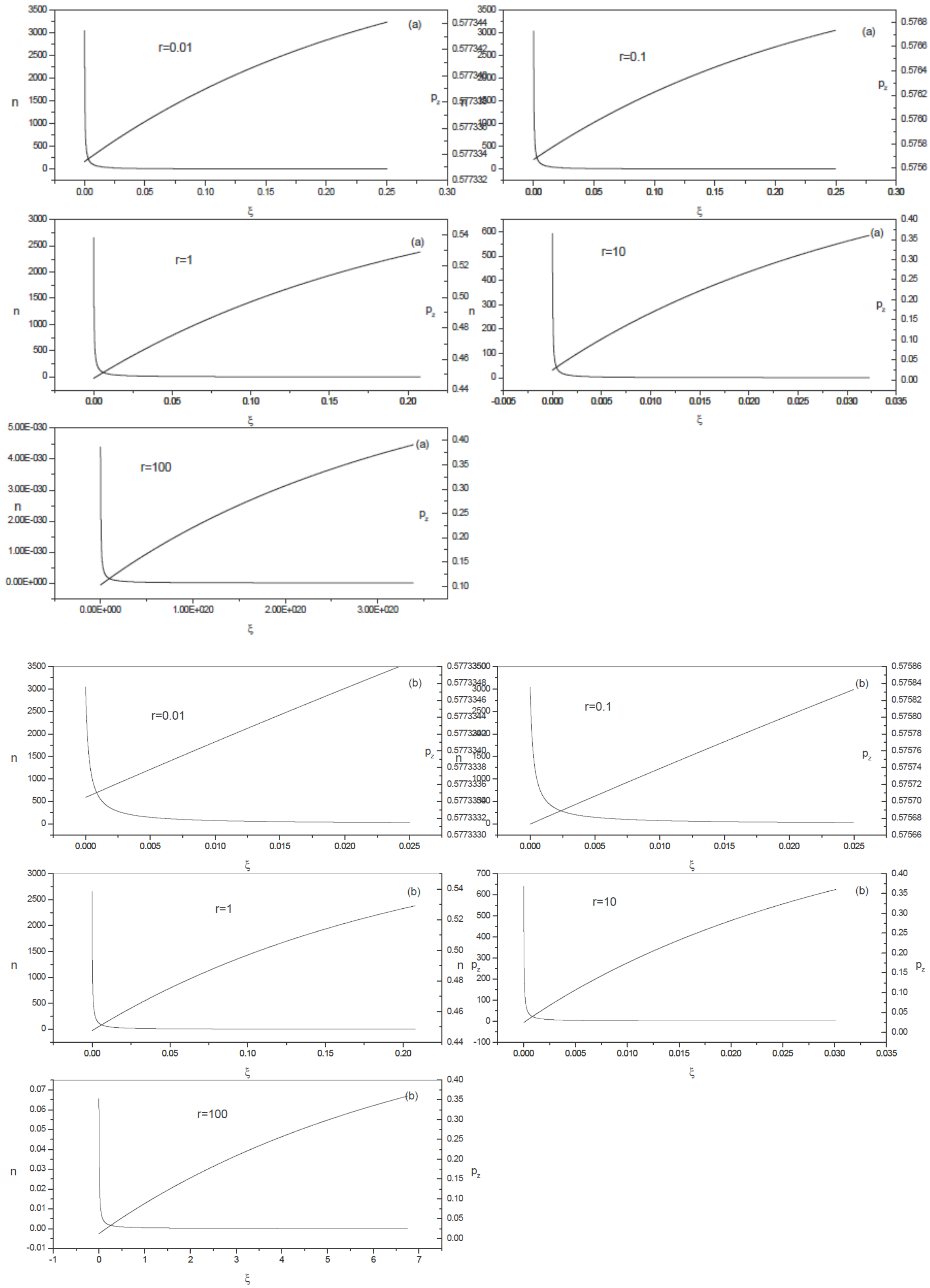

Fig. (2). Examples of density profile (n-profile) of a unmagnetized electron beam and corresponding p_z-profile, where $\eta = 1/2$, $\partial_r\beta + \frac{\beta}{r} = \exp(-r^2/R_\perp^2)$ with $R_\perp = 20$ in (**a**) and $\partial_r\beta + \frac{\beta}{r} = \frac{R_\perp^2}{R_\perp^2+r^2}$ with $R_\perp = 20$ in (**b**). Moreover, corresponding bondary conditions for Eq. (23) are $p_z|_{\xi=0} = 0$ and $\partial_z p_z|_{\xi=0} = -0.01$. Here, p_z is in unit of $m_e c$, n is in unit of $\frac{8.85*0.511}{1.6} * 10^7[\mu m]^{-3}$, E is in unit of $0.511 * 10^6[Volt/\mu m]$, and R is in unit of μm.

Above theoretical and numerical results indicate that even though all particles in the beam have a same charge, their collective self-consistent fields could confine themselves to form a globally-moving, small-volume and hard-core charged ``cloud". Here, the word ``hard-core" refers to that the density at the inside region of the cloud is far larger than that at the outside region.. A correct knowledge on a charged particles beam is very valuable to correctly understand its interaction with matter. As pointed out by Chao *et al.* [26], it is *quite counterintuitively* to understand that *such a narrowly focused* (charged particles) *beam is preserved in the presence of strong space charge forces*. Our above theory could answer this doubt to some extent.

2.3. Beam-wave Interaction

When a unmagnetized electron beam is injected into a pre-existing plasma ES wave, their respective self-electric fields will strongly perturb each other. As shown previously (for example, Eq. (24)), every ES structure is usually described by its phase velocity $\frac{1}{\eta}c$ and shape (which is determined by the boundary condition). Before making detailed study, we first discuss the phase velocity.

