Enhanced Algorithm for MIESM

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Abstract: The link adaptation technique based on MIESM (Mutual Information based exponential SNR Mapping) has been extensively used in the literature for 802.16 based systems. The previous work on MIESM uses equal modulation order for all the subcarriers in an OFDM block. We have proposed the concept of unequal modulation for the subcarriers in the single OFDM block, and derived a mathematical model based on bivariate Gaussian distribution, based on this a mathematical equation has been derived for joint PDF and the average probability of performance equation has been obtained. The difficulty in generating a probability of error for bivariate Gaussian is the motivation for this paper. Results show that for bivariate case the performance is related to the correlation parameter.

Keywords: 802.16, MIESM, OFDM, link adaptation.

I. INTRODUCTION

Next generation cellular systems support multiple transmission modes, which can be used to improve the performance of such systems by adapting to current channel conditions. This process is referred to as link adaptation. Typically, these transmission modes include different modulation and coding, schemes (MCS) and different multiple antenna arrangements modes – such as beam-forming, space-time coding and spatial multiplexing – as the transmission becomes multidimensional in space, time, and frequency domain. Orthogonal Frequency Division Multiplexing (OFDM) is the air interface for 802.11, 802.16 (WiMAX), and 3GPP Long Term Evaluation (LTE) systems. The resources typically referred to as subcarriers, available in an OFDM frame, can be defined on a time frequency grid [1]. The performance of a binary code depends on the channel condition obtained over the allocated subcarriers. Typically, the channel is frequency selective, and a mean SNR metric is only sufficient to obtain a long term performance metric of the channel. On the other hand, short term performance metrics, which are also key to obtaining performance enhancements with feedback, are obtained from the actual instantaneous channel realization.

A well-known approach to link performance modeling and link quality prediction is the Effective Exponential SINR Metric (EESM) method, which computes an effective SNR (also referred to as AWGN equivalent SNR) metric by taking as input the individual sub carrier SNRs and using an exponential combining function. Once computed, the block error rate is obtained from looking up an AWGN performance curve. This approach has been widely applied to OFDM link layers and is based on the performance approximation by asymptotic union bounds [2].

One of the disadvantages of the EESM approach is that a normalization parameter (usually represented by a scalar, β) must be computed for each modulation and coding (MCS) scheme. In particular, for broader link-system mapping applications, it can be inconvenient to use EESM when combining codeword mapped onto different modulation types, where the EESM method can require the use of so-called symbol de-mapping penalties. Seeking a means to overcome some of the shortcomings of EESM, we focus here on the Mutual Information based approach to link performance prediction [3].

Approaches based on mutual information (MI) are proposed in literature [4, 5], which provides advantages over parametric EESM approaches. However, most of the link performance prediction methods proposed so far are based on mapping from SNR to an associated MI metric, by which new algorithm for MIESM Link Adaptation with unequal modulation has been proposed. It has been found by the literature survey [5, 6] that the Doppler spreading of OFDM systems follows joint probability distribution function implies the transmission will also be dependent on Bivariate Gaussian distribution and gives better PDF, CDF. By the concept of unequal modulation technique, the performance of 802.16 networks can be enhanced as the difference in modulation leads to different channel gain. MMIB has been worked for fast link adaptation in 802.16m systems where bit interleaved coded modulation is used. The MI is dependent upon the code rate not on the modulation order. But it has been found that for higher modulation the performance of MMIB to BLER does a not match well and again there is a bound limit for this assumption. The encoded bits follow different modulation in the channel but at the receiver, it uses the same decoder, hence, the decoder performance will be different for soft decision bits but will not be a problem with hard code decision bits [6].

Hence, the MMIB to BLER mapping must be dependent on the modulation as well as the code rate for perfect performance results. Hence, the problem of using different modulation in the same channel for different symbols can be considered as two dimensional that is bivariate Gaussian distribution.

Keeping this concept as motivating point the unequal modulation is designed by bivariate distribution where two modulation orders are used, expressions has been derived for PDF and average probability of error performance. This paper is organized as follow: Section II explains the link

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1877-6124/09

adaptation, Section III explains the unequal modulation in 802.16, and conclusion is presented in section IV.

