In our analysis, we assumed that particles followed geodesics of KS metric: however, it is important to point out that this is true if matter is minimally and universally coupled to the metric, which is not necessarily true in HL gravity, where, as we said above, the role of matter has not been yet clarified. In this paper, starting from the same assumption, we focus on the effects induced by the examined solution on the orbital period P_b of a test particle, on an extra solar system environment. We will explicitly work out the consequent correction $P_{\omega 0}$ to the usual third Kepler law in Section 2. In Sections 3-4 we compare it with the observations of the transiting extrasolar planet HD209458b "Osiris". We point out that the resulting constraints are to be considered as preliminary and just order-of-magnitude figures because, actually, the entire data set of HD209458b should be re-processed again by explicitly modeling the effect of the KS gravity; however, this is outside the scopes of the present paper. Section 5 is devoted to the conclusions.

2. KS CORRECTIONS TO THE THIRD KEPLER LAW

As shown in [25], from [26]

$$\ddot{x}^i = -\frac{1}{2}c^2 h_{00,i} - \frac{1}{2}c^2 h_{ik} h_{00,k} + h_{00,k}\dot{x}^k \dot{x}^i + \left(h_{ik,m} - \frac{1}{2}h_{km,i}\right)\dot{x}^k \dot{x}^m, \quad i = 1,2,3, \tag{1}$$

it is possible to obtain the following radial acceleration acting upon a test particle at distance d from a central body of mass M

$$\vec{A}_{\omega 0} \approx \frac{4(GM)^4}{\omega_0 c^6 d^5}\hat{d}, \tag{2}$$

valid up to terms of order $\mathcal{O}(\upsilon^2/c^2)$. Its effect on the pericentre of a test particle have been worked out in [25]; here we want to look at a different orbital feature affected by eq. (2) which can be compared to certain observational determinations.

The mean anomaly is defined as

$$\mathcal{M} \doteq n(t - t_p); \tag{3}$$

in it $n = \sqrt{GM/a^3}$ is the Keplerian mean motion, a is the semimajor axis and t_p is the time of pericentre passage. The anomalistic period P_b is the time elapsed between two consecutive pericentre passages; for an unperturbed Keplerian orbit it is $P_b = 2\pi/n$. Its modification due to a small perturbation of the Newtonian monopole can be evaluated with standard perturbative approaches. The Gauss equation for the variation of the mean anomaly is, in the case of a radial perturbation A_d to the Newtonian monopole [27],

$$\frac{d\mathcal{M}}{dt} = n - \frac{2}{na}A_d\left(\frac{d}{a}\right) + \frac{\left(1-e^2\right)}{nae}A_d\cos f, \tag{4}$$

where e is the eccentricity and f is the true anomaly counted from the pericentre position. The right-hand-side of eq. (4) has to be valuated onto the unperturbed Keplerian orbit given by (see [28])

$$d = \frac{a\left(1-e^2\right)}{1+e\cos f}. \tag{5}$$

By using (see [28])

$$df = \left(\frac{a}{d}\right)^2\left(1-e^2\right)^{1/2} d\mathcal{M} \tag{6}$$

and

$$\int_0^{2\pi}\left(1+e\cos f\right)^2\left[2 - \frac{\left(1+e\cos f\right)}{e}\cos f\right]df = \pi\left(1+\frac{5}{4}e^2\right), \tag{7}$$

it is possible to work out the correction to the Keplerian period due to eq. (2); it is

$$P_{\omega 0} = \frac{4\pi(GM)^4(1+\frac{5}{4}e^2)}{\omega_0 c^6 n^3 a^6 (1-e^2)^{5/2}} = \frac{4\pi(GM)^{5/2}(1+\frac{5}{4}e^2)}{\omega_0 c^6 a^{3/2}(1-e^2)^{5/2}}. \tag{8}$$

Note that eq. (8) retains its validity in the limit $e \rightarrow 0$ becoming equal to

$$P_{\omega 0} \rightarrow \frac{4\pi(GM)^{5/2}}{\omega_0 c^6 d^{3/2}}, \tag{9}$$

where d represents now the fixed radius of the circular orbit. It turns out that eq. (9) is equal to the expression that can be easily obtained by equating the centripetal acceleration $\Omega^2 d$, where Ω is the particle's angular speed, to the total gravitational acceleration $GM/d^2 - 4(GM)^4/\omega_0 c^6 d^5$ with the obvious assumption that the Newtonian monopole is the dominant term in the sum.

