The Knee in the Cosmic Ray Energy Spectrum: a Pulsar, Supernova Origin?

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Abstract: The origin of the prominent 'knee' in the cosmic ray energy spectrum at an energy of several PeV is still uncertain. A recent mechanism has shown promise, however; this involves particles from a very young pulsar interacting with the radiation field from its associated very young supernova remnant. The ensuing nuclear reaction of the particles with the photons by way of e^+e^- production then causes the characteristic knee.

In an earlier paper we argued that the mechanism would imply only one source of a very rare type - if it were to explain the spectral shape. Here we examine the mechanism in more detail and conclude that for even a single source to work its characteristics would need to be so unusual that the mechanism would not be possible for any known type of pulsar-supernova combination.

1. INTRODUCTION

There are only two features of the cosmic ray energy spectrum which depart significantly from smoothness: the 'knee' at a few PeV and the 'ankle' at a few EeV. (1 PeV = 10^{15} eV, 1 EeV = 10^{18} eV). The origin of both is still subject to argument. Here we examine the first feature.

The 'knee', a comparatively sudden increase in slope, has been known since the pioneering work of Kulikov and Khristiansen [1] and it has been confirmed by many workers since. As time has gone on and with improved precision of measurement it has become realised that it is very sharp, particularly so if measurements can be confined to a single nuclear component (e.g. protons).

In order to place the present work in perspective we address, necessarily briefly, the various models which have been put forward to account for the knee.

The first class comprises models in which cosmic rays (CR) are produced with a power law in energy with a constant exponent to well beyond the knee. An early suggestion was that the knee then resulted from a "Change in the elementary act" (Nikolski and Pomanski [2] and later publications). This suggestion has found little favour and the latest CERN LHC data militate against the idea [3]. Still within this class are models involving loss of particles by way of Galactic diffusion, [4, 5] introduced the idea of

magnetised clouds. Modern analyses, using Kolmogorov (and other) forms for the spectrum of magnetic irregularities in the interstellar medium, owe much to the early ideas of Ginzburg and Syrovatsky [6]. Problems for this class include lack of knowledge of specific sources having constant exponents and the expected, but not observed, spatial anisotropies at high energies (see Ref. [7] for a recent summary).

Turning to the second class, supernova remnants (SNR) have long been regarded as likely accelerators for energies up to the Z PeV region (where Z is the nuclear charge). Much detailed modelling has been carried out by, e.g. Lucek and Bell [8], Berezhko *et al.* [9] and later publications), and fits obtained to very moderate accuracy. The knee would now correspond to the (average) maximum SNR (CR) energy with particles above the knee coming from "super-supernovae" or pulsars (see e.g. [10, 11]). Other models involve acceleration by Galactic shocks and other large scale structures.

It would seem that the second class of models is superior but a problem arises with the observed sharpness of the knee. It is too sharp for it to be due to an assembly of SNR (Erlykin and Wolfendale [12] and later publications), and these authors proposed the Single Source Model (SSM) in which a single recent local SNR causes the sharp knee. The latest, very precise, measurement from the Tibet array [13] confirms the high degree of sharpness. Another, more recent, model is that due to [14] in which particles from a rapidly rotating pulsar interact with radiation from the parent SNR. We refer to this model as PSNR and later publications).

The aim of the present work is to follow up our earlier work Ref. [15] in which we examined the PSNR Model as

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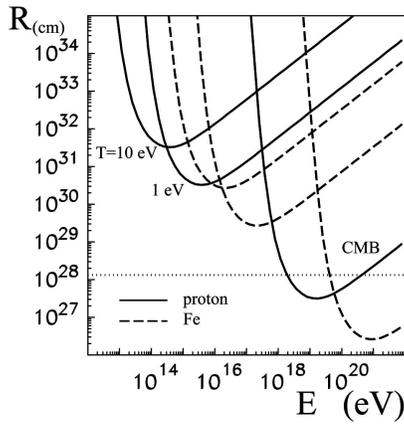
Fig. (1). Absorption length vs energy for protons and iron nuclei interacting with photon fields of mean 'temperature' as shown. The energy density of all but the CMB is 1 eV cm^{-3}.

originally proposed in [14], to see if the knee caused by the particle -SNR photon interactions was a universal property of all pulsar, SNR pairs. We found that it was not but that there was hope for a single rare pair having the required properties. This would be identified as the single source of the SSM although the mechanism would differ from that in the original SSM which involved SNR alone. Firstly we examine the needed properties of the pulsar, SNR pair in more detail.