II. 802.16 LINK ADAPTATION

For communication systems like OFDM where multiple channel states may be obtained on a transmitted codeword, link performance prediction, in general, is based on determining a function $I\ (SINR_1,\ SINR_2,\ldots)$, which maps multiple physical SINR observations into a single "effective SINR" metric $SINR_{eff}$ (or equivalent), which can then be input to a second mapping function $B\ (SINR_{eff})$ to generate a block error rate (BLER) estimated for a hypothesized codeword transmission. We assume the access to a set Ω of N SINR measures, denoted $SINR_n$, $0 \le n <$ N. Note that the precise definition of these observations will depend on the SISO/MIMO transmission mode and a receive type, but for the simple SIOS case, the SINR measures may be assumed to correspond to SINR observations of individual data subcarriers (and therefore, of associated QAM symbols) transporting the hypothesized codeword of interest. Fig. (**1**) shows the BICM model used in this paper [7, 8].

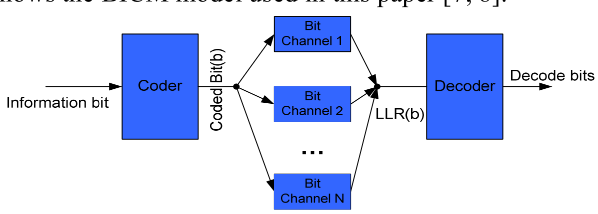

Fig. (1). BICM model.

The first mapping function *I,* and effective SINR metric *SINR eff*, may be generally defined as:

$$\Gamma = I\left(\frac{SINR_{eff}}{\alpha_1}\right) = \frac{1}{N}\sum_{n=1}^{N}\left(\frac{SINR_{eff}}{\alpha_2}\right) \qquad (1)$$

where α_1 and α_2 are constants (and maybe constrained to be equal), which may be MCS-specific, and Γ may correspond to a defined statistical measure. *I* (.) is a reference function usually selected to represent a performance model. Exponential ESM is derived by using an exponential function, which is based on using Chernoff approximation to the union bounds on the code performance.

Similarly, other performance measures like capacity or mutual information can be used. The accuracy of the model to some extent is dependent on how closely the reference model represents the code performance (with sufficient parameterization, a given model can yield a reasonably good accuracy as in EESM). In the method proposed here, Γ is the ***mean mutual information per coded bit*** (**MMIB**), or simply denoted as *M,* and α_1 and α_2 are discarded (i.e., set to unity).

$$M = I\left(SINR_{eff}\right) = \frac{1}{N}\sum_{n=1}^{N} I_m\left(SINR_n\right) \qquad (2)$$

$$\left(SINR_{eff}\right) = I^{-1}\left(\frac{1}{N}\sum_{n=1}^{N} I_m\left(SINR_n\right)\right) \qquad (3)$$

where I_m (.) is a function that depends on the modulation type identified by m and the associated bit labeling in the constellation, where $m \in \{2, 4, 6\}$ corresponding to QPSK, 16- QAM, 64-QAM, respectively. I_m (.) maps the sub-carrier SINR to the mean mutual information between the log- likelihood ratio and the binary codeword bits comprising the QAM symbol.

$$M = I_m\left(SINR\right) = \frac{1}{m}\sum_{i=1}^{m} I_{m,i}\left(SINR\right) \qquad (4)$$

We will refer to the above quantity as Mutual Information per coded Bit or MIB, with the understanding that it is derived by averaging over the "m" bit channels. Furthermore, mean mutual information per bit (MMIB) is used to refer to the mean obtained over different channel states or SNR measures. Thus, the modulation order follows Gaussian distribution and is shown in Figs. (**2**, **3**), for the QPSK and 16 QAM. We can note that, as expected, for BPSK, the LLR distribution is Gaussian with mean $2/\sigma_n^2 = 4\ E_s/N_o$ =12.65 (SNR = 5 dB). Predictably for QPSK, the distribution is also Gaussian with a mean, which is one half of the BPSK mean. The plot between SNR and MMIB is given by the CDF of Gaussian curve is shown in Figs. (**4**, **5**).