3. CONFRONTATION WITH THE OBSERVATIONS

In the scientific literature there is a large number of papers (see, e.g., [29-41]) in which the authors use the third Kepler law to determine, or, at least, constrain un-modeled dynamical effects of mundane, i.e. due to the standard Newtonian/Einsteinian laws of gravitation, or non-standard, i.e. induced by putative modified models of gravity. As explained below, in many cases such a strategy has been, perhaps, followed in a self-contradictory way, so that the resulting constraints on, e.g., new physics, may be regarded as somewhat "tautologic".

Let us briefly recall that the orbital period P_b of two point-like bodies of mass m_1 and m_2 is, according to the third Kepler law,

$$P^{\rm Kep} = 2\pi\sqrt{\frac{a^3}{GM}} \tag{10}$$

where a is the relative semi-major axis and $M \doteq m_1 + m_2$ is the total mass of the system. Let us consider an unmodeled dynamical effect which induces a non-Keplerian (NK) correction to the third Kepler law, i.e.

$$P_b = P^{\mathrm{Kep}} + P^{\mathrm{NK}}, \quad (11)$$

where

$$P^{\mathrm{NK}} = P^{\mathrm{NK}}(M, a, e; p_j), \quad (12)$$

is the analytic expression of the correction to the third Kepler law in which p_j, $j = 1,2 \ldots N$, are the parameters of the NK effect to be determined or constrained. Concerning standard physics, P_{NK} may be due to the centrifugal oblateness of the primary, tidal distortions, General Relativity; however, the most interesting case is that in which P_{NK} is due to some putative modified models of gravity. As a first, relatively simple step to gain insights into the NK effect one can act as follows. By comparing the measured orbital period to the computed Keplerian one it is possible, in principle, to obtain preliminary information on the dynamical effect investigated from $\Delta P \doteq P_b - P^{\mathrm{Kep}}$. Actually, one should re-process the entire data set of the system considered by explicitly modeling the non-standard gravity forces, and simultaneously solving for one or more dedicated parameter(s) in a new global solution along with the other ones routinely estimated. Such a procedure would be, in general, very time-consuming and should be repeated for each models considered. Anyway, it is outside the scopes of the present paper, but it could be pursued in further investigations.

Concerning our simple approach, in order to meaningfully solve for p_j in

$$\Delta P = P^{\mathrm{NK}} \quad (13)$$

it is necessary that

- In the system considered a measurable quantity which can be identified with the orbital period and directly measured independently of the third Kepler law itself, for example from spectroscopic or photometric measurements, must exist. This is no so obvious as it might seem at first sight; indeed, in a N-body system like, e.g., our solar system a directly measurable thing like an "orbital period" simply does not exist because the orbits of the planets are not closed due to the non-negligible mutual perturbations. Instead, many authors use values of the "orbital periods" of the planets which are retrieved just from the third Kepler law itself. Examples of systems in which there is a measured orbital period are many transiting exoplanets, binaries and, e.g, the double pulsar. Moreover, if the system considered follows an eccentric path one should be careful in identifying the measured orbital period with the predicted sidereal or anomalistic periods. A work whose authors are aware of such issues is [42].
- The quantities entering P^{Kep}, i.e. the relative semimajor axis a and the total mass M, must be known independently of the third Kepler law. Instead, in many cases values of the masses obtained by applying just the third Kepler law itself are used. Thus, for many exoplanetary systems the mass $m_1 \doteq M_\bigstar$ of the hosting star should be taken from stellar evolution models and the associated scatter should be used to evaluate the uncertainty $\delta M_\bigstar$ in it, while for the mass $m_2 = m_{\mathrm{p}}$ of the planet a reasonable range of values should be used instead of straightforwardly taking the published value because it comes from the mass function which is just another form of the third Kepler law. Some extrasolar planetary systems represent good scenarios because it is possible to know many of the parameters entering P^{Kep} independently of the third Kepler law itself, thanks to the redundancy offered by the various techniques used. Such issues have been accounted for in several astronomical and astrophysical scenarios in, e.g., [43-47].

4. THE TRANSITING EXOPLANET HD209458B

Let us consider HD 209458b "Osiris", which is the first exoplanet[1] discovered with the transit method [48, 49]. Its orbital period P_{b} is known with a so high level of accuracy that it was proposed to use it for the first time to test General Relativity in a planetary system different from ours [50]; for other proposals to test General Relativity with different orbital parameters of other exoplanets, see [51-54].