2.3.1. Phase Velocity

From Eq. (8) at $B \equiv 0$ case, we know that u-value at the zero point of a moving E just corresponds to a constant characteristic velocity. Considering that the phase velocity $\frac{1}{\eta}c$ is also a a constant characteristic velocity, for convenience, we choose two constant characteristic velocities being equal and hence $\frac{1}{\eta}c$ is defined as the fluid velocity at the zero point of the moving (E, B) fields.

For every ES structure, its $P(u)$-profile and E-profile are power series of $z - \int_0^t S(t')dt'$

$$E = \sum_{i\dagger 0} cE_i \left(z - \int_0^t S(t')dt'\right)^i ; P(u) = \sum_{i\dagger 0} cPu_i \left(z - \int_0^t S(t')dt'\right)^i. \quad (36)$$

where $S = \frac{1}{\eta}$ and $cPu_0 = \frac{S}{\sqrt{1-S^2}}$. Note that because $\partial_t P(u) = -E$, there is $cE_0 = 0$ if $d_t S = 0$. Obviously, for an ES structure free from perturbation, there are $d_t S = 0$ and $cE_0 = 0$. Namely, along a trajectory defined by $P(z,t) = \frac{S}{\sqrt{1-S^2}}$, there is always $E(z,t) = 0$. Thus, $[P(z,t), E(z,t)] = \left[\frac{S}{\sqrt{1-S^2}}, 0\right]$ defines a phase-velocity section of an ES structure, or $z = \int_0^t S(t')dt'$ section in which $E(\int_0^t S(t')dt', t) = 0$ exists.

2.3.2. Momentum Exchange

When two structures A and B encounter, their phase-velocity sections are not always at a same position. Namely, at the phase-velocity section of A structure, there might be $E_B \neq 0$ and vice versa. Thus, a momentum exchange will occur and can be described by

$$-d_t \frac{S_A}{\sqrt{1-S_A^2}} = -d_t p_A \left(\int_0^t S_A(t')dt', t\right) = E_A \left(\int_0^t S_A(t')dt', t\right) + E_B \left(\int_0^t S_A(t')dt', t\right)$$
$$= E_B \left(\int_0^t S_A(t')dt', t\right) = \sum_{i\dagger\geq 0} cEB_i * \left(\int_0^t S_A(t')dt' - \int_0^t S_B(t')dt'\right)^i ; \quad (37)$$