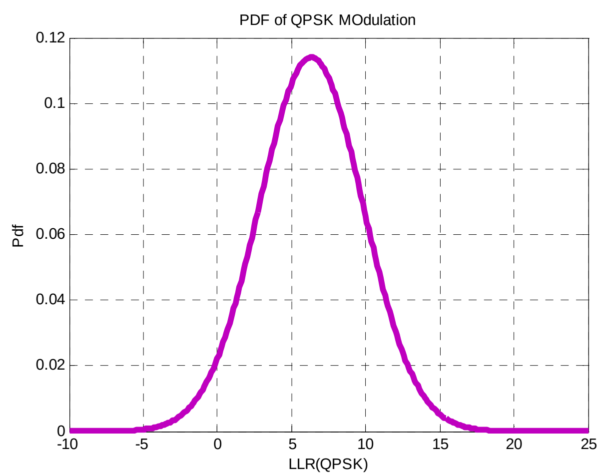

Fig. (2). QSPK bit-wise conditional LLR distributions.

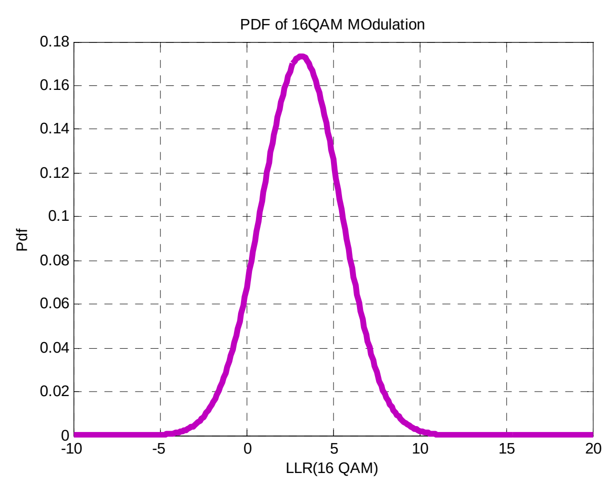

Fig. (3). 16 QAM bit-wise conditional LLR distributions.

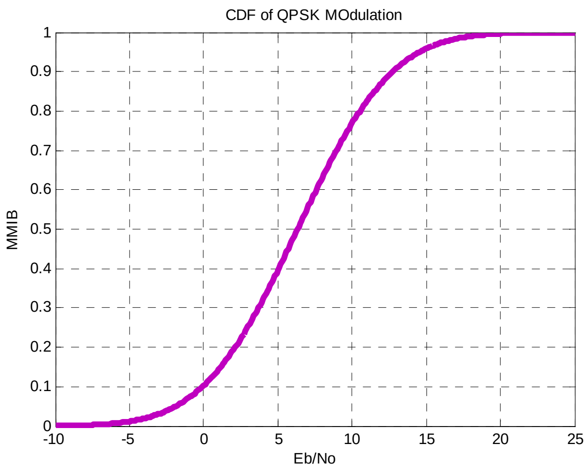

Fig. (4). MIB *vs* Es/No for QPSK.

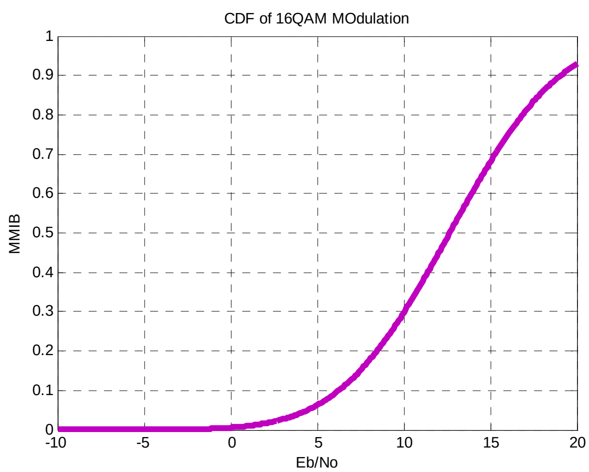

Fig. (5). MIB *vs* Es/No for 16 QAM.