In the present case, the system's parameters entering the Keplerian period i.e. the relative semimajor axis a, the mass $M_\bigstar$ of the host star and the mass m_p of the planet, can be determined independently of the third Kepler law itself, so that it is meaningful to compare the photometrically measured orbital period P_{b} = 3.524746 d [55] to the computed Keplerian one P^{Kep}: their difference can be used to put genuine constraints on KS solution which predicts the corrections of eq. (2.8) to the third Kepler law. Indeed, the mass $M_\bigstar = 1.119 \pm 0.033\ \mathrm{M}_\odot$ and the radius $R_\bigstar = 1.555^{+0.014}_{-0.016}\mathrm{R}_\odot$ of the star [55], along with other stellar properties, are fairly straightforwardly estimated by matching direct spectral observations with stellar evolution models since for HD 209458 we have also the Hipparcos parallax π_{Hip} = 21.24±1.00 mas [56]. The semimajor axis-to-stellar radius ratio $a/R_\bigstar$ = 8.76±0.04 is estimated from the photometric light curve, so that $a = 0.04707^{+0.00046}_{-0.00047}\mathrm{AU}$ [55]. The mass m_p of the planet can be retrieved from the parameters of the photometric light curve and of the spectroscopic one entering the formula for the planet's surface gravity g_p (eq. (6) in [55]). As a result, after having computed the uncertainty in the Keplerian period by summing in quadrature the errors due to δa, $\delta M_\bigstar$, δm_p, it turns out

$$\Delta P \doteq P_b - P^{\mathrm{Kep}} = 204 \pm 5354\ \mathrm{s}; \quad (14)$$

the uncertainties $\delta M_\bigstar$, δa, δm_p, contribute 4484.88 s, 2924.77 s, 2.66 s, respectively to $\delta(\Delta P)$ = 5354 s.

The discrepancy ΔP between P_{b} and P^{Kep} of eq. (14) is statistically compatible with zero; thus, eq. (14) allows to constrain the parameter ω_0 entering $P_{\omega 0}$. Since

[1] See on the WEB http://www.exoplanet.eu/

$$P^{\rm NK} \doteq P_{\omega 0} = \frac{\mathcal{K}}{\omega_0}, \tag{15}$$

with

$$\mathcal{K} \doteq \frac{4\pi(GM)^{5/2}}{c^6 d^{3/2}} = 8\times 10^{-15}\ {\rm s}, \tag{16}$$

by equating the non-Keplerian correction $P_{\omega 0}$ to the measured ΔP one has

$$\omega_0 = \frac{\mathcal{K}}{\Delta P}. \tag{17}$$

Since ΔP is statistically compatible with zero, the largest value of ω_0 is infinity; from eq. (4) a lowerbound on $|\omega_0|$ can be obtained amounting to

$$|\omega_0| \geq 1.4\times 10^{-18}. \tag{18}$$

A confrontation with the solar system constraints[2] Our previous paper [25] shows that such a lower bound is at the level of those from Jupiter and Saturn, while it contradicts the possibility that values of ω_0 as small as those allowed by Uranus, Neptune and Pluto ($|\omega_0| \geq 10^{-24} - 10^{-22}$). may exist. However, tighter constraints are established by the inner planets for which $|\omega_0| \geq 10^{-15} - 10^{-12}$.

5. CONCLUSIONS

We have investigated how the third Kepler law is modified by the KS solution, whose Newtonian and lowest order post-Newtonian limits coincides with those of GR, by using the standard Gauss perturbative approach. The resulting expression for $P_{\omega 0}$, obtained from the Gauss equation of the variation of the mean anomaly $\mathcal{M}$, in the limit $e \rightarrow 0$ reduces to the simple formula which can be derived by equating the centripetal acceleration to the Newton+KS gravitational acceleration for a circular orbit.

Then, after having discussed certain subtleties connected, in general, with a meaningful use of the third Kepler law to put on the test alternative theories of gravity, we compared our explicit expression for $P_{\omega 0}$ to the discrepancy ΔP between the phenomenologically determined orbital periods $P_{\rm b}$ and the computed Keplerian ones $P^{\rm Kep}$ for the transiting extrasolar planet HD209458b “Osiris”. Since ΔP is statistically compatible with zero, it has been possible to preliminary obtain the lower bound $|\omega_0| \geq 1.4\times 10^{-18}$ on the dimensionless KS parameter. However, the entire data set of HD209458b should be re-processed by including KS gravity as well, and a dedicated, solve-for parameter should be estimated as well. The previously reported constraint rules out certain smaller values allowed by the lower bounds obtained from the perihelia of Uranus, Neptune and Pluto ($|\omega_0| \geq 1.4\times 10^{-12}$). On the other hand, our exoplanet bound still leaves room for values of ω_0 too small according to the constraints from the perihelia of Mercury, Venus and the Earth ($|\omega_0| \geq 10^{-15} - 10^{-12}$).

[2] To avoid confusions with the perihelion ω, the KS parameter is dubbed ψ_0 in [25].

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