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Non-Darcy Mixed Convection with Thermal Dispersion-Radiation in a Saturated Porous Medium

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Abstract: In the present work melting with thermal dispersion and radiation on non-Darcy, mixed convective heat transfer from an infinite vertical plate embedded in a saturated porous medium is studied. Both aiding and opposing flows are examined. Forchheimer extension for the flow equations in steady state is considered Similarity solution for the governing equations is obtained .The equations are numerically solved using Runge-kutta fourth order method coupled with shooting technique. The effects of melting (M), thermal dispersion (D), radiation(R) inertia (F) and mixed convection (Ra/Pe) on velocity distribution, temperature and Nusselt number are examined. It is observed that the Nusselt number decreases with increase in melting parameter and increases with increase in the combined effect of thermal dispersion and radiation.

Keywords: Porous medium, melting, thermal dispersion, thermal radiation, mixed convection.

INTRODUCTION

Heat transfer in porous media is seen both in natural phenomena and in engineered processes .It is replete with the features that are influences of the thermal properties and volume fractions of the materials involved. These features are seen of course as responses to the causes that force the process in to action. For instance many biological materials, whose outermost skin is porous and pervious, saturated or semi saturated with fluids give- out and take- in heat from their surroundings. Industrial fluids in interaction with heat supply agencies and flowing over porous beds carry convective heat to different regions of their field of flow near and distant from the heat source.

The study of radiative heat and mass transfer in convection flow is found to be most important in industrial and technologies. The applications are often found in situation the such as fiber and granules insulation, geothermal systems in the heating and cooling chamber, fossil fuel combustion, energy processes and Astro-physical flows. Further, the magneto convection place an important role in the control of mountain iron flow in the steady industrial liquid metal cooling in nuclear reactors and magnetic separation of molecular semi conducting materials. A classical example using the nuclear power stations is separation of Uranium U_{235} from U_{238} by gases diffusion. When mass transfer takes the place in a fluid rest, the mass is transformed purely by molecular diffusion a result identified from concentration gradient. For low concentration of the mass in the fluid and low mass transfer the convective heat and mass transfer processes are similar in nature. Further, the problem of convective heat transfer plays an exhaustive role in applications in soil physics, geothermal energy extraction, chemical engineering, glass production, furnace design, space technology application, flight aerodynamics, plasma physics which operates at extremely high temperature radiation effects are found to be highly prominent and significant. In all above applications understanding the boundary layer development and heat transfer characteristics are of primary requirement to investigate the problem more exhaustively and intensively in detail.

Extensive analysis had been carried out with combined heat and mass transfer under the assumption of different physical real life situation. Radiation in free convection is also examined by several investigators due to their important applications in the areas of prime importance. This is due to the significant role of thermal radiation in surface heat transfer when convection heat is assumed to be relative small particularly in free convection problems involving and absorbing emitting fluids.

In early studies on porous media Darcy's law is employed which is a linear empirical relation between the velocity and the pressure drop across the porous medium. Subsequently, Darcian law has been modified to include the effects of inertia. Nakayama and pop [1] while Lai and Kulacki [2] analyzed the inertial effects on mixed convection along a vertical wall using the Forchheimer flow model. A review of both natural and mixed convection boundary-layer flows in Darcian and non-Darcian fluid saturated porous media is given in Nield and Bejan [3].

In studies of heat transfer associated with melting in a porous medium Kazmierczak *et al.* [4] examined the velocity, temperature and Nusselt number in the melting region from a flat plate in the presence of steady natural convection.