The BS can store the AWGN reference curves for different MCS levels in order to map the MMIB to BLER [9]. Another alternative is to approximate the reference curve with a parametric function. For example, we consider a Gaussian cumulative model with 3 parameters, which provides a close fit to the AWGN performance curve, parameterized as

$$y = \frac{a}{2}\left[1 - erfc\left(\frac{x-b}{\sqrt{2}c}\right)\right],\ c \neq 0 \qquad (5)$$

where a is the "transition height" of the error rate curve, b is the "transition center", and c is related to the "transition width" (transition width = 1.349 c) of the Gaussian cumulative distribution. In the linear BLER domain, the parameter a can be set to 1, and the mapping requires only two parameters as shown in Fig. (**6**).

III. 802.16 UNEQUAL MODULATIONS – BIVARIATE DISTRIBUTION

In a bit-interleaved coded modulation (BICM) system, the bit interleaver breaks the memory of the modulator such that the equivalent channel in Fig. (**11**) is represented by an equivalent model of parallel independent bit channels as shown in Fig. (**1**). Note that for higher-order modulations, the bit positions in the signal constellation are asymmetric; therefore, each bit location of one symbol will experience a different bit channel and will have different statistical properties [8, 9].

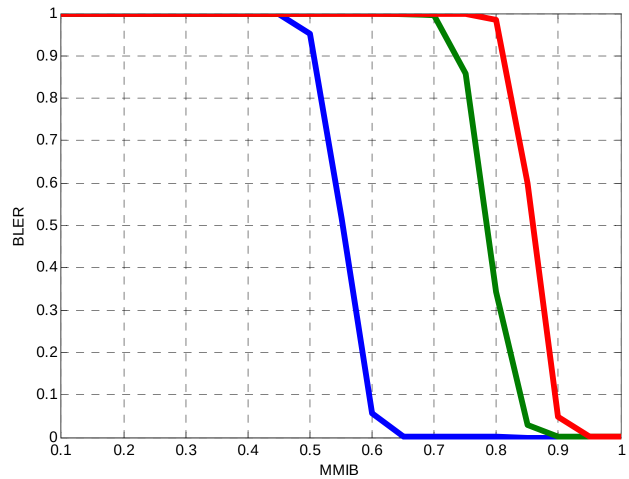

Fig. (6). Mapping of MMIB to BLER.

Taking into account the above remarks and assuming the bits in a symbol have equal probabilities, the mutual information per coded bit (MIB) is considered as the average of the m bit-wise mutual information measures, m being the number of bits per symbol [10]:

$$I_m = \frac{1}{m}\sum_{i=1}^{m} I_{m,i}\left(c_i, LLR\left(c_i\right)\right)$$

where $I_{m,i}\left(c_i, LLR\left(c_i\right)\right)$ is the MI between the input to the 2^m-QAM mapper and the output LLR for the i^{th} bit position in the symbol. $I_{m,i}\left(c_i, LLR\left(c_i\right)\right)$ depends on the SINR of the QAM symbol, therefore, equation becomes:

$$I_m\left(SINR\right) = \frac{1}{m}\sum_{i=1}^{m} I_{m,i}\left(SINR\right)$$

From the above equation, considering the codeword is transmitted over N channel symbols, the mean mutual information per coded bit (MMIB) can be computed as:

$$MMIB = \frac{1}{N}\sum_{n=1}^{N} I_{m(n)}\left(SINR_n\right)$$

where $m(n)$ is the modulation order of the n^{th} symbol and $SINR_n$ is the corresponding SINR.

It is also shown in [10] that I_m can be numerically approximated, and then stored for later use for link performance prediction. The MI per coded bit is given by:

$$I_{m,i}\left(c_i, LLR\left(c_i\right)\right) = \frac{1}{2}\sum_{c_i \in \{0,1\}} \int_{-\infty}^{+\infty} p_{LLR}\left(r \mid c_i\right) \cdot \log_2\left(\frac{2p_{LLR}\left(r \mid c_i\right)}{p_{LLR}\left(r \mid c_i = 0\right) + p_{LLR}\left(r \mid c_i = 1\right)}\right) dr$$

where r is a LLR integration variable and $p_{LLR}\left(r \mid c_i\right)$ is the LLR's conditional PDF for the i^{th} bit position in the QAM symbol.