2. THE BASIS OF THE PSNR MODEL

2.1. The Characteristics of the Interactions

As remarked already, the philosophy of the source process is a composite pulsar-SNR system with the pulsar-accelerated particles interacting with the photon field of the SNR. Under certain circumstances a sharp knee will result due to particle, photon $\rightarrow e^+ e^-$ interactions. We start by examining the general properties of the interaction in terms of the mean energy of the photons.

An important characteristic of the 'knee' in the energy spectrum is its 'sharpness', S. We follow Ref. [12] and define the sharpness as

$$S = -\frac{\partial^2 (\log IE^3)}{\partial (\log E)^2}, \tag{1}$$

where I is the cosmic ray flux. The factor E^3 has no particular meaning, but it is kept according to the convention of presenting the very steep ($\sim E^{-3}$) spectrum of cosmic rays in the way which allows the sharpness S to be seen clearly as a particular change of this steepness. The results presented in this work were obtained with the numerical differentiation of the spectra tabulated using extensive simulations of the propagation of cosmic ray particles in the assumed magnetic and photon fields in the vicinity of the sources.

Fig. (**1**) sets the scene for the attenuation length, *R*, of protons and iron nuclei interacting with black body photons. The energy density of the photons is 1 eV cm^{-3} for all but the Cosmic Microwave Background (CMB), where it is the usual 0.24 eV cm^{-3}. The minimum absorption length is inversely proportional to the mean photon energy, *T*. (1 eV corresponds to 1.16×10^4 K, ie a mean wavelength of 12,400

Å). We note that the middle of the visible range, yellow light, has wavelength ~ 5000 Å ie a mean energy of 2.48 eV.

The effect of the photon field on the energy spectrum of the emerging particles depends on the energy density of the photons at the time of their passage through the field; for a low energy density there is no effect and for a high energy density the interactions are catastrophic, with no particles emerging. As an illustration, with the object of finding S - values in the required range, we choose a combination of average energy density and 'trapping time' such as to give the minimum R in Fig. (**1**) for $T = 1$ eV ($R = 3 \times 10^{30}$ cm). The limits to the time over which the interactions occur, t_0 and t_1, are discussed in the next section. So as to maximise the PSNR 'effect' we choose the smallest value of t_0 and the largest t_1 permissible.

2.2. The Limiting Time Parameters for the Interactions and the Two Models

The lower limit to the interaction time, t_0, is set by energy losses by the pulsar-accelerated particles on the environmental gas. This latter is enhanced very considerably by the ejecta from the SN; it is this material, too, that obscures the optical radiation for a period of days. For a typical massive progenitor of a Type II SN (and a 'magnetar') the mass ejected is of order 5 $M_\odot$ [16]. The expansion velocity at short times after the expansion can only be found by reference to model calculations; a commonly quoted value is $\sim c/10$. Some support for a value of this order comes from Ref. [17]: $c/30$ some 30 years after the expansion of a shell remnant (43.31, +592) in Messier 82.

If the density of the ejecta were constant, an attenuation length for protons of 120 g cm^{-2} (as required for considerable absorption of the particle beam) would be achieved in $\sim 1.6 \times 10^6$ s, neglecting the ISM swept up. If concentrated in a thin shell the time needed would be $\sim 10^6 s$. In fact, there will be considerable clumpiness, reducing this time by perhaps an order of magnitude; indeed, observations of (much older) SNR show very variable densities. Here, as a rather extreme limit (a factor 30 smaller), we adopt $t_0 = 3 \times 10^4$ s. A measure of support comes from the fact that, occasionally, amongst the very many SNR light curves, one encounters `light break through' at such early times.