$$-d_t \frac{S_B}{\sqrt{1-S_B^2}} = -d_t p_B \left(\int_0^t S_B(t')dt', t\right) = E_A \left(\int_0^t S_B(t')dt', t\right) + E_B \left(\int_0^t S_B(t')dt', t\right)$$
$$= E_A \left(\int_0^t S_B(t')dt', t\right) = \sum_{i\dagger\geq 0} cEA_i * \left(\int_0^t S_B(t')dt' - \int_0^t S_A(t')dt'\right)^i, \quad (38)$$

which could be written as

$$d_{tt} \frac{S_A}{\sqrt{1-S_A^2}} = -[S_A - S_B] * \sum_{i\dagger\geq 1} cEB_i * \left(\int_0^t S_A(t')dt' - \int_0^t S_B(t')dt'\right)^{i-1} ; \quad (39)$$

$$d_{tt} \frac{S_B}{\sqrt{1-S_B^2}} = -[S_B - S_A] * \sum_{i\dagger\geq 1} cEA_i * \left(\int_0^t S_B(t')dt' - \int_0^t S_A(t')dt'\right)^{i-1}. \quad (40)$$

Some examples of the solutions of Eqs.(39,40) are presented in Fig. (**3**), which indicates $\max(S_A, S_B)$ decreasing and $\min(S_A, S_B)$ increasing and hence a momentum exchange between two structures. Such an exchange will continue until $S_A - S_B$ becoming 0 and imply two structures being merged into a new structure. Note that the relative ratio between cEA_i and cEB_i affects the saturated value of S. Moreover, how soon to arrive at this saturated value is determined by the values of cEA_i and cEB_i.

2.3.3. Shape Distortion

A periodic structure can be described by a first integral of p_z and its β-profile is restricted to meet $\partial_r \beta + \frac{\beta}{r} + \frac{1}{r}\partial_\theta \lambda = 0$, while an aperiodic one cannot and its β-profile is restricted to meet $\partial_r \beta + \frac{\beta}{r} + \frac{1}{r}\partial_\theta \lambda \neq 0$. If A is aperiodic and B is periodic, their different detailed features in shapes determine that even in the merged new structure $A + B$, two subsystems still affect each other. Noting that the fluid velocity of this $A + B$ structure is

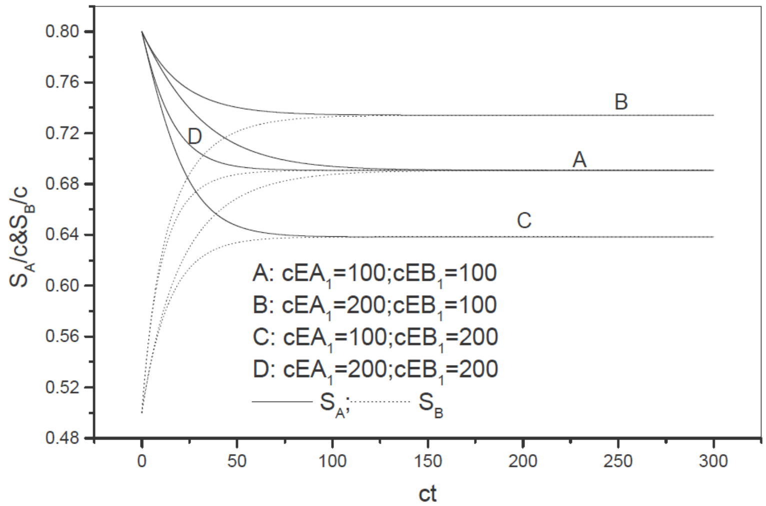

Fig. (3). Examples of the histories of S_A and S_B, where cEA_1, as well as cEB_1, is in unit of $0.511 * 10^6[Volt/\mu m]/\mu m$.

$$u_{A+B} = \frac{n_A u_A + n_B u_B}{n_A + n_B}. \tag{41}$$

Moreover, from

$$\partial_t p(u_B) = \partial_t p(u_A) = \partial_t p(u_{A+B}) = -E, \tag{42}$$

we have

$$p(u_A) - p(u_{A+B}) = C_A(r, \theta, z); \tag{43}$$

$$p(u_B) - p(u_{A+B}) = C_B(r, \theta, z), \tag{44}$$

where $C_A(r, \theta, z)$ and $C_B(r, \theta, z)$ are both t-independent.

