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An Experimental Investigation into the Effective Permeability of Porous Media whose Matrices are Composed of Obstacles of Different Sizes

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Abstract: A comprehensive experimental investigation has been conducted to determine the effective permeability of fluid saturated porous media consisting of small and large spherical particles. Small and large particles were mixed uniformly at a certain mixture ratio so as to form a vertical porous column. Water was drawn from a reservoir to flow through the vertical tube filled with particles of different sizes. The resulting pressure drops between the inlet and outlet sections of the test column were measured at various flow rates to determine the effective permeability. The results were compared with the present theoretical model generalized on the basis of the analysis reported by Liu, Sano and Nakayama for fractured porous media. It has been confirmed that the present mathematical model agrees very well with the experimental data and thus can be used to estimate the permeability of porous media consisting of obstacles of different sizes.

Keywords: Permeability, porous media, pressure measurement, Darcy's law.

INTRODUCTION

The measure of the flow conductance, namely, the permeability, is one of the most important parameters, when we investigate transport phenomena in saturated porous media. The permeability can be determined from the application of the first principles of mass and momentum conservation to flow of viscous fluids at the pore scale. Such theoretical attempts have been made by a number of investigators including Eidsath *et al*. [1], Couland *et al*. [2], Fowler and Bejan [3], Larson and Higdon [4] and Nakayama *et al*. [5]. A comprehensive review on this topic may be found in Nakayama and Kuwahara [6]. However, when the medium consists of particles of different sizes, numerical approaches for the effective permeability based on the first principles become quite formidable due to complexity in describing geometrical details at the pore scale.

For such complex porous media consisting obstacles of different sizes, Liu *et al*. [7] introduced an elegant mathematical model to determine the effective permeability, extending the analytical solution [8] based on the Brinkman-Darcy model to the cases of fractured porous media. They focused on the equivalent continuum model and proposed a rational mathematical model for determining the equivalent permeability of the fractured porous medium. The fractured porous medium is modeled as an array of square blocks placed in a fluid saturated porous medium. The absolute permeability of the blocks (i.e. rocks) is assumed much smaller than that of the fractures. Upon assuming a horizontal flow through fractures, an analytical expression was derived for determining the equivalent permeability of the fractured porous media. They also conducted a series of numerical calculations using a single structural unit with periodic boundary conditions. Then, the equivalent permeability was evaluated from the microscopic velocity and pressure fields so as to examine the validity of the analytical expression. Subsequently, their three-dimensional model was extended to a three dimensional case in which the cubic rocks are arranged in a cubic array. The resulting analytical expression for the equivalent permeability has been found to agree very well with both existing formula and microscopic numerical simulation.

We shall extend their expression for the fractured porous media to estimate the effective permeability of porous media consisting of obstacles of different sizes, since such porous media, whose effective permeability is comparatively low, are often found in engineering applications. In order to examine the proposed general formula, we shall conduct an exhaustive experiment using water saturated porous column to measure the effective permeability of the porous media consisting of spherical glass particles of different sizes. The experimental data thus obtained from pressure drop measurements are compared against the extended formula to elucidate the validity of the proposed general formula.

GENERALIZATION OF THE LIU-SANO-NAKAYAMA MODEL FOR PERMEABILITY

Liu *et al*. [7] consider a dual structured porous medium as shown in Fig. (**1**). The cubic blocks of size D are arranged in a cubic array, whereas the horizontal and vertical fractures of aperture d are filled with smaller particles, characterized by the absolute permeability K_1 and porosity ε_1.