Turning to the maximum time, t_1, its' derivation is easier. Inspection of Fig. (**1**) shows that for a photon 'temperature' of 1eV and an energy density of 1 eV cm^{-3}, interactions will cease when R $\sim 10^{30}$ cm. This can be converted to an appropriate radius for the environment of a SNR of known total photon energy and its time dependence.The highest energy output achievable is $\sim 10^{43}$ erg s^{-1} [18] yielding, in 10^6 s : 10^{49} erg.

We now come to an important problem: the extent to which the pulsar-accelerated particles are trapped in the remnant before escaping. Alternatives are adopted here, as follows:
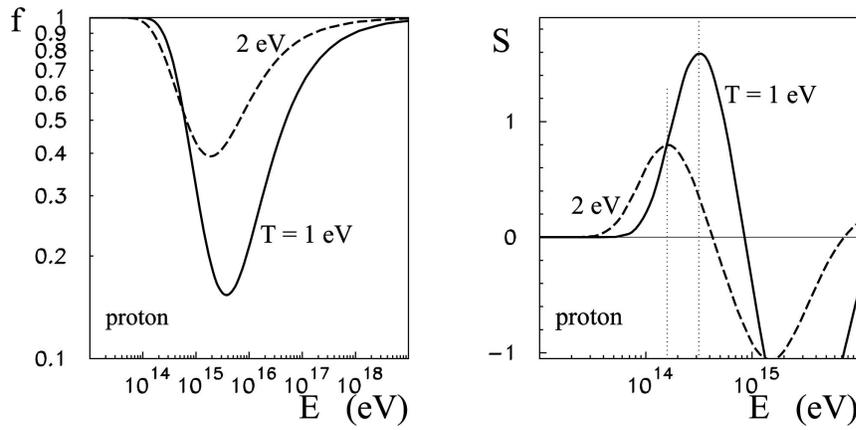
Fig. (2). (**a**) The reduction factor, to be applied to the proton spectrum due to interactions with the photon field chosen (see text) for two 'temperatures'. (**b**) Sharpness values derived from the above.
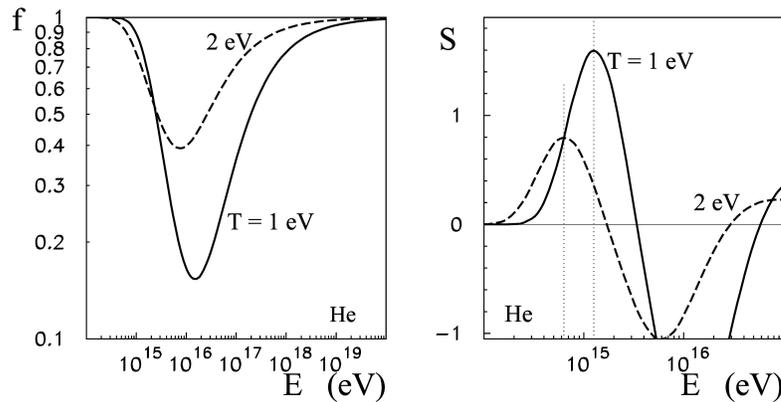
Fig. (3). (**a**),(**b**) As for Fig. (**2**) but for Helium nuclei.

- The particles are trapped by the tangled magnetic fields and diffuse out with the remnant (Model 1), and

- The particles are not trapped but travel out from the pulsar with the speed of light (Model 2) Predictions will be made for each case and then common problems will be addressed.

3. RESULTS FOR MODEL 1

3.1. Model 1 - Particle Diffusion

SNR commonly expand at a velocity of $\sim c/10$ as already remarked and an expansion time of 10^6 s yields a radius of 3×10^{15} cm. Assuming that the CR are trapped in the remnant for this time the required total distance traveled for the photon energy achieved corresponds to $R \simeq 10^{30}$ cm for 1 eV cm^{-3}. The energy density for our remnant follows as $\simeq 5 \times 10^{13}$ eV cm^{-3}.

