Gravity Effects in Inclined Air Showers Induced by Cosmic Neutrinos

A.V. Kisselev*

Institute for High Energy Physics, 142281 Protvino, Russia

Abstract: The Randall-Sundrum model with a small curvature is considered in which five-dimensional Planck scale lies in the TeV region. The cross sections for interactions of ultra-high energy cosmic neutrinos with nucleons are calculated. It is shown that effects related with Kaluza-Klein graviton excitations can be detected in deeply penetrating inclined air showers induced by these neutrinos. The expected number of the inclined air showers at the Auger Observatory is estimated as a function of two parameters of the model.

1. WARPED EXTRA DIMENSION WITH THE SMALL CURVATURE

One of the most important problems of the modern particle physics is the hierarchy problem, i.e. unnaturally large ratio of the gravity scale (10^{19} \text{GeV}) to the electroweak scale (10^2 \text{GeV}). To solve this problem, a number of theories with large spacial extra dimensions have been proposed [1]. However, they could only explain the huge value of the Planck mass by introducing another large scale, namely, the size of extra flat dimensions. Thus, the hierarchy problem was not really solved, but reformulated in terms of this new scale.

The model which does solve the problem most economically is the Randall-Sundrum (RS) model [2] with a single extra dimension and warped background metric [3]:

\[ ds^2 = e^{2\pi r_0 y} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \]  

(1)

Here \( y = r, \theta (-\pi \leq \theta \leq \pi), r_c \) is the “radius” of the extra dimension, while \( \{x^i\}, \mu = 0, 1, 2, 3, \) are the coordinates in four-dimensional space-time. The parameter \( k \) defines the scalar curvature in five dimensions. Note that the points \( (x^i, y) \) and \( (x^i, -y) \) are identified, and the periodicity condition, \( (x^i, y) = (x^i, y + 2\pi r_c) \), is imposed. The tensor \( \eta_{\mu\nu} \) is the Minkowski metric.

It is assumed that there are two 3-dimensional branes with equal and opposite tensions located at the points \( y = 0 \) (called the Plank brane) and \( y = \pi r_c \) (referred to as the TeV brane). All SM fields are confined to the TeV brane, while the gravity propagates in five dimensions. The following relation between the 4-dimensional (reduced) Planck mass, \( \overline{M}_{Pl} \), and (reduced) gravity scale in five dimensions, \( \overline{M}_5 \), can be derived:

\[ \overline{M}_{Pl}^2 = \frac{\overline{M}_5^2}{k} (e^{k \pi r_c} - 1). \]  

(2)

The masses of the Kaluza-Klein (KK) graviton excitations are proportional to the curvature parameter \( k \):

\[ m_n = \lambda_k \kappa, \quad n = 1, 2, \ldots \]  

(3)

where \( x_n \) are zeros of the Bessel function \( J_1(x) \). On the TeV brane, the zero graviton mode, \( h^{(0)}_{\mu\nu} \), and massive graviton modes, \( h^{(n)}_{\mu\nu} \), are coupled to the energy-momentum tensor of the matter, \( T^{\mu\nu} \), as follows:

\[ \mathcal{L}_{\text{int}} = -\frac{1}{\overline{M}_{Pl}} T^{\mu\nu} h^{(0)}_{\mu\nu} - \frac{1}{\Lambda_\pi} T^{\mu\nu} \sum_{n=1}^\infty h^{(n)}_{\mu\nu} \]  

(4)

with

\[ \Lambda_\pi = \left( \frac{\overline{M}_5}{k} \right)^{1/2} \]  

(5)

being the physical scale on this brane. Let us note that the Planck mass ( \( \overline{M}_5 \sim \kappa \sim \overline{M}_{Pl} \) ). Moreover, the size of the warped extra dimension should be extremely small (\( r_c \lesssim 60 l_P \)) [3]. Thus, in order to explain the huge value of \( \overline{M}_{Pl} \), in such a scheme, one has to introduce new mass scales of the same order, namely, \( \overline{M}_5, \kappa, \) and \( r_c^{-1} \).

However, the hierarchy problem can be successfully solved in the RS scenario, but with the metric (1). The equation (2) allows us to consider the small curvature option of the RS model [3-5]:

\[ \kappa \sim 1 \text{ GeV}, \quad \overline{M}_5 \sim 1 \text{ TeV}. \]  

(6)

In such a case, we get an almost continuous spectrum of low-mass graviton excitations with the lightest mass equal to 3.83 \( \kappa \), and small mass splitting \( \Delta m \approx \pi \kappa \). Note that in the standard scenario of the RS model [2] one has a series of KK graviton resonances with the lightest one having the mass around 1 TeV.