They analytically derived the following expression for the effective permeability K_2 :

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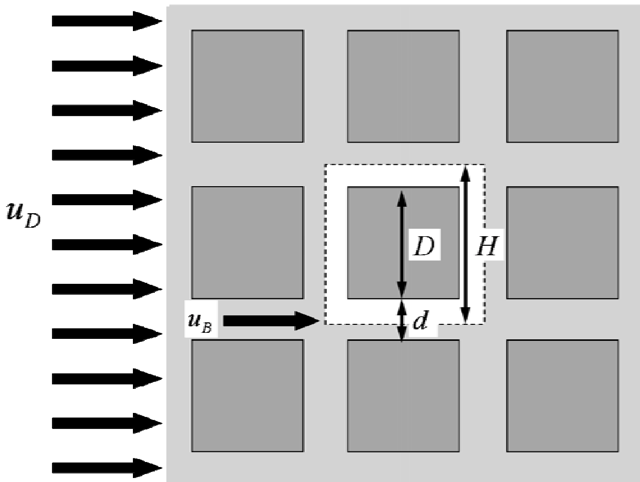

Fig. (1). Mathematical model having both vertical and horizontal fractures.

$$K_2 = \frac{\mu u_D}{\left(-\dfrac{dp}{dx}\right)} = f\left(\varepsilon_2, \varepsilon_1, \alpha_2\right) D^2$$
$$= \left(\frac{\left(1-\left(1-\varepsilon_2\right)^{1/3}\right)^2 \left(1-\left(1-\varepsilon_2\right)^{2/3}\right)\varepsilon_1}{4\left(1-\varepsilon_2\right)^{2/3}} \left(\frac{\alpha_2 \cosh\alpha_2 - \sinh\alpha_2}{\alpha_2^3 \cosh\alpha_2} \right) \right) D^2 \tag{1}$$

where

$$\alpha_2 = \frac{d}{2}\sqrt{\frac{\varepsilon_1}{K_1}} \tag{2}$$

is the dimensionless number related to the fracture aperture d and square root of absolute fracture permeability K_1. Furthermore, μ is the viscosity while u_D is the Darcian velocity. Moreover, $\varepsilon_2 = \left(H^3 - D^3\right)/H^3$ is the volume fraction occupied by the fracture space (i.e. fracture porosity), such that the volume fraction of the void, namely, the effective porosity is given by $\varepsilon_1\varepsilon_2$. For the limiting case of $\alpha_2 \to 0$ and $\varepsilon_1 \to 1$, Equation (2) reduces to:

$$K_2\big|_{\alpha_2 \to 0} = f\left(\varepsilon_2, 1, 0\right) D^2 = \frac{\left(1-\left(1-\varepsilon_2\right)^{1/3}\right)^2 \left(1-\left(1-\varepsilon_2\right)^{2/3}\right)}{12\left(1-\varepsilon_2\right)^{2/3}} D^2 \tag{3}$$

They also conducted an exhaustive series of three-dimensional computations for a wide range of the porosity, using a collection of cubes, and found that the foregoing analytical formula closely agrees with the empirical Carman-Kozeny equation for the packed beds [9]:

$$K_2\big|_{\alpha \to 0} = \frac{\varepsilon_2^3}{180\left(1-\varepsilon_2\right)^2} D^2 : \text{Carman-Kozeny} \tag{4}$$

and the numerical results using a collection of cubes.

Note that their equation (3) is not for the spherical particles but for the cubes of size D. However, the equation is found to serve equally well to estimate the permeability of the spherical obstacles of diameter D. Similar evidence can be found in Nakayama and Kuwahara [6] in which they carried out a series of two- and three-dimensional computations using various collections of square rods, circular rods, cubes and spheres, and reported that all resulting expressions as function of the porosity and obstacle size turned out to be close to one another. Thus, their equation (1) for the case of the fractured porous media may well be generalized for the case of the porous media consisting of spherical obstacles of different sizes.