To summarise: our (rather extreme) limits are $t_0 = 3 \times 10^4$ s and $t_1 = 10^6$ s; the corresponding SNR radii are 10^{14} cm and 3×10^{15} cm.

3.2. Model 1 - The Derived CR Spectrum after the Interactions

Calculations have been made using the above parameters, for protons in the first instance. The results are shown in Fig. (**2**), for both the spectral reduction factors, f, defined as a ratio of the cosmic ray flux calculated for the model and the one which would be observed if the were no interactions

within the vicinity of the source, the conventional production spectrum, and sharpness values, S. In order to study the sensitivity to the mean photon energy, the calculations are made for both $T = 1$ eV and 2 eV.

It should be noted that the S-values are upper limits in that we ignore the particles which are accelerated after the interactions with the SNR-photons have effectively ceased. Such particles will dilute the knee and reduce S.

Fig. (**3**) gives the corresponding plots for Helium; it will be seen that the S-values are rather similar - the main difference being, as expected, in the energy at which S is a maximum.

3.3. Model 1 - The Ensuing Knee Position

Inspection of Fig. (**2**) and (**3**) shows that the CR energies for S_{max} are, for protons: E^*(p): 0.31 PeV (1 eV) and 0.15 PeV (2 eV) and for helium nuclei: E^* (He): 1.2 PeV (1 eV) and 0.62 PeV (2 eV), where the value in brackets is the mean photon energy.

The ratio of the energies for S_{max} for helium and protons are seen to be ~ 4, as would be expected since it is energy per nucleus that is important in determining the threshold energy for photon, particle interactions.

Comparison can now be made with experiment. The energy of the knee in the CR energy spectrum is model-dependent and values range from about 3 to 8 PeV. There is evidence that Helium nuclei predominate at the knee (Refs. [19, 20]) and if so, there is clearly a factor 10 difference in
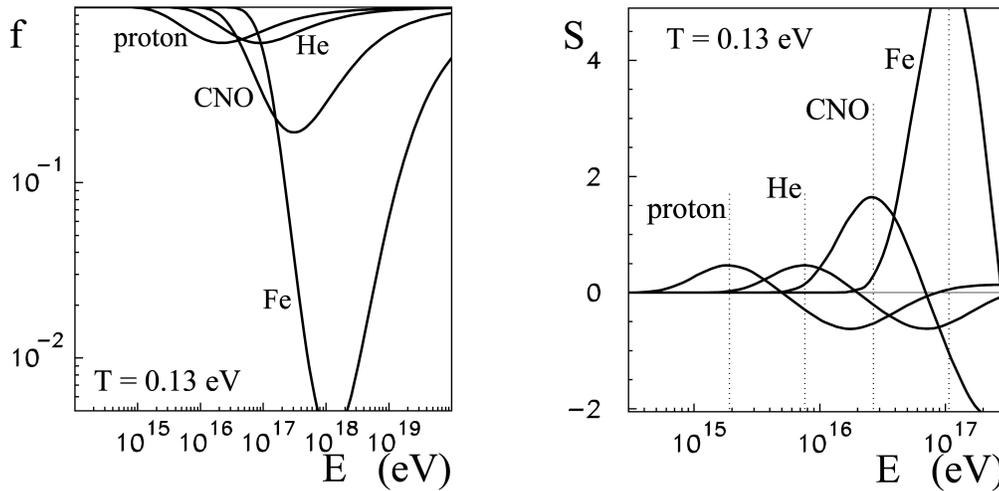
Fig. (4). (**a**) f -values for the nuclei shown as a function of particle energy for $T = 0.13$ eV, the temperature needed to give knee positions for protons and He near those found experimentally. (**b**) sharpness values derived from the above.
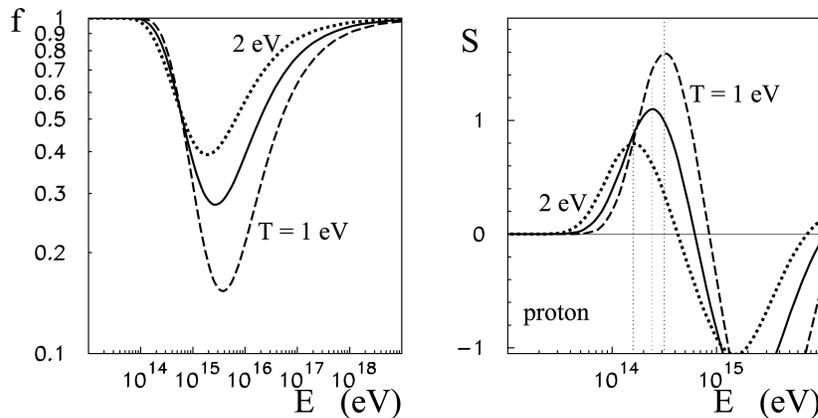
Fig. (5). (**a**) f -values for a model in which the T -value fell smoothly from 2 eV to 1 eV during the interaction period (full line), in comparison with constant values of 2 eV and 1 eV. (**b**) Sharpness values derived from the above.