The RS model with the large extra dimension has been checked by the DELPHI Collaboration [6]. The gravity
The measured obtained \[6\] is:

\[ \text{sequence of trajectories enumerated by the KK number} \]

The Regge trajectory of the graviton is splitting into an infinite set of gravi-Reggeons, i.e. reggeized gravitons in the kinematical region (10), elastic scattering amplitude is given by

In the eikonal approximation which is valid in the trans-Planckian region the gravity contribution to the scattering of the SM fields in the trans-Planckian mass KK gravitons (3) is to look for their contributions to the scattering of the SM fields in the trans-Planckian region. It is also assumed that inequality \(s > M_3\) is satisfied. As we will see below, in the trans-Planckian region the gravity contribution to the scattering of UHE cosmic neutrinos off the atmospheric nucleons can dominate the SM contribution.

In the eikonal approximation which is valid in the trans-Planckian region (10), elastic scattering amplitude is given by the sum of gravi-Reggeons, i.e. reggeized gravitons in the \(t\)-channel. Because of the presence of extra dimension, the Regge trajectory of the graviton is splitting into an infinite sequence of trajectories enumerated by the KK number \(n\) [11]:

\[ \alpha_s(t) = 2 + \alpha_s' t - \alpha_s' m_n^2, \quad n = 0, 1, ... \]  

In string theories, the slope of the gravi-Reggeons is universal, and \(\alpha_s = M_s^{-2}\), where \(M_s\) is the string scale. For more details, see Refs. [11].

Correspondingly [3], the gravity Born amplitude for the neutrino scattering off a point-like particle looks like:

\[ A_{\text{grav}}^B(s,t) = \frac{\pi \alpha_s^2}{2 \Lambda_s} \sum_{n=0}^{\infty} \left( 1 - \cot \frac{\pi \alpha_s'(t)}{2} \right) \left( s \frac{m_n^2}{M_s^2} \right)^{\alpha_s'(t) - 2}. \]  

The differential neutrino-proton cross section is of the form:

\[ \frac{d\sigma}{dy} = \frac{1}{16\pi s} |A_{\text{grav}}^B(s,t)|^2. \]  

The inelasticity \(y = -t/s\) defines the fraction of the neutrino energy transferred to the nucleon. \(A_{\text{grav}}^0\) is the neutrino-proton amplitude which is related to the eikonal:

\[ A_{\text{grav}}^0(s,t) = 4\pi s \int_0^1 dq_1 q_1 J_0(q_1 b) A_{\text{grav}}^B(s,t). \]

In its turn, the eikonal is given by the equation:

\[ \chi(s, b) = \frac{1}{4\pi s^5} \int dq_0 q_0 J_0(q_0 b) A_{\text{grav}}^B(s,t). \]

The calculations show that the imaginary part of the eikonal is negligible with respect to its real part, since \(\Im \chi(\Re \chi = 0(s/M_3)\). That is why we can omit the contribution from inelastic interactions.

The hadronic Born amplitude in (15) is defined by the gravity amplitude (12) and skewed (\(t\)-dependent) parton distributions \(F_i(x, t)\):

\[ A_{\text{grav}}^B(s,t) = \sum_{i,q} \int_0^1 dx A_{\text{grav}}^{B_i} (x s, t) F_i(x, t). \]

The 
\(t\)-dependent distributions have the Regge-like form [3]:

\[ F_i(x, t) = f_i(x) \exp[k r_0^2 - \alpha_p \ln x], \]

where \(\alpha_p\) is the Pomeron slope, while \(f_i(x)\) is the distribution of the parton of the type \(i\) inside the proton. The values of the parameters are [12]:

\[ r_0^2 = 0.62 \text{ GeV}^{-2}, \quad \alpha_p = 0.094 \text{ GeV}^{-2}. \]

We will use the set of parton distribution functions \(f_i(x)\) from Ref. [13].

In Fig. (1) and Fig. (2) we present total neutrino-nucleon cross sections calculated by using Eqs. (12)-(16) for two values of the curvature \(\kappa\) and different values of the reduced fundamental gravity scale \(M_3\). For comparison, the SM prediction for the neutrino total cross section is presented in both figures which was calculated by using the following expression from Ref. [14].

\[ a \] Remember that the KK gravitons interact universally with the SM fields (4).

\[ b \] Since \(A^B - s^2\), the integral converges rapidly at \(x = 0\).

\[ c \] This expression is valid for \(10^7 \text{ GeV} \lesssim E_N \lesssim 10^{12} \text{ GeV within 10\%} \] [14].
\[ \sigma_{\nu N}(vN) = 7.84 \cdot 10^{-36} \text{ cm}^2 \left( \frac{E_{\nu}}{1 \text{ GeV}} \right)^{0.363} \]  

(19)

It varies from \(2.72 \cdot 10^{-33} \text{ cm}^2\) at \(E_{\nu} = 10^7 \text{ GeV}\) up to \(1.78 \cdot 10^{-31} \text{ cm}^2\) at \(E_{\nu} = 10^{12} \text{ GeV}\).