Suppose that we fill every void space in a packed bed of large spherical particles of size D_2 with sufficiently small spherical particles of size D_1 to construct a porous medium consisting of small and large particles. Then, its dimensionless parameter α_2 may be estimated as follows:

$$\alpha_2 = \frac{d}{2}\sqrt{\frac{\varepsilon_1}{K_1}} = \frac{1-\left(1-\varepsilon_2\right)^{1/3}}{2\left(1-\varepsilon_2\right)^{1/3}} \sqrt{\frac{\varepsilon_1}{f\left(\varepsilon_1, 1, 0\right)}} \frac{D_2}{D_1}$$
$$= \left(\frac{1-\varepsilon_1}{1-\varepsilon_2}\right)^{1/3} \frac{1-\left(1-\varepsilon_2\right)^{1/3}}{1-\left(1-\varepsilon_1\right)^{1/3}} \sqrt{\frac{3\varepsilon_1}{1-\left(1-\varepsilon_1\right)^{2/3}}} \frac{D_2}{D_1} \tag{5}$$

where we used the geometrical relationship $d/D_2 = \left(1-\left(1-\varepsilon_2\right)^{1/3}\right)/\left(1-\varepsilon_2\right)^{1/3}$ to eliminate d in favor of D_2. Substituting α_2 as given above into Equation (1), we readily obtain the effective permeability of the porous medium consisting large particles of size D_2 and small particles of size D_1.

It is easy to generalize the foregoing results further to estimate the effective permeability K_n of the porous medium consisting of n sets of spherical particles of different sizes, namely, $D_n > D_{n-1} > \ldots > D_2 > D_1$, as follows:

$$K_i = \left(\frac{\left(1-\left(1-\varepsilon_i\right)^{1/3}\right)^2 \left(1-\left(1-\varepsilon_i\right)^{2/3}\right)\varepsilon_{i-1}}{4\left(1-\varepsilon_i\right)^{2/3}} \left(\frac{\alpha_i \cosh\alpha_i - \sinh\alpha_i}{\alpha_i^3 \cosh\alpha_i} \right) \right) D_i^2 \quad \text{for}$$
$$i = 2, 3, , , n \tag{6}$$

where

$$\alpha_i = \frac{1-\left(1-\varepsilon_i\right)^{1/3}}{2\left(1-\varepsilon_i\right)^{1/3}} \sqrt{\frac{\varepsilon_{i-1}}{K_{i-1}}} D_i \quad \text{for } i = 2, 3, , , n \tag{7}$$

and

$$K_1 = \frac{\left(1-\left(1-\varepsilon_1\right)^{1/3}\right)^2 \left(1-\left(1-\varepsilon_1\right)^{2/3}\right)}{12\left(1-\varepsilon_1\right)^{2/3}} D_1^2 \tag{8}$$

Note that the volume fraction of the particle of size D_n is given by $1-\varepsilon_n$ whereas that of the particle of size D_i (for

$i = 2,, n-1$) is given by $\varepsilon_{i+1}\left(1-\varepsilon_i\right)$. Naturally, the effective porosity is given by $\varepsilon_n \varepsilon_{n-1} \cdots \varepsilon_2 \varepsilon_1$.

The effective permeability K_3 for the case of n=3, for example, may be evaluated as

$$K_3 = \left(\frac{\left(1-\left(1-\varepsilon_3\right)^{1/3}\right)^2 \left(1-\left(1-\varepsilon_3\right)^{2/3}\right)\varepsilon_2}{4\left(1-\varepsilon_3\right)^{2/3}} \left(\frac{\alpha_3 \cosh\alpha_3 - \sinh\alpha_3}{\alpha_3^3 \cosh\alpha_3} \right) \right) D_3^2 \tag{9}$$

where

$$\alpha_3 = \frac{1-\left(1-\varepsilon_3\right)^{1/3}}{2\left(1-\varepsilon_3\right)^{1/3}} \sqrt{\frac{\varepsilon_2}{K_2}} D_3 \tag{10}$$

$$K_2 = \left(\frac{\left(1-\left(1-\varepsilon_2\right)^{1/3}\right)^2 \left(1-\left(1-\varepsilon_2\right)^{2/3}\right)\varepsilon_1}{4\left(1-\varepsilon_2\right)^{2/3}} \left(\frac{\alpha_2 \cosh\alpha_2 - \sinh\alpha_2}{\alpha_2^3 \cosh\alpha_2} \right) \right) D_2^2 \tag{11}$$

and

$$\alpha_2 = \left(\frac{1-\varepsilon_1}{1-\varepsilon_2} \right)^{1/3} \frac{1-\left(1-\varepsilon_2\right)^{1/3}}{1-\left(1-\varepsilon_1\right)^{1/3}} \sqrt{\frac{3\varepsilon_1}{1-\left(1-\varepsilon_1\right)^{2/3}}} \frac{D_2}{D_1} \tag{12}$$

EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental set-up used in the study is schematically illustrated in Fig. (**2**). Water was pumped upward through a vertical acrylic tube using the variable speed pump (SANSO PMD 1511BZE) with the voltage inverter (YAMABISHI S130-20). The test section of the tube whose inner diameter is 39 mm was filled with spherical glass particles, using two or three sets of particles of different diameter. Four sets of glass particles were prepared to construct the column consisting particles of different diameter. Their diameters are 0.5mm, 1.0mm, 2.3mm and 5.0mm, respectively. Different sets of particles were mixed evenly to form a macroscopically uniform porous medium with constant porosities, ε_1, ε_2 and ε_3. These particles were held within the test section using wire meshes. The Darcian velocity u_D was determined from measuring the flow rate, and was set sufficiently low in the range from 0.001 to 0.01 m/s such that non-Darcian effects can be eliminated from the pressure measurements. The pressure drops were measured using a pair of pressure transducers (Tokyo Sokki Kenkyujo PW500KPA and PW100KPA) placed at both inlet and outlet of the test section. The output signals from the transducers were passed through the signal conditioners (Tokyo Sokki Kenkyujo TD-91). Subsequently, their outputs were fed into the data logger (M-SYSTEM Giken R1M-GH), which were then processed by the personal computer. Since the resistance due to the presence of meshes and tube wall is negligibly small, the effective permeability may easily be obtained from the pressure drop measurements:

$$K_2 = \frac{\mu u_D L}{\Delta p - \rho g L} \tag{13}$$

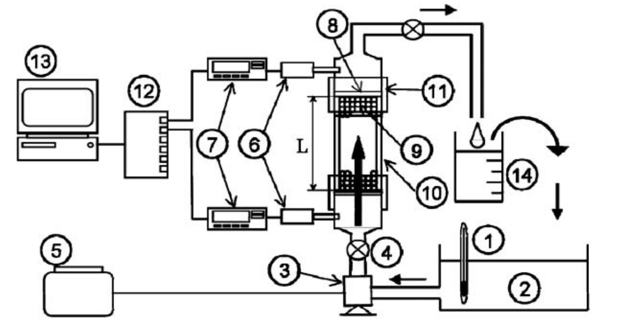

Fig. (2). Expremental set-up.

where Δp is the pressure drop between the inlet and outlet of the test section, at a distance L =517 mm apart.

RESULTS AND DISCUSSION

In the Darcy flow regime, the pressure drop Δp increases linearly with the Darcian velocity u_D following Equation (13). A typical linear relationship between the reduced pressure $\left(\Delta p - \rho g L\right)$ drop and Darcian velovity u_D is illustrated in Fig. (**3**) for the case of ε_1 =0.48, ε_2 = 0.43, D_1= 0.5 mm and D_2= 5 mm. The corresponding effective permeability can easily be determined by fitting these experimental data to the linear function using the least squares method and reading its slope. For each flow condition, three consecutive runs were made to construct $\left(\Delta p - \rho g L\right)$- u_D plot for determining the effective permeability. The experimental uncertainty in the effective permeability was found less than 10%.