the S_{max} -energies, between observed and expected, for $T = 1$ eV. A value nearer $T = 0.1$ eV (wavelength ~ 120,000 Å: for infra-red) is required. This is a serious drawback for Model 1.

3.4. Model 1 - A Variant to fit the CR Spectral Knee

Fig. (**4**) shows the situation where T is reduced to 0.13 eV so that the proton knee (peak) is now at E^* (p) $\simeq$ 2 PeV. The Helium knee (peak) is at E^* (He) ~ 7.8 PeV. Both can be regarded as, perhaps, just acceptable.

Two aspects need consideration, now. The first concerns the position of the knee (peak) for iron of the same rigidity for protons and He, i.e. at 100 PeV. This is close to the 70 PeV where recent work [21] gives rather strong evidence for a peak from an analysis of the results from the EAS arrays. This is a good feature of the model. The second relates to the need for T ~ 0.13 eV; with the conversion 1 eV correspond to 1.16×10^4 K, the effective temperature is in the region of 1,500 K, ie in the infrared, as already remarked. Although there is, inevitably, an infrared flux, from visible radiation absorbed by the SNR material, its intensity is too low to give sufficient energy density for our purpose and this is a bad feature.

3.5. Model 1 - The Effect of a Non-Uniform SNR Temperature

Inevitably, the temperature of the remnant will fall with time and a reduction in S will occur. Fig. (**5**) shows the situation for the likely case of a 'conventional SNR' (e.g. [22]) where the temperature falls by a factor 2 over the operative period of particle-photon interactions. Interestingly, although finite, the reduction in S is not large. However, for more dramatic SNR, such as are required here, the change in temperature can be much bigger. In Ref. [18] temperatures versus time are given for the following progenitors: red supergiants, blue supergiants and Wolf-Rayet stars. The ratios of $T(3 \times 10^4$ s) to $T(10^6$ s) are, respectively, (for typical stars within these types): 7.1, 8.4 and 8.4. The exponents of the power laws are -0.56, -0.61 and -0.61. It is true that the luminosity falls with time, too, but with much smaller exponents: -0.17, -0.35 and -0.35. In consequence, the reduction in S in practice for SNR from such progenitors will be much smaller than shown in Fig. (**5**).