It can be demonstrated that for BPSK and QPSK modulations the bit LLRs are Gaussian distributed and their mean value has the property $\mu_{LLR} = \sigma_{LLR}^2/2$:

$$p_{LLR}\left(r \mid c_i\right) = \frac{1}{\sqrt{2\pi\sigma_{LLR}^2}}\exp\left(-\frac{\left(r-\mu_{LLR}c_i\right)^2}{2\sigma_{LLR}^2}\right)$$

$$I_{m,i}\left(c_i, LLR\left(c_i\right)\right) = 1 - \int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi\sigma_{LLR}^2}}\exp\left(-\frac{\left(r-\mu_{LLR}c_i\right)^2}{2\sigma_{LLR}^2}\right)\cdot\log_2\left(1+e^{-r}\right)dr \triangleq J\left(\sigma_{LLR}\right)$$

where $J(\cdot)$ is a nonlinear function, which can be approximated by [13]:

$$J(x) \approx \begin{cases} a_1x^3 + b_1x^2 + c_1x, & for\ 0 < x < 1.6363 \\ 1-\exp\left(a_2x^3 + b_2x^2 + c_2x + d_2\right), & for\ x \geq 1.6363 \end{cases}$$

where the coefficients are:

$a_1 = -0.04210661 \quad b_1 = 0.209252 \quad c_1 = -0.00640081$

$a_2 = 0.00181492 \quad b_2 = -0.142675 \quad c_2 = -0.0822054 \quad d2 = 0.0549608$

For BPSK, we have $\sigma_{LLR}^2 = 8E_s/N_0$, while for QPSK $\sigma_{LLR}^2 = 4E_s/N_0$. This leads to the following expression of the MI per coded bit for BPSK:

$$I_1\left(SINR\right) = I_{1,1}\left(c_1, LLR\left(c_1\right)\right) = J\left(\sqrt{8SINR}\right)$$

While for QPSK we have equation 'a':

$$I_2\left(SINR\right) = \frac{1}{2}\sum_{i=1}^{2} I_{2,i}\left(c_i, LLR\left(c_i\right)\right) = \frac{1}{2}\sum_{i=1}^{2} J\left(\sqrt{4SINR}\right) = J\left(\sqrt{4SINR}\right)$$

The problem with higher-order modulations is that they cannot be expressed in a closed form like above. But in [10], an approximation of the MIBs for these modulations is given, based on the observation that the LLR conditional PDFs can be approximated by a mixture of Gaussian distributions that are non-overlapping at high SINR values. Thus, the MIB is expressed as a sum of $J(\cdot)$ functions in equation 'b'

$$I_m\left(SINR\right) = \sum_{k=1}^{K} a_k J\left(b_k\sqrt{SINR}\right) \quad \text{with} \quad \sum_{k=1}^{K} a_k = 1$$

[10] States that limiting the sum in equation 'a' to 3 terms (3 dominant Gaussians) gives a good approximation for the MIB function. In this way, the following expressions are obtained for 16QAM and 64QAM:

$$I_4\left(SINR\right) = \frac{1}{2}J\left(b_1^{(4)}\sqrt{SINR}\right) + \frac{1}{4}J\left(b_2^{(4)}\sqrt{SINR}\right) + \frac{1}{4}J\left(b_3^{(4)}\sqrt{SINR}\right)$$

$$I_6\left(SINR\right) = \frac{1}{3}J\left(b_1^{(6)}\sqrt{SINR}\right) + \frac{1}{3}J\left(b_2^{(6)}\sqrt{SINR}\right) + \frac{1}{3}J\left(b_3^{(6)}\sqrt{SINR}\right)$$

The b_k coefficients are obtained by numerical simulation and curve fitting by following these steps [10, 11]:

1. Obtain through numerical simulation, the LLR conditional PDFs for each bit of a specific modulation at each SNR in an AWGN channel.
2. Compute the MIB by numerical integration using above equations.

Perform curve fitting using the approximation functions in equation 'b'.

Fig. (**12**) presents the MIB versus SNR curves. Note that if the MIB approximations for 64QAM are not accurate enough, LUTs, which store the MIB *vs* SINR, could be used.

Having the $I_m(\cdot)$ functions for the modulation types of interest allows the computation of MMIB for a specific channel realization over the coded symbols [12, 13].