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Bakier [5] studied the melting effect on mixed convection from a vertical plate of arbitrary wall temperature both in aiding and opposing flows in a fluid saturated porous medium. It was observed that, the melting phenomena decrease the local Nusselt number at solid-liquid Interface. The problem of mixed convection in melting from a vertical plate of uniform temperature in a saturated porous medium has been extensively studied by Gorla *et al.* [6]. It was noticed that, the melting process is analogous to mass injection and blowing near the boundary and thus reduces the heat transfer through solid liquid interface. In [7] Tashtoush analysed the magnetic and buoyancy effects on melting from a vertical plate by considering the Forchheimer's extension. It was noticed that, the velocity and temperature profiles and heat transfer rate of melting phenomena associated with uniform wall temperature using the collocation finite element method. In such an analysis, the effect of inertial forces on flow and heat transfer was noticed prominently. Recently, Cheng and Lin [8] studied the melting effect on mixed convective heat transfer form a solid porous vertical plate with uniform wall temperature embedded in the liquid saturated porous medium by using Runge-kutta Gill method and Newton's iteration for similarity solutions. They had established the criteria for $\left(G_r / R_e\right)$ values for forced mixed and free convection form an isothermal vertical plate in porous media with aiding and opposing external flows in melting process. Effect of melting and thermo-diffusion on natural convection heat mass transfer in a non-newtonian fluid saturated non-Darcy porous medium was studied by R.R.Kairi and P.V.S.N.Murthy [9]. It is noted that the velocity, temperature and concentration profiles as well as the heat and mass transfer coefficients are significantly affected by the melting phenomena and thermal-diffusion in the medium.

Study of thermal dispersion effects becomes, prevalent in the porous media flow region. Hong and Tien [10] examined analytically the effect of transverse thermal dispersion on natural convection from a vertical, heated plate in a porous medium. Their results show that due to the better mixing of the thermal dispersion effect, the heat transfer rate is increased. Plumb [11] modeled thermal dispersion effects over a vertical plate. Murthy [12] analyzed the non-Darcy mixed convection flow and heat transfer about an isothermal vertical wall embedded in a fluid saturated porous medium considering the effects of thermal dispersion and viscous dissipation in both aiding and opposing flows. It was noticed that, the thermal dispersion effect enhances the heat transfer rate and the effect of viscous dissipation is to increase with increasing values of the dispersion parameter.

Further, at high temperatures thermal radiation can significantly affect the heat transfer and the temperature distribution in the boundary layer flow of participating fluid. Murthy *et al.* [13] has studied the combined effect of radiation and mixed convection from a vertical wall with suction / injection in a non – Darcy porous medium. It was established that Nusselt number increases with increase in radiation parameter and also with increase in fluid suction parameter. Mohammadien & El-Amin [14] studied the thermal dispersion and radiation effects on Non-Darcy natural convection in a fluid saturated porous medium considering Forchheimer extension. They found that neglecting thermal dispersion there is no variation in heat transfer rate with varying Rayleigh number. However, in non – Darcy case, the increase in value of Rayleigh number reduced the heat transfer rate.

The present paper is aimed at analyzing the effect of melting with thermal dispersion and radiation on non-Darcy (Forchheir flow model) mixed convective heat transfer from a vertical plate embedded in a saturated porous medium. Both aiding and opposing flows are considered in the study. The mathematical model of thermal dispersion adopted in this paper is that of Plumb [11]. The inclusion of thermal dispersion modifies energy equation and also the condition at the plate. By denoting the x component of velocity u at large distance of the plate as u_∞ we can recover the melting conditions without thermal dispersion from the present mathematical formulation of the problem. The radiation heat flux is approximated with the Rosseland approximation. The governing equations are solved by employing fourth order Runge-Kutta method along with the shooting technique.

Mathematical Formulation

A mixed convective heat transfer in a non-Darcian porous medium saturated with a homogeneous Newtonian fluid adjacent to a vertical plate, with a uniform wall temperature is considered. This plate constitutes the interface between the liquid phase and the solid phase during melting inside the porous matrix at steady state. The plate is at a constant temperature T_m at which the material of the porous matrix melts. Fig. (**1**) shows the coordinates and flow model. The x-coordinate is measured along the plate and the y-coordinate normal to it. The solid phase is at temperature $T_0 < T_m$. A thin boundary layer exists close to the right of the vertical plate and temperature changes smoothly through this layer from T_m to T_∞ $\left(T_m < T_\infty\right)$ which is the temperature of the fluid phase.

Taking into account the effect of thermal dispersion and thermal radiation, the governing equations, for steady non-Darcy flow in a saturated porous medium, can be written as follows.