>From initial condition, we can find that $\beta_{A+B} = \frac{[n_A u_A + n_B u_B]_r}{[n_A u_A + n_B u_B]_z}$, as well as $\lambda_{A+B} = \frac{[n_A u_A + n_B u_B]_\theta}{[n_A u_A + n_B u_B]_z}$, usually does not meet $\partial_r \beta + \frac{\beta}{r} + \frac{1}{r}\partial_\theta \lambda = 0$. Therefore, $p_z(u_{A+B})$ will be aperiodic and cannot correspond to a first integral. Following the same procedure of deriving Eq. (14) and (24), we obtain a similar equation for $p_z(u_{A+B})$

$$\left[1 - \frac{\eta(1 + \beta_{A+B}^2 + \lambda_{A+B}^2)p_z}{\sqrt{1 + (1 + \beta_{A+B}^2 + \lambda_{A+B}^2)p_z^2}}\right]\partial_{\xi\xi} p_z$$

$$= \left\{\left[\left(\partial_r \beta_{A+B} + \frac{\beta_{A+B}}{r} + \frac{1}{r}\partial_\theta \lambda_{A+B}\right)\partial_\xi p_z\right] - ZN_i\right\}$$

$$\frac{p_z}{\sqrt{1 + (1 + \beta_{A+B}^2 + \lambda_{A+B}^2)p_z^2}}. \tag{45}$$

After solving $p_z(u_{A+B})$ from this equation, we could know $p(u_A)$ and $p(u_B)$ according to Eqs. (43, 44). Once the histories of $p(u_A)$ and $p(u_B)$ are known, the evolutions of n_A and n_B can be strictly calculated from respective continuity equations.

Therefore, when two ES structures encounter, their interaction are displayed by two aspects. As shown in Eqs.(39, 40), their phase velocities are approaching to a common value. On the other hand, their respective shapes are also distorted. For instance, Eq. (45) indicates that the shape of a periodic structure is distorted because of another aperiodic structure.

2.4. Phase Space Structure

First, we consider $B \neq 0$ case. The VE can be written as

$$0 = [\partial_t + v \cdot \nabla - [E + v \times B] \cdot \partial_{p(v)}]f$$

$$= [\partial_t - E \cdot \partial_{p(v)}]f + v \cdot [\nabla - B \times \partial_{p(v)}]f. \tag{46}$$

Moreover, if f is a strict solution of the VE, any mono-variable function of f, or $g(f)$, is also a strict solution.

For the case in which E and B are running waves of a phase velocity $\frac{1}{\eta}c$, i.e. $E = E(r - \frac{1}{\eta}ct)$ and $B = B(r - \frac{1}{\eta}ct)$, we should note a relation between E and B: $E = -\frac{1}{\eta}c \times B + \nabla\Phi(r - \frac{1}{\eta}ct)$, which arises from $\nabla \times E = -\partial_t B$. Here, $\Phi(r - \frac{1}{\eta}ct)$ is a scalar function but cannot be simply taken as ES potential (because $-\frac{1}{\eta}c \times B$ also has divergence or $\nabla \cdot \left(-\frac{1}{\eta}c \times B\right) = \frac{1}{\eta}c \cdot \nabla \times B \neq 0$). In this case, the VE can be further written as

$$0 = [\partial_t - E \cdot \partial_{p(v)}]f + v \cdot [\nabla - B \times \partial_{p(v)}]f$$

$$= \left[\partial_t + \frac{1}{\eta}c \times B \cdot \partial_{p(v)}\right]f + v \cdot [\nabla - B \times \partial_{p(v)}]f - \nabla\Phi \cdot \partial_{p(v)}f$$

$$= \left[\partial_t + \frac{1}{\eta}c \cdot B \times \partial_{p(v)}\right]f + v \cdot [\nabla - B \times \partial_{p(v)}]f - \nabla\Phi \cdot \partial_{p(v)}f$$

$$= \left(v - \frac{1}{\eta}c\right) \cdot [\nabla - B \times \partial_{p(v)}]f - \nabla\Phi \cdot \partial_{p(v)}f. \tag{47}$$