In the basic link adaptation for 802.16 based on MIESM, the mutual information follows the Gaussian distribution, in which the OFDM block uses a single modulation. We have adapted the same concept in which the OFDM block uses 2 modulation schemes: a low order and a high order. The purpose of such a multilevel modulation is that the carriers in an OFDM block under deep fading can be allotted a low modulation order and the others high level modulation and as a result the SNR performance can be improved much better than equal modulation. From theory, two univariate marginal distributions following Gaussian distribution can be modeled under the same roof as bivariate joint distribution given by

$$f_{X_1X_2}(x_1,x_2) = \frac{(x_1,x_2)^{(\alpha-1)/2}}{\Gamma(\alpha)(\beta_1)(\beta_2)^{(\alpha+1)/2}(1-\rho)\rho^{((\alpha-1)/2)}}$$

$$\times exp\left(-\frac{x_1/\beta_1 + x_2/\beta_2}{1-\rho}\right)$$

$$\times I_{\alpha-1}\left(\frac{2\sqrt{\rho}}{1-\rho}\sqrt{\frac{x_1x_2}{\beta_1\beta_2}}\right) U(x_1)(x_2)$$

where r(.)is gamma function and I(.) is a modified Bessel function of first kind, β1 and β1 are scaling parameters, x1 and x2 are random variables for different marginal distributions, and ρ is the correlation coefficient, When ρ → 0 the joint pdf tends to be the product of two univariate gamma distribution [13]. Consider two OFDM modulation techniques, 16QAM and 64QAM, and the joint probability distribution for them is shown in Figs. (**7**, **8**).

Nakagami proposed a model that corresponds to the scenarios of dual-diversity reception over correlated Nakagami-*m* channels, which are not necessarily identically distributed. This may therefore, apply to independent fading channels among signal paths.

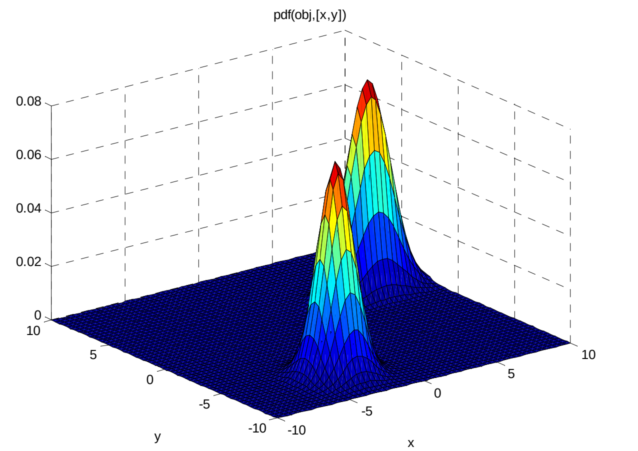

Fig. (7). PDF for Bivariate normal distribution for 16QAM and 64QAM.

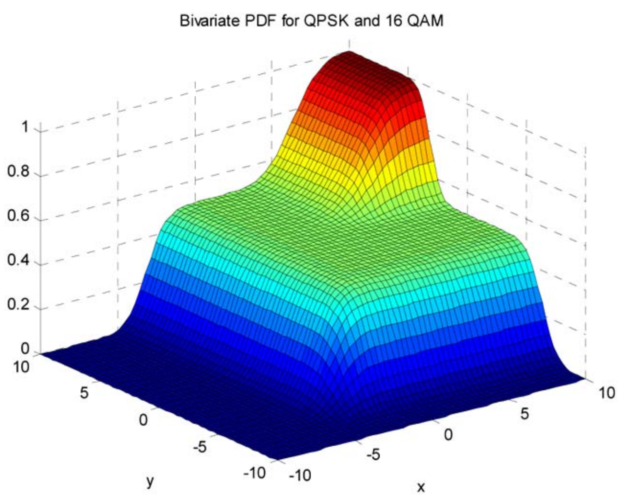

Fig. (8). CDF for bivariate normal distribution for 16QAM and 64QAM.