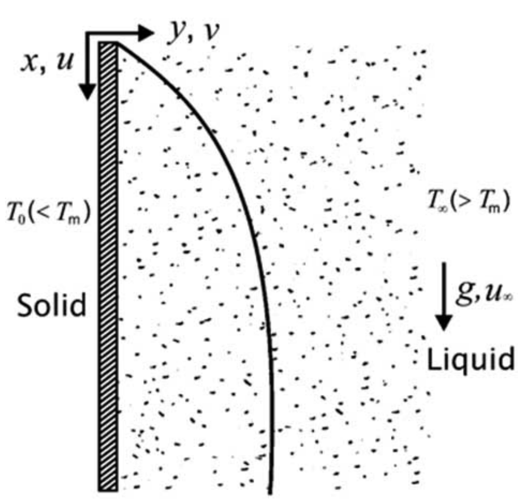

Fig. (1). Schematic diagram of the problem.

The continuity equation is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

The momentum Equation [7] is

$$\frac{\partial u}{\partial y}+\frac{2C\sqrt{K}}{\upsilon}u\frac{\partial u}{\partial y}=-\frac{Kg\beta}{\upsilon}\frac{\partial T}{\partial y} \tag{2}$$

The Energy Equation [12, 13] is

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y}\right)-\frac{1}{\rho c_P}\frac{\partial q}{\partial y} \tag{3}$$

Here u and v are the velocities along the x- y-directions respectively, T is the temperature in the thermal boundary layer, K is the permeability, C is the empirical constant, β is the coefficient of thermal expansion υ is the kinematic viscosity, ρ is the density, c_p is the specific heat at constant pressure, g is the acceleration due to gravity, and thermal diffusivity $\alpha = \alpha_m + \alpha_d$, where α_m is the molecular diffusivity and α_d is the dispersion thermal diffusivity due to mechanical dispersion. As in the linear model proposed by Plumb [11] the dispersion thermal diffisivity α_d is proportional to the velocity component that is $\alpha_d = \gamma ud$, where γ is the dispersion coefficient and d is the mean particle diameter.

The radiative heat flux term q is written using the Rosseland approximation (Sparrow and Cess [15], Raptis [16]) as

$$q=-\frac{4\sigma_R}{3a}\frac{\partial T^4}{\partial y} \tag{4}$$

Where σ_R is the Stefan - Boltzmann constant and 'a' is the mean absorption coefficient.

The necessary boundary conditions for this problem are

$$y=0,\ T=T_m,\ k\frac{\partial T}{\partial y}=\rho\left[h_{sf}+c_s\left(T_m-T_0\right)\right]v \tag{5}$$

$$\text{and}\quad y\to\infty,\ \ T\to T_{\infty,}\ \ u=u_\infty \tag{6}$$

Where h_{sf} and c_s are latent heat of the solid and specific heat of the solid phases, respectively and u_∞ is the assisting external flow velocity, $k=(\rho c_p)\alpha$ is the effective thermal conductivity of the porous medium.

The boundary condition (5) means that the temperature on the plate is constant and thermal flux of heat conduction to the melting surface is equal to the sum of the heat of melting and the heat required for raising the temperature of solid T_0 to its melting temperature T_m.

Introducing the stream function ψ with $u=\frac{\partial\psi}{\partial y}$, and $v=-\frac{\partial\psi}{\partial x}$

The continuity equation (1) will be satisfied and the Eqs. (2) & (3) transform to

$$\frac{\partial^2\psi}{\partial y^2}+2\frac{C\sqrt{K}}{\upsilon}\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)=-\frac{Kg\beta}{\upsilon}\frac{\partial T}{\partial y} \tag{7}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left[\left(\alpha_m+\gamma\frac{\partial\psi}{\partial y}d\right)\frac{\partial T}{\partial y}\right]+\frac{4\sigma_R}{3\rho c_P a}\frac{\partial}{\partial y}\left[\frac{\partial T^4}{\partial y}\right] \tag{8}$$