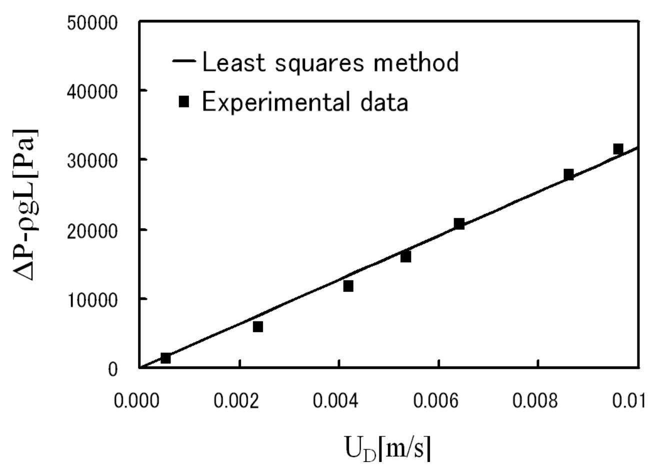

Fig. (3). Pressure-Darcian velocity relationship.

In order to investigate the generality acquired in the present expression for the effective permeability, a series of the pressure measurements were carried out for various sets of parameters. We mixed large and small particles evenly to fill the test column space, forming a homogeneous porous medium there. The mixing ratios of large and small particles were varied to change the parameter α_2 which is the function of the porosities ε_1 and ε_2. These values ε_1 and ε_2 were determined noting that the volume fractions of the large and small particles to the total volume of the column are given by $(1-\varepsilon_2)$ and $\varepsilon_2(1-\varepsilon_1)$, respectively. For the given set of ε_1 and ε_2, the corresponding parameter α_2 was evaluated according to Equation (12). The values of the effective permeability obtained from the pressure drop measurements are plotted against the dimensionless parameter α_2 in Fig. (**4**) for the case of D_1= 0.5 mm and D_2= 2.3 mm. In the figure, the theoretical curve generated by Equations (6) to (8) is presented along with the experimental data. The figure shows an excellent agreement between the present expression and experiment.

All experimental data obtained for three sets of small and large particle mixture, namely, (D_1, D_2)= (0.5 mm, 1.0 mm), (0.5 mm, 2.3 mm) and (0.5 mm, 5.0 mm), are assembled and presented together in Fig. (**5**). The three sets of the experimental data closely follow their corresponding curves generated by Equations (6) to (8) based on the present theoretical model. The experimental and theoretical values of the effective permeability are tabulated in Table **1** for perusal.

Pressure measurements have been also conducted using three sets of particles of different sizes, namely, (D_1, D_2, D_3)=(0.5 mm, 2.3 mm, 5.0 mm), to see if the general expression given by Equations (9) to (12) works equally well for determining the effective permeability of the porous medium consisting of three sets of spherical particles of different sizes. The measured values of the effective permeability K_3 are compared against the theoretical values in Table **2**, which indicates fairly good agreement between the experiment and theory, substantiating the applicability of the present formula for the porous media consisting of more than two sets of particles of different sizes. In the same table, the values based on Dullien's formula are presented for comparison. The present formula gives a fairly good agreement while Dullien's formula tends to overestimate the permeability to some extent.

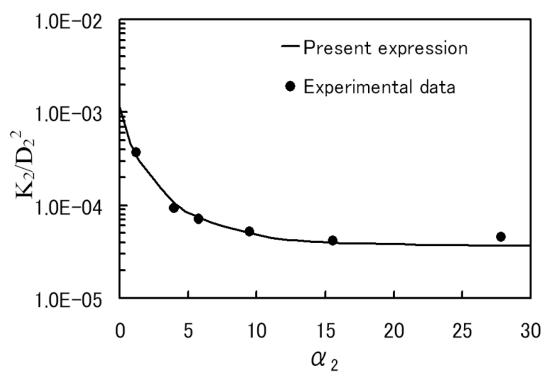

Fig. (4). Variation of effective permeability for the case of D_1 = 0.5 mm and D_2 = 2.3 mm.