In this case, the PDF of the combined signal envelope, $pa(rt)$, is given by

$$p_{a(r_t)} = \frac{2r_t\sqrt{\pi}}{\Gamma(m)\left[\sigma_1\sigma_2(1-\rho)\right]^m}\left[\frac{r^2{}_t}{2\beta}\right]^{m-1/2} \tag{7}$$

$$I_{m-1/2}(\beta r^2{}_t)e^{-\alpha r^2{}_t} \quad , \qquad r_t \geq 0$$

where $I\upsilon(\cdot)$ denotes the υth-order modified Bessel function

$$p_a(\gamma_t) = \frac{1}{2\sqrt{\rho\bar{\gamma}}}\left\{\exp\left[-\frac{\gamma_t}{(1+\sqrt{\rho})\bar{\gamma}}\right]\right\} - \exp\left[-\frac{\gamma_t}{(1-\sqrt{\rho})\bar{\gamma}}\right]$$

$$\rho = \frac{\mathrm{cov}(r_1^2, r_2^2)}{\sqrt{\mathrm{var}(r_1^2)\,\mathrm{var}(r_2^2)}} \ , \ 0 \leq \ \rho \geq 1 \tag{8}$$

Is the envelope correlation coefficient between the two signals and the parameters σd (d = 1, 2), α, and β are defined as follows:

$$\sigma_d = \frac{\Omega_d}{m} \ , \ d = 1,2, \ldots$$

$$\alpha = \frac{\sigma_1 + \sigma_2}{2\sigma^2{}_1\sigma^2{}_2(1-\rho)} \tag{9}$$

$$\beta^2 = \frac{(\sigma_1 - \sigma_2)^2 + 4\sigma_1\sigma_2\rho}{4\sigma^2{}_1\sigma^2{}_2(1-\rho)^2}$$

where Ωd, d = 1, 2, is the average fading power of the dth channel. By using a standard transformation of random variables, it can be shown that the PDF of the combined SNR per symbol, $pa(\gamma t)$, is given by

$$p_{a(\gamma_t)} = \frac{\sqrt{\pi}}{\Gamma(m)}\left[\frac{m^2}{\bar{\gamma}_1\bar{\gamma}_2(1-\rho)}\right]^m\left(\frac{\gamma_t}{2\beta'}\right)^{m-1/2} \tag{10}$$

$$I_{m-1/2}(\beta'\gamma_t)e^{-\alpha'\gamma_t} \ , \ \gamma_t \geq 0$$

where the parameters α' and β' are normalized versions of the parameters α and β, and are given by

$$\alpha' = \frac{\alpha}{E_s / N_0} = \frac{m(\bar{\gamma}_1 + \bar{\gamma}_2)}{2\bar{\gamma}_1\bar{\gamma}_2(1-\rho)} \tag{11}$$

$$\beta' = \frac{\beta}{E_s / N_0} = \frac{m(\bar{\gamma}_1 + \bar{\gamma}_2) - 4\bar{\gamma}_1\bar{\gamma}_2(1-\rho)^{1/2}}{2\bar{\gamma}_1\bar{\gamma}_2(1-\rho)}$$

And finally the equation reduces to

$$p_a(\gamma_t) = \frac{1}{2\sqrt{\rho\bar{\gamma}}}\left\{\exp\left[-\frac{\gamma_t}{(1+\sqrt{\rho})\bar{\gamma}}\right]\right\} - \exp\left[-\frac{\gamma_t}{(1-\sqrt{\rho})\bar{\gamma}}\right], \ \gamma_t \geq 0 \tag{12}$$

Using the Laplace transform it can be shown after some manipulations that the MGF of $pa(\gamma t)$ is given by

$$M_a\left(s:\bar{\gamma}_1,\bar{\gamma}_2:m:\rho\right) = M_a(s) = \left[1 - \frac{(\bar{\gamma}_1+\bar{\gamma}_2)}{m}s + \frac{(1-\rho)\bar{\gamma}_1\bar{\gamma}_2}{m^2}s^2\right]^{-m} \ , \ s \geq 0 \tag{13}$$

It should be noted that the previous leads to an *exact* expression for average probability of error. For exponential correlation and average probability of error, performance is plotted for different values of correlation coefficient, clearly the BER performance degrades with correlation values shown in Figs. (**9**, **10**).