Introducing the similarity variables as

$$\psi=f(\eta)\left(\alpha_m u_\infty x\right)^{\frac{1}{2}},\ \eta=\left(\frac{u_\infty x}{\alpha_m}\right)^{\frac{1}{2}}\left(\frac{y}{x}\right)\ \text{and}$$

$$\theta(\eta)=\frac{T-T_m}{T_\infty-T_m} \tag{9}$$

The momentum Equation (Eq. (7)) and energy Equation (Eq. (8)) are reduced to

$$\left(1+Ff'\right)f''+\frac{Ra_x}{Pe_x}\theta'=0 \tag{10}$$

$$\left(1+Df'\right)\theta''+\left(\frac{1}{2}f+Df''\right)\theta'+\frac{4}{3}R\left[\left(\theta+c_r\right)^3\theta''+3\theta'^2\left(\theta+c_r\right)^2\right]=0 \tag{11}$$

where the prime symbol denotes the differentiation with respect to the similarity variable η and $\frac{Ra_x}{Pe_x}$ is the mixed convection parameter, which is taken as positive when the buoyancy is aiding the external flow and as negative when the buoyancy is opposing the external flow,. $c_r=\frac{T_m}{T_\infty-T_m}$ is the temperature ratio which assumes very small values by the definition (taken as 0.1), $Ra_x=\frac{g\beta K\left(T_\infty-T_m\right)x}{\upsilon\alpha_m}$ is the local Rayleigh number, $Pe_x=\frac{u_\infty x}{\alpha_m}$ is the loacal pecklet number, $F=\frac{2C\sqrt{K}u_\infty}{\upsilon}$ is the flow inertia coefficient, $D=\frac{\gamma du_\infty}{\alpha_m}$ is the thermal dispersion parameter and $R=\frac{4\sigma_R\left(T_\infty-T_m\right)^3}{ka}$ is the radiation parameter.

Taking into consideration, the thermal dispersion effect together with melting, the boundary conditions Eq. (5) and Eq. (6) take the form

$$\eta=0,\ \theta=0,\ f(0)+\left\{1+Df'(0)\right\}2M\theta'(0)=0 \tag{12}$$

$$\text{and}\quad \eta\to\infty,\ \theta=1,\ \ f'=1 \tag{13}$$

where $M=\frac{c_f\left(T_\infty-T_m\right)}{h_{sf}+c_s\left(T_m-T_0\right)}$ is the melting parameter.

The local heat transfer rate from the surface of the plate is given by

$$q_w = -k\left[\frac{\partial T}{\partial y}\right]_{y=0} \tag{14}$$

The Nusselt number is

$$Nu = \frac{hx}{k} = \frac{q_w x}{k\left(T_m - T_\infty\right)} \tag{15}$$

Where h is the local heat transfer coefficient and k is the effective thermal conductivity of the porous medium which is the sum of the molecular thermal conductivity k_m and the dispersion thermal conductivity k_d [11]. Substituting Eqs. (4), (9) and (14) in (15), the modified Nusselt number is obtained as

$$\frac{Nu_x}{\left(Pe_x\right)^{\frac{1}{2}}} = \left[1 + \frac{4}{3}R\left(\theta(0) + c_r\right)^3 + Df'\left(0\right)\right]\theta'\left(0\right) \tag{16}$$

Solution Procedure

The dimensionless equations (10) and (11) together with the boundary conditions (12) and (13) are solved numerically by means of the fourth order Runge-kutta method coupled with the shooting technique. The solution thus obtained is matched with the given values of $f'\left(\infty\right)$ and $\theta\left(0\right)$. In addition, the boundary condition $\eta \to \infty$ is approximated by $\eta_{\max} = 7$ which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties. Numerical computations are carried out for F = 0, F=1 and $0 \le R, D \le 1$;Ra/Pe=-1,-1.5,1,2,5,20;M=0.0.8,2.