The number of neutrino induced air showers is given by

\[ \frac{dN}{dt} = \int_{E_{\nu}}^{E_{\nu}^{\text{max}}} dE_{\nu} \int_{0}^{\pi} dy (E_{\nu} - E_{\nu}^{th}) \frac{d\sigma(E_{\nu})}{dy} \Phi(E_{\nu}) A_{\text{eff}}(E_{\nu}^{th}, E_{\nu}) \]  

(20)

where \(E_{\nu}\) is the energy of the cosmic neutrino, \(\Phi(E_{\nu})\) denotes its flux, and \(E_{\nu}^{th}\) is the air shower energy. The effective aperture for the UHE neutrinos is defined by the neutrino flux attenuation \(\text{att}(E_{\nu})\) and detector efficiency \(P(E_{\nu})\) and \(A_{\text{eff}}(E_{\nu}^{th}, E_{\nu})\):  

\[ A_{\text{eff}}(E_{\nu}^{th}, E_{\nu}) = \text{att}(E_{\nu}) P(E_{\nu}) A_{\nu}(E_{\nu}) \]  

(22)

The attenuation \(\text{att}(E_{\nu})\) depends (besides neutrino-nucleon total cross section) on \(X_{\text{obs}}\), the depth within which air shower is visible for the ground array detector, and \(X_{\text{max}}\), the minimum atmospheric depth a neutrino must reach in order to induce an observable shower to these detectors.

Previously, low-scale gravity effects in cosmic neutrino interactions were calculated in models with compactified extra dimensions (see [15, 16] and references therein). Recently, the gravity effects on the neutrino-nucleon cross sections in the eikonal approximation were estimated for the case of infinitely thin branes embedded in five extra dimensions [17]. The black hole production cross sections in cosmic neutrino interactions were also calculated (see, for instance, Refs. [18]).

By comparing Figs. (1, 2) with figures from Refs. [17], one can see that the neutrino cross sections in the small curvature scenario of the RS model and those in the ADD model have different energy dependence. The formers are significantly smaller at \(E_{\nu} \leq 10^9 \text{ GeV}\), but exceed the ADD cross sections at \(E_{\nu} \geq 10^{10} \text{ GeV}\) (at comparable values of gravity scale \(\mathcal{M}_5\) in both models).

The threshold energy in (20) is taken to be \(E_{\nu}^{th} = 5 \cdot 10^7 \text{ GeV}\), and the maximum energy \(E_{\nu}^{\text{max}} = 10^{12} \text{ GeV}\). The result of our calculations for the Waxman-Bahcall neutrino flux [22] is presented in Fig. (3). It shows the rate of the inclined air showers at the Auger detector as a function of two parameters of the model.

<table>
<thead>
<tr>
<th>(N_{\nu})</th>
<th>(M_5), TeV</th>
<th>(\kappa), GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>7.5</td>
<td>1.5</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

\*It corresponds to \(X_{\text{max}} \geq 2500 \text{ g/cm}^2\), where \(X_{\text{max}}\) is the shower maximum.
The number of the inclined air showers at $\bar{M}_5 = 7$ TeV, $\bar{M}_5 = 8$ TeV, and $\bar{M}_5 = 9$ TeV are presented in Table 1 for three different values of the parameter $\kappa$. Our SM prediction is 0.13 events per year. It can be compared with the estimate from Ref. [15], 0.22 SM events per year for $\theta_{\text{zenith}} \geq 70^\circ$.

Table 1. The Number of the Inclined Neutrino Induced Air Showers (in yr$^{-1}$) at Several Values of the Parameters $\bar{M}_5$ and $\kappa$

<table>
<thead>
<tr>
<th>$\kappa$ (GeV)</th>
<th>$\bar{M}_5 = 7$ TeV</th>
<th>$\bar{M}_5 = 8$ TeV</th>
<th>$\bar{M}_5 = 9$ TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>2.69</td>
<td>2.53</td>
<td>1.54</td>
</tr>
<tr>
<td>1.0</td>
<td>1.32</td>
<td>1.17</td>
<td>0.68</td>
</tr>
<tr>
<td>1.5</td>
<td>1.18</td>
<td>0.77</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Taking into account that systematic errors of SM cross section can be factor 3, we conclude that the search limit of the Auger Observatory for the 5-dimensional Planck scale $\bar{M}_5$ varies from 7 TeV to 9 TeV when $\kappa$ varies from 1.5 GeV to 0.5 GeV. These values are closed to the discovery limit of the LHC which was derived recently in the framework of the same scenario (see Eq. (9)).

Since our scheme has only one extra dimension, the sum over KK excitations is UV-finite, contrary to the ADD scheme with two or more extra dimensions (see, for instance, Sec. II.C of the first reference in [18]). In any case, a possible UV cutoff is irrelevant, since it should be equal to (or larger than) the scale $\bar{M}_5$, while the masses of the first KK states are much smaller than $\bar{M}_5$.

Thus, our predictions (Figs. 1-3) depend only on the 5-dimensional gravity scale $\bar{M}_5$ and curvature parameter $\kappa$.

REFERENCES

*Remember that we imposed stronger condition $\theta_{\text{zenith}} \geq 75^\circ$ during our calculations.