It is easy to verify that any function of $p + \int E(r - \frac{1}{\eta}ct)dt$ will meet

$$0 = [\partial_t - E \cdot \partial_{p(v)}]g\left(p + \int E(r - \frac{1}{\eta}ct)dt\right)$$

$$= [-\frac{1}{\eta}c \cdot \nabla + \frac{1}{\eta}c \cdot B\times \partial_{p(v)}]g$$

$$= -\frac{1}{\eta}c \cdot [\nabla - B\times \partial_{p(v)}]g, \tag{48}$$

where we have used the property $\partial_t \int E(r - \frac{1}{\eta}ct)dt = -\frac{1}{\eta}c \cdot \nabla \int E(r - \frac{1}{\eta}ct)dt$. Thus, if $\nabla\Phi \equiv 0$, any mono-variable function of $p + \int E(r - \frac{1}{\eta}ct)dt$, or $g\left(p + \int E(r - \frac{1}{\eta}ct)dt\right)$, will be a strict solution of the VE.

On the other hand, for more general $\nabla\Phi$, we can find that any mono-variable function of $\sqrt{1 + \frac{p^2}{c^2}} - \frac{1}{\eta}c \cdot p + \Phi$, or $g\left(\sqrt{1 + \frac{p^2}{c^2}} - \frac{1}{\eta}c \cdot p + \Phi\right)$, is a strict solution of the VE. According to Eq.(47), $\partial_p\left[\sqrt{1 + \frac{p^2}{c^2}} - \frac{1}{\eta}c \cdot p\right]$ will contribute a vector parallel to $\left(v - \frac{1}{\eta}c\right)$ and hence make the operator $\left(v - \frac{1}{\eta}c\right) \cdot B\times \partial_{p(v)}$ has zero contribution.

Therefore, for coherent self-consistent fields $E = E(r - \frac{1}{\eta}ct)$ and $B = B(r - \frac{1}{\eta}ct)$, the phase space distribution, if $\nabla\Phi \equiv 0$, can be described by a positive-valued function of $p + \int E(r - \frac{1}{\eta}ct)dt$, for example $\exp\left[-\left(p + \int E(r - \frac{1}{\eta}ct)dt\right)^2\right]$, $\sin^2(\exp\left[-\left(p + \int E(r - \frac{1}{\eta}ct)dt\right)^2\right])$, etc. We can further pick out reasonable solutions according to the definition of u

$$u = \frac{\int \frac{p}{\sqrt{1+p^2}} g\left(p+\int E(r-\frac{1}{\eta}ct)dt\right)d^3p}{\int g\left(p+\int E(r-\frac{1}{\eta}ct)dt\right)d^3p}. \tag{49}$$

Likewise, same procedure exists for more general $\nabla\Phi$ and $g\left(\sqrt{1 + \frac{p^2}{c^2}} - \frac{1}{\eta}c \cdot p + \Phi\right)$.

Actually, a set of macroscopic functions (E, B, u) can have multiple microscopic solutions of corresponding VE. Therefore, usually we know the phase space distribution from the initial condition of the microscopic distribution $f(r, p, t = 0)$. From the function dependence of $f(r, p, t = 0)$ on p, we can obtain the function dependence of g on $\sqrt{1 + \frac{p^2}{c^2}} - \frac{1}{\eta}c \cdot p$ and hence determine detailed function form of g.