RESULTS AND DISCUSSION OF A PARAMETRIC STUDY

The parameters governing the physics of the present study are the melting (M), the mixed convection $\left(\frac{Ra}{Pe}\right)$, the inertia (F), thermal dispersion (D) and thermal radiation(R), parameters. The non-Darcian nature of the medium is reflected quantitatively in values we ascribed to F. The Darcian medium is referred as F = 0. In fixing the parametric value ranges of $M, \frac{Ra}{Pe}, F, D, R$ quantitatively we followed earlier workers and carried out the computations. We illustrate our parametric study for both aiding and opposing buoyancies.

The combined effect of dispersion, radiation with melting on the velocity distribution across the boundary layer in both aiding and apposing flows is shown in Fig. (**2**) for a given non-Darcy medium fixing F=1, and varying values of R,D and M in their respective ranges for fixed $\left|\frac{Ra}{Pe}\right|$=1. From the figure it is clear that in aiding flow the velocity is increasing due to increase in radiation and dispersion. It is also clear that velocity increases with increase in the melting parameter M. Also the effect of melting with thermal dispersion and radiation is seen in the increase of velocity in aiding flow. In the opposing flow the velocity is seen to decrease with melting. A similar phenomenon is observed with increase of combined effect of radiation and dispersion.

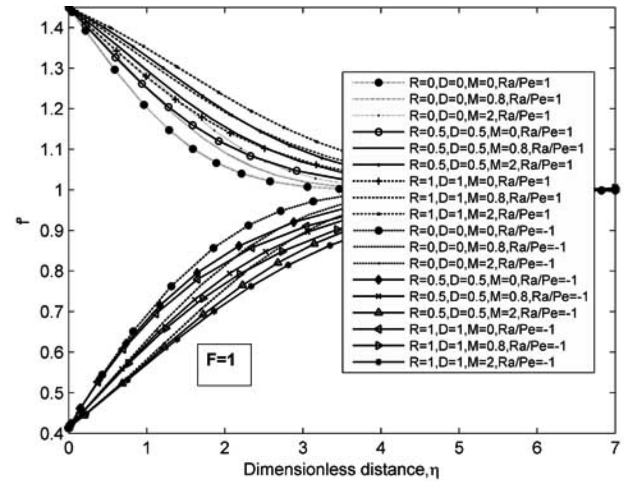

Fig. (2). The effect of thermal dispersion-radiation and melting on velocity distribution in both aiding and opposing flows.

Fig. (**3**) shows the dependence of dimension less flow velocity distribution on melting strength (M) for a fixed inertia strength (F) and for values of $\frac{Ra}{Pe}$=1,2,5 in 3(a) and $\frac{Ra}{Pe}$= -1,-1.5 in 3(b) for (1) R = 0, D = 0 (ii) R=0.5, D=0.5 (iii) R =1 , D = 1. The Fig. (**3**(**a**)) shows that the velocity increases with increasing mixed convection parameter both in the presence and absence of the radiation and dispersion. In the presence of radiation and dispersion, velocity increases with the increasing melting but this increment increases with increase in mixed convection. Melting with the combined effect of radiation and dispersion increases velocity more significantly than when melting occurs without radiation and dispersion. In opposing flows as shown in Fig. (**3**(**b**)) the velocity decreases with increase in opposing buoyancy both in the presence and absence of radiation and dispersion. In the presence of radiation and dispersion, velocity decreases with increase in melting.

In Fig. (**4**) the effects of inertia and melting on the velocity for R = 0, D=0; R=0.5, D=0.5; R=1, D=1 are presented. In aiding flow increase in inertia – from Darcian to non-Darcian – decreases velocity as it suppresses the flow, when compared to Darcy conditions. The combined effect of radiation and dispersion, increases the velocity with melting in both Darcy and non-Darcy conditions, but the increment is less in non-Darcy conditions. The effect is reversed in opposing flow.

Fig. (**5**) shows the effect of melting on temperature distribution for aiding and opposing flows. It is observed that the temperature decreases with increasing melting parameter as well as with increase in dispersion and radiation in the thermal boundary layer in both the cases.