CONCLUSIONS

Exhaustive pressure measurements were conducted using a vertical tube filled with particles of different sizes. Water

Table 1. Effective Permeability K_2, Porosities (ε_1, ε_2) and Dimensionless Parameter α_2

(D_1,D_2) = (0.5 mm, 1.0 mm)							
ε_1		0.86	0.76	0.73	0.66	0.53	0.49
ε_2		0.44	0.47	0.51	0.57	0.67	0.76
α_2		0.85	1.49	1.90	2.84	6.15	9.81
$(K_2/D_2^2)\times10^4$	Experiment	5.18	4.99	6.62	5.83	3.02	2.61
	Present formula	7.83	6.59	5.73	4.65	3.26	2.73
(D_1,D_2) = (0.5 mm, 2.3 mm)							
ε_1		0.89	0.66	0.58	0.52	0.46	0.42
ε_2		0.37	0.43	0.47	0.56	0.65	0.78
α_2		1.18	3.94	5.75	9.46	15.6	27.7
$(K_2/D_2^2)\times10^5$	Experiment	37.3	9.42	7.08	5.20	4.20	4.59
	Present formula	34.6	10.8	6.89	5.53	3.94	3.66
(D_1,D_2) = (0.5 mm, 5.0 mm)							
ε_1		0.48	0.48	0.42	0.42	0.40	0.39
ε_2		0.43	0.46	0.47	0.64	0.72	0.79
α_2		15.3	17.6	23.8	38.3	52.8	69.6
$(K_2/D_2^2)\times10^6$	Experiment	7.46	6.44	6.03	7.30	7.35	8.79
	Present formula	6.24	6.56	4.35	5.67	5.56	6.09

Table 2. Effective Permeability K_3, Porosities (ε_1, ε_2, ε_3) and Dimensionless Parameter α_3

(D_1,D_2,D_3) = (0.5 mm, 2.3 mm, 5.0 mm)							
ε_1		0.53	0.61	0.56	0.52	0.50	0.53
ε_2		0.80	0.57	0.68	0.71	0.74	0.60
ε_3		0.54	0.72	0.78	0.79	0.81	0.84
α_2		20.9	7.27	12.2	15.4	18.1	10.4
α_3		30.1	40.4	60.7	72.9	83.3	89.5
$(K_3 / D_3^2)\times 10^6$	Experiment	8.27	12.3	12.0	9.96	10.1	9.92
	Dullien's formula	12.8	15.0	16.7	13.0	12.8	10.7
	Present formula	7.80	13.2	12.4	9.50	9.14	8.94

was pumped from a reservoir to flow upward through the vertical column composed of the particles. Four sets of glass particles were used to fill the vertical column to make various porous media consisting particles of different diameter. The values of effective permeability were obtained reading the slope of the measured pressue-velocity plot. They are found to agree very well with the theoretical model generalized on the basis of the analysis reported by Liu, Sano and Nakayama for fractured porous media. It has been found that the present mathematical model is quite useful for estimating the permeability of porous media consisting of even more than two sets of particles of different sizes.

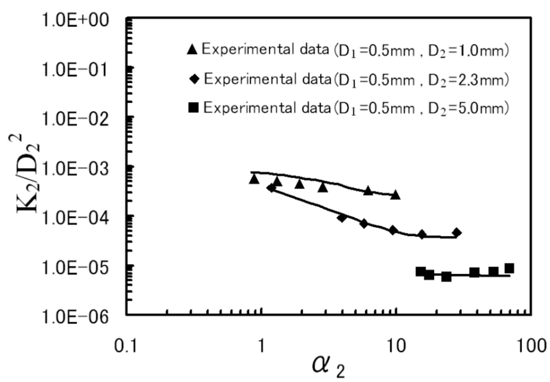

Fig. (5). Effect of α_2 on effective permeability.

NOMENCLATURE

d = Fracture aperture

D_i = Size of particles (i=1,2,3)

g = Acceleration of gravity

H = Size of unit structure

L = Distance between the pressure transducers

K_i = Permeability (i=1,2,3)

p = Pressure

u_D = Darcian velocity

x = Streamwise coordinate

ε = Porosity

α = $\alpha = (d/2)\sqrt{\varepsilon/K}$, dimensionless parameter

ρ = Fluid density

μ = Viscosity

Subscripts

e = Effective

i = Index of particles, i=1 for the smallest particles

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