Detailed procedure of determining function form of g is described as follows [32]: We can seek for special space position R in which $E(R, 0) = -\frac{1}{\eta}c\times B(R, 0)$, or $\nabla\Phi(r, 0)|_{r=R} = 0$, exists. The initial distribution at R, i.e., $f(R, p, t = 0)$, is thus a mono-variable function p. At the same time, two expressions are equivalent and hence there is $f(R, p, t = 0) = g\big(K + \Phi(R, 0)\big) = g(K)$. where $K = \sqrt{1 + \frac{p^2}{c^2}} - \frac{1}{\eta}c \cdot p$ and $\Phi(R, 0) = 0$ (if $\nabla\Phi(r, 0)|_{r=R} = 0$). Because of certain relation between p and K, once the expansion coefficients c_i in $f(R, p, t = 0) = \sum_i c_i p^i$ is known, the expansion coefficients d_i in $g(K) = \sum_i d_i K^i$ is also easy to be calculated.

Then, we consider $B \equiv 0$ case. Clearly, BGK modes [7, 19] are analytic strict solutions of the VE in $B \equiv 0$ case

$$0 = \left[\partial_t - E\left(r - \frac{1}{\eta}ct\right) \cdot \partial_{p(v)}\right]f + v \cdot \nabla f, \tag{50}$$

whose solutions are mono-variable functions of $\phi\left(r - \frac{1}{\eta}ct\right) + \sqrt{1 + p^2} - \frac{1}{\eta}c \cdot p$, where $\phi\left(r - \frac{1}{\eta}ct\right)$ is scalar potential and $E = -\nabla\phi$. Moreover, there is a similar procedure of determining function form of g.

We should note that K is a nonlinear function of v and the maximum value of K, or $K_{\max}$ is reached at $v = \frac{1}{\eta}c$. Thus, even g is a Dirac function of $K + \Phi$, g cannot be a Dirac function of v, i.e. $g\sim\delta\big(v - u(r, t)\big)$. The nonlinear function relation between K and v determines that g is at least a summation of two Dirac components: $g = f_1(r, t)\delta\big(v - u_1(r, t)\big) + f_2(r, t)\delta\big(v - u_2(r, t)\big) + \ldots$. This agree with previous conclusion that functions of a general form $f_1(r, t)\delta\big(v - u_1(r, t)\big)$ cannot meet VE.

We should also note that, because of nonlinear function relation between g and $K + \Phi$, the maximum of g, or $g_{\max}$, is usually reached at $K + \Phi \neq K_{\max} + \Phi$. Namely, if $g_{\max}$ is reached at $K = K_{g\max}$, this $K_{g\max}$ usually corresponds to two values of v. In contrast, $K_{\max}$ merely corresponds to a value of v. Thus, the contour plot of g in the phase space will take on complicated structures, such as hole, island etc. Some examples of such a complicated phase space structure is shown in Fig. (**4**).

3. SUMMARY

By a universal equation set of (E, B, u), we study the 3-D nonlinear plasma ES wave beyond the A-P model. The first integral of the nonlinear wave are presented. Also, we explain why an unmagnetized charged particles beam must have aperiodic density profile. The interaction between a periodic ES structure and an aperiodic one are studied by this universal equation set. Moreover, we also present a standard procedure for constructing strict solution of VE.

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4. APPENDIX

The fluid velocity is usually defined as

$$u(r_i(t), t) \equiv \frac{\sum_m \delta\big(r_i(t) - r_m(t)\big)d_t r_m(t)}{\sum_m \delta\big(r_i(t) - r_m(t)\big)}.$$