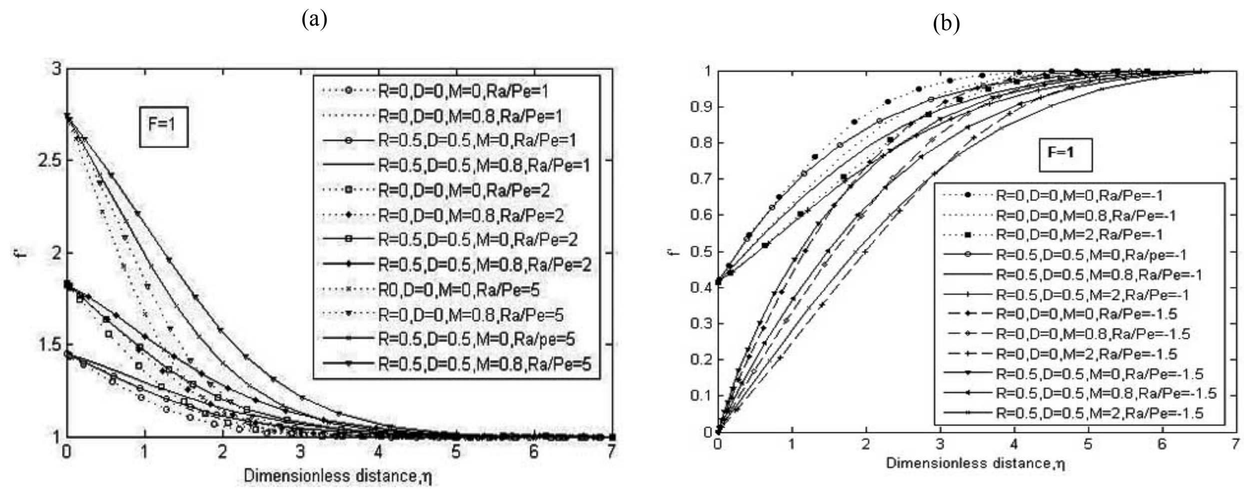

Fig. (3). Variation in f^{1} (η) with η for different values of thermal dispersion - radiation, melting and mixed convection parameters. (**a**) aiding flow (**b**) opposing flow.

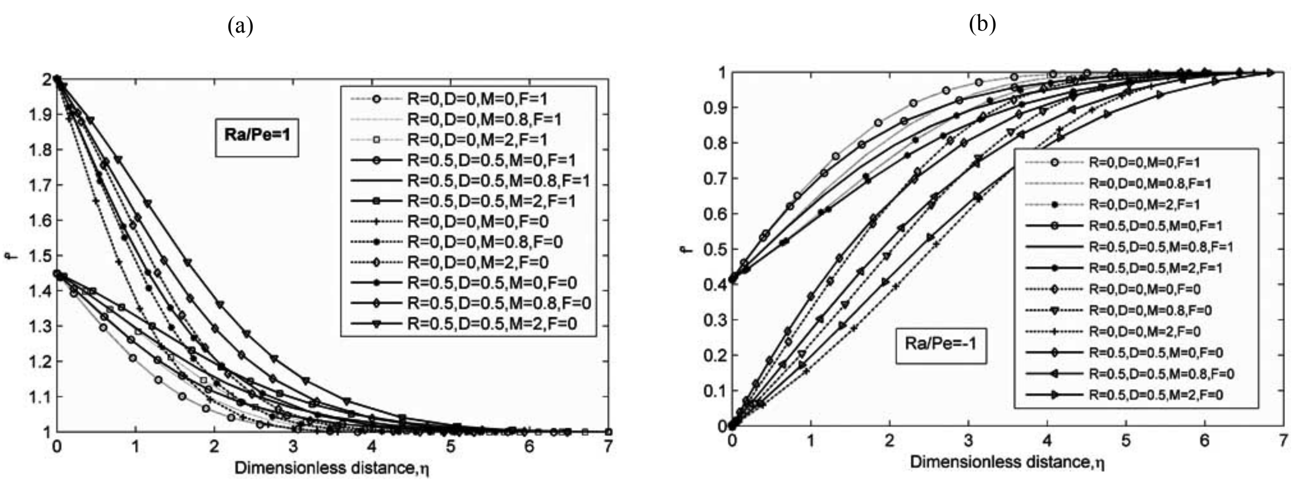

Fig. (4). The variation in $f^{'}$ (η) with 'η' for different values of inertia, thermal dispersion - radiation, and melting parameters. (**a**) aiding flow (**b**) opposing flow.