IV. CONCLUSION

The concept of unequal modulation has been modeled and studied for 802.16 based systems. Analysis shows that unequal modulation follows bivariate distribution. Mathematical expressions have been derived correlation parameters and results have been plotted for average probability of error with correlation coefficient.

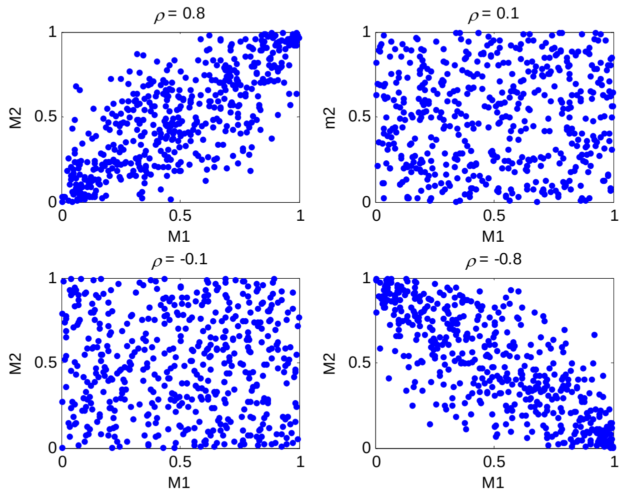

Fig. (9). BER for different values of ρ.

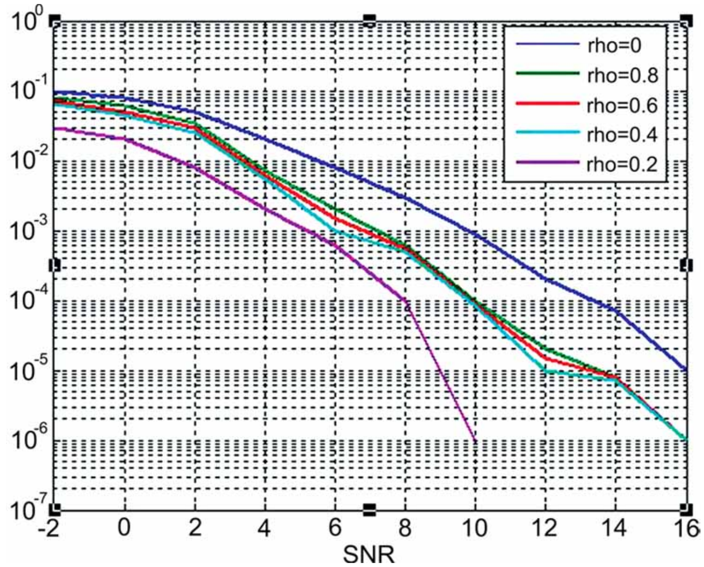

Fig. (10). Performance of SNR *vs* BER for different values of ρ.

Fig. (11). Equivalent information channel.

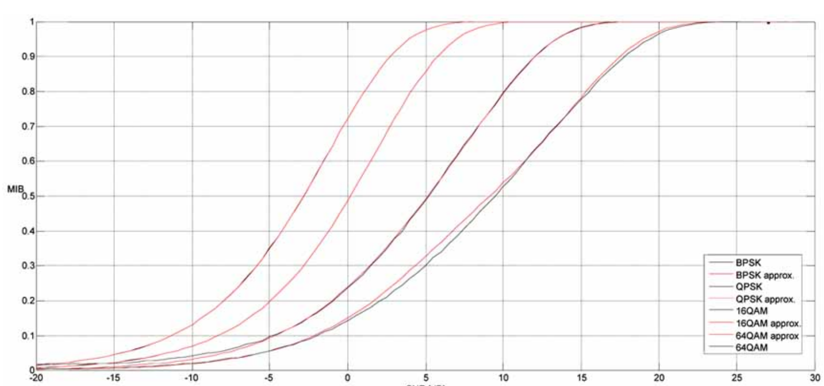

Fig. (12). MIB *vs* SNR; numerical simulation (black) and approximation (red).

ACKNOWLEDGEMENTS

The work presented in this paper was supported by the European project ICT-CODIV.

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Received: June 15, 2009 Revised: July 9, 2009 Accepted: August 13, 2009