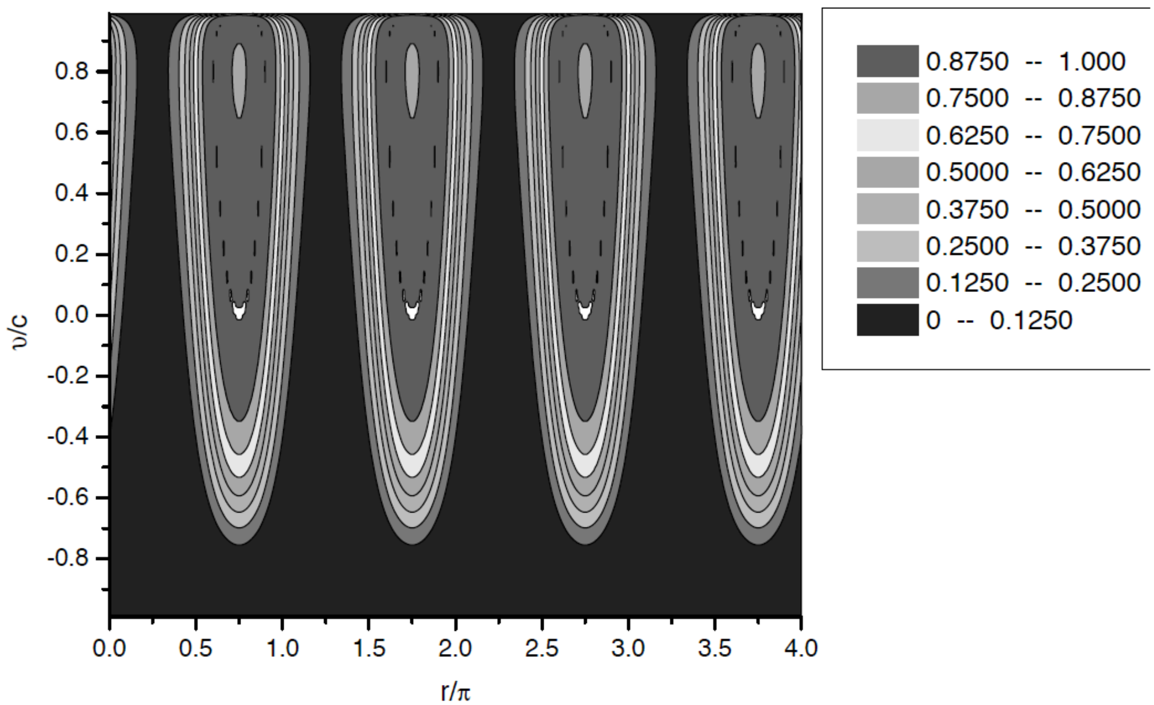

Fig. (4). Example of hole structure and island structure in the phase space contour plot of normalized microscopic distribution, where the self-consistent fields are periodic and have a phase velocity $0.8c$.

We can also use a definition RV

$$RV(r_i(t),t) \equiv \sum_m \delta\big(r_i(t)-r_m(t)\big) * [d_t r_m(t) - d_t r_i(t)]$$

and re-write $u(r_i(t),t)$ as

$$u(r_i(t),t) \equiv \frac{\sum_m \delta\big(r_i(t)-r_m(t)\big) * [d_t r_m(t) - d_t r_i(t)]}{\sum_m \delta\big(r_i(t)-r_m(t)\big)} + d_t r_i(t)$$

$$= \frac{R(r_i(t),t)}{\sum_m \delta\big(r_i(t)-r_m(t)\big)} + d_t r_i(t).$$

Clearly, there is

$$\nabla_{r_i} u(r_i(t),t)$$

$$= \frac{\sum_m \sum_n [\nabla_{r_i}\delta\big(r_i(t)-r_m(t)\big)] * \delta\big(r_i(t)-r_n(t)\big) * [d_t r_m(t) - d_t r_n(t)]}{\big[\sum_m \delta\big(r_i(t)-r_m(t)\big)\big]^2}$$

$$= \frac{-\sum_m \sum_n [\nabla_{r_m}\delta\big(r_i(t)-r_m(t)\big)] * \delta\big(r_i(t)-r_n(t)\big) * [d_t r_m(t) - d_t r_n(t)]}{\big[\sum_m \delta\big(r_i(t)-r_m(t)\big)\big]^2}$$

$$= \frac{-\sum_m \sum_n \nabla_{r_m}\{\delta\big(r_i(t)-r_m(t)\big) * \delta\big(r_i(t)-r_n(t)\big) * [d_t r_m(t) - d_t r_n(t)]\}}{\big[\sum_m \delta\big(r_i(t)-r_m(t)\big)\big]^2}$$

$$= \frac{-\sum_m \nabla_{r_m}\{\delta\big(r_i(t)-r_m(t)\big) * \sum_n \{\delta\big(r_i(t)-r_n(t)\big) * [d_t r_m(t) - d_t r_n(t)]\}\}}{\big[\sum_m \delta\big(r_i(t)-r_m(t)\big)\big]^2}$$