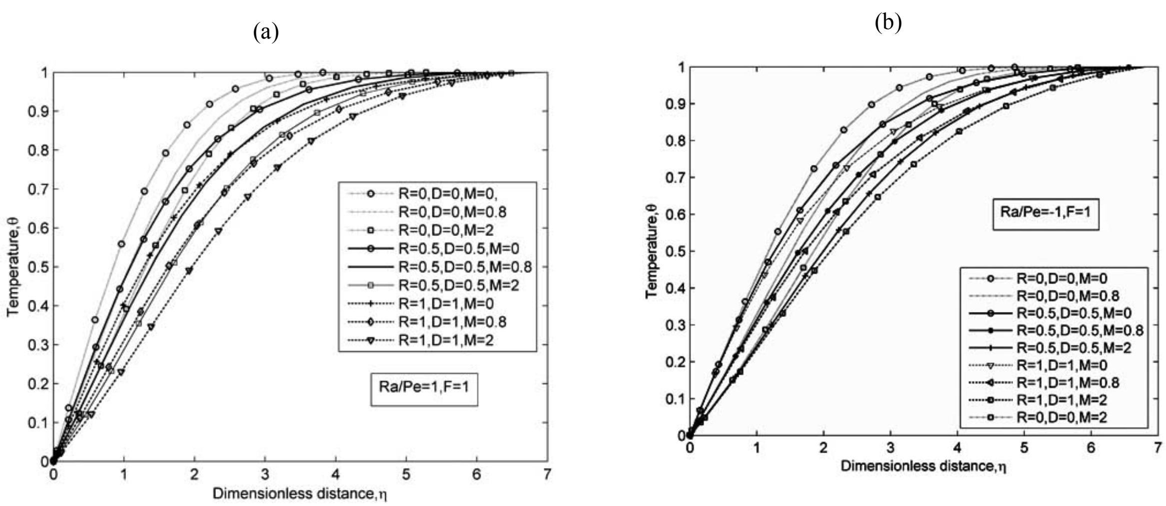

Fig. (5). The effect of melting and thermal dispersion - radiation on temperature distribution. (**a**) aiding flow (**b**) opposing flow.

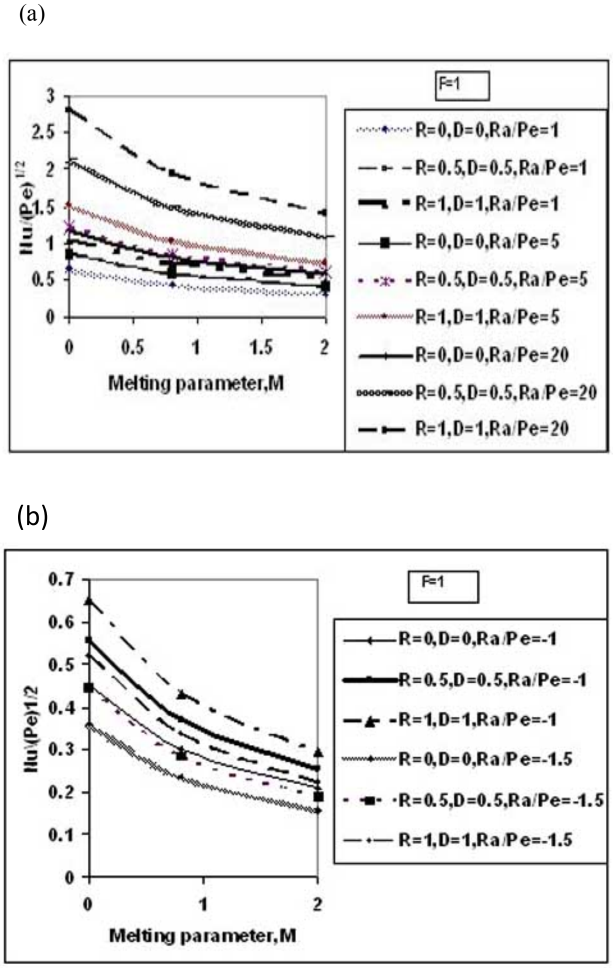

Fig. (6). Variation in the local Nusselt number with the melting parameter for different values of thermal dispersion-radiation, and mixed convection parameters (**a**) aiding flow (**b**) opposing flow.

The effect of melting strength, mixed convection and thermal dispersion, radiation on heat transfer rate is shown in Fig. (**6**) in terms of Nusselt number defined in equation (15). This shows that Nusselt number decreases significantly with the increasing metling strength (M). Further it is seen that the Nusselt number increases with the increase in thermal dispersion and radiation. In aiding flows as $\frac{Ra}{Pe}$ increases Nusselt number increases but this increase is less in absence of the thermal dispersion and radiation than in the presence of thermal dispersion and radiation. In opposing flow the Nusselt number is found to decrease as $\left|\frac{Ra}{Pe}\right|$ increases.

It is clear from Fig. (**7**) that the Nusselt number is decrasing / increasing with the increasing inertia effect in aiding/opposing flow. But this decrease/increase is less in the absence of thermal dispersion and radiation than in the presence of thermal dispersion and radiation.

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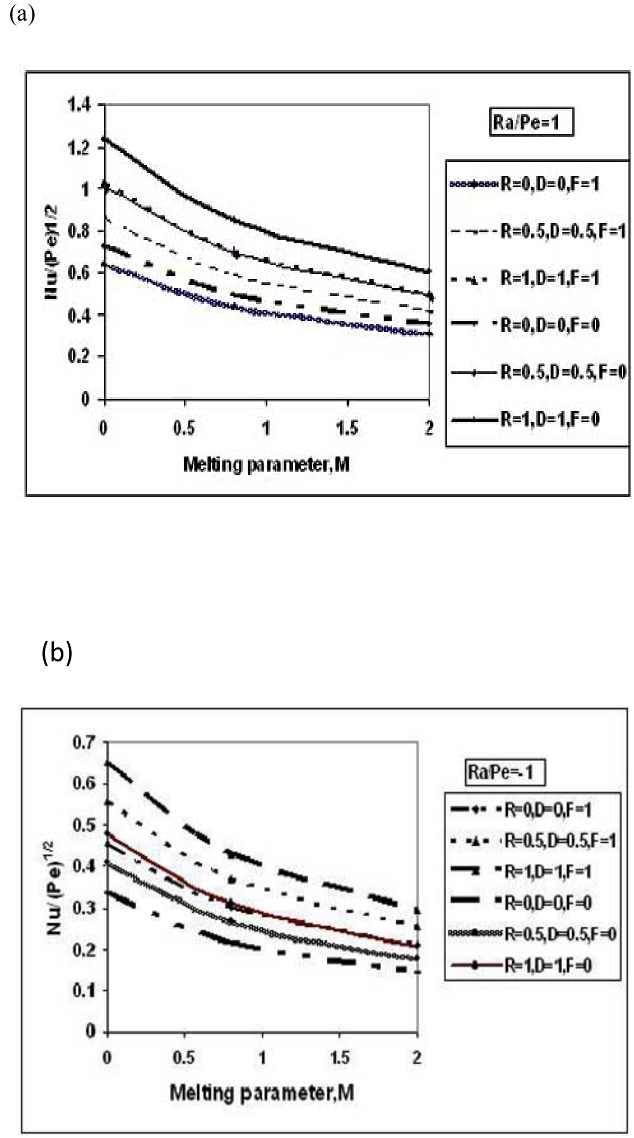

Fig. (7). Variation in the local Nusselt number with the melting parameter for different values of thermal dispersion, radiation, and inertia parameters (**a**) aiding flow (**b**) opposing flow.

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