$$= \frac{-\sum_m \nabla_{r_m}\{\delta\big(r_i(t)-r_m(t)\big) * [d_t r_m(t) - d_t r_i(t)] * \sum_n \delta\big(r_i(t)-r_n(t)\big)\}}{\big[\sum_m \delta\big(r_i(t)-r_m(t)\big)\big]^2}$$

$$- \frac{-\sum_m \nabla_{r_m}\{\delta\big(r_i(t)-r_m(t)\big) * RV(r_i,t)\}}{\big[\sum_m \delta\big(r_i(t)-r_m(t)\big)\big]^2}$$

$$= \frac{\sum_n \delta\big(r_i(t)-r_n(t)\big) \sum_m \nabla_{r_i}\{\delta\big(r_i(t)-r_m(t)\big) * [d_t r_m(t) - d_t r_i(t)]\}}{\big[\sum_m \delta\big(r_i(t)-r_m(t)\big)\big]^2}$$

$$- \frac{\sum_m \nabla_{r_m}\{\delta\big(r_i(t)-r_m(t)\big) * RV(r_i,t)\}}{\big[\sum_m \delta\big(r_i(t)-r_m(t)\big)\big]^2}$$

$$= \frac{\nabla_{r_i} RV(r_i,t)}{\big[\sum_m \delta\big(r_i(t)-r_m(t)\big)\big]} - \frac{\sum_m \nabla_{r_m}\{\delta\big(r_i(t)-r_m(t)\big) * RV(r_i,t)\}}{\big[\sum_m \delta\big(r_i(t)-r_m(t)\big)\big]^2},$$

where we have used relations

$$\nabla_{r_i}\delta\big(r_i(t)-r_m(t)\big) = -\nabla_{r_m}\delta\big(r_i(t)-r_m(t)\big);$$

$$\nabla_{r_i} d_t r_i(t) = d_t \nabla_{r_i} r_i(t) = d_t 1 = 0;$$

$$\nabla_{r_m} \sum_n \{\delta\big(r_i(t)-r_n(t)\big) * [d_t r_m(t) - d_t r_n(t)]$$

$$= \sum_n \nabla_{r_m}\{\delta\big(r_i(t)-r_n(t)\big) * [d_t r_m(t) - d_t r_n(t)]\} = 0;$$

$$\sum_n \{\delta\big(r_i(t)-r_n(t)\big) * [d_t r_m(t) - d_t r_n(t)]\}$$

$$= [d_t r_m(t) - d_t r_i(t)] \sum_n \delta\big(r_i(t)-r_n(t)\big) + RV(r_i(t),t);$$

$$\nabla_{r_m}\{\delta\big(r_i(t)-r_m(t)\big) * [d_t r_m(t) - d_t r_i(t)] * \sum_n \delta\big(r_i(t)-r_n(t)\big)\}$$

$$= \sum_n \delta\big(r_i(t)-r_n(t)\big) * \nabla_{r_m}\{\delta\big(r_i(t)-r_m(t)\big) * [d_t r_m(t) - d_t r_i(t)]\};$$

The term ``zero-temperature" means that at any position, all particles at same time-space point have a same velocity and hence

$$RV(r_i(t),t) \equiv \sum_m \delta\big(r_i(t)-r_m(t)\big) * [d_t r_m(t) - d_t r_i(t)] = 0,$$

$$u(r_i(t),t) = d_t r_i(t),$$

Thus, above long formulus will suggest

$$\nabla_{r_i} u(r_i(t),t) = \frac{\nabla_{r_i} 0}{\left[\sum_m \delta\big(r_i(t) - r_m(t)\big)\right]^2} - \frac{\sum_m \nabla_{r_m}\{\delta\big(r_i(t) - r_m(t)\big) * 0\}}{\left[\sum_m \delta\big(r_i(t) - r_m(t)\big)\right]^2} = 0.$$

This clearly explains why ``zero-temperature'' means the absence of the so-called convective term.

CONFLICT OF INTEREST

The author confirms that this article content has no conflicts of interest.

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