For young magnetars, potential SN for the mechanism considered here, the light curves are a rather strong function of the magnetic field of the magnetar and often continue for hundreds of days [23] Predictions are not available for the
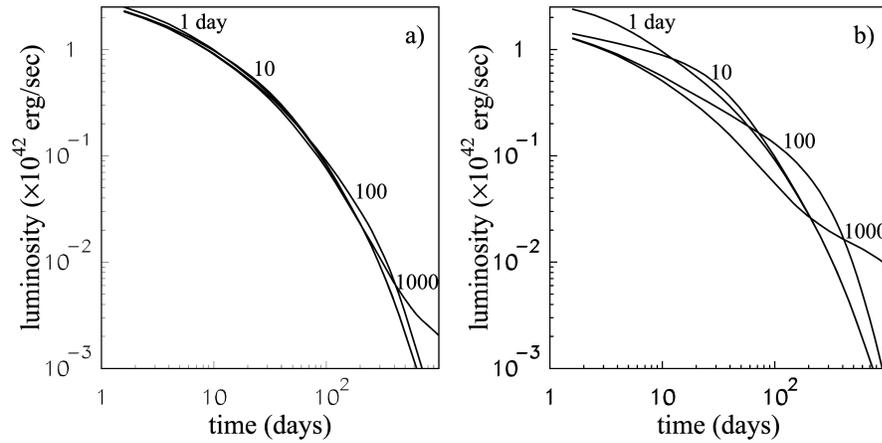
Fig. (6). Luminisity for 10% (**a**) and 50% (**b**) absorption.

short period from SN birth (0.3 to 10 days) needed here so that conclusions cannot be drawn.

In summary, falling temperatures appear not to be a problem for the model.

4. RESULTS FOR MODEL 2

Inspection of many published SNR `luminosity curves' shows a rather wide spread but Fig. (**6a**) shows the general form,for the case where the absorption by the SNR shell material is only 10%. Fig. (**6b**) shows the light curve for an absorption of 50%. The parameter for the lines is the `cooling time', i.e. the mean exponential decay time of the radiation.

Calculations have been made by us for a variety of values for the absorption efficiency, the cooling time and the time over which the source is active (the `source activity time').Demanding that the sharpness should exceed S=2 gives the results shown in Fig. (**7**). It will be noted that there is a minimum cooling time of 40 days and the source activity time must be greater than about 30 days.

5. DISCUSSION

5.1. Parameters to be Discussed

A number of parameters need examination, principally:

The nuclear masses needed for injection,

The total photon energy,

The products of the photon-nucleus interactions and, most importantly,

The characteristics needed for the pulsar.

These parameters will be considered, in turn.

5.2. The Nuclear Masses Needed for Injection

It is apparent that the masses of nuclei needed: protons, Helium, Iron (?) are not expected as products of pulsar acceleration. Rather, proton-rich or iron-rich neutron-star surface elements would be expected. Although not an impossible situation, the needed composition does not add confidence to the model.

5.3. The Total Photon Energy

The total photon energy in the photon field adopted (which we assume effectively ceases after 10^6 s) is $\sim 10^{49}$
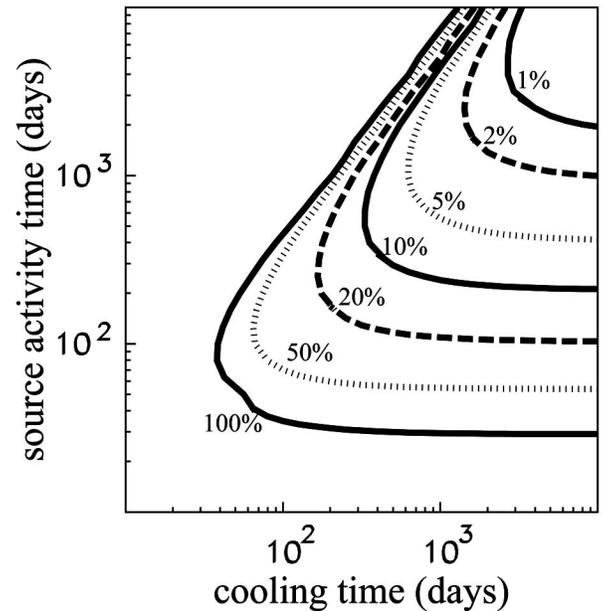
Fig. (7). Model-2 in which the pulsar-accelerated particles travel in straight lines. The percentages refer to the fraction of the SNR radiation absorbed by the ejecta shell. 'Source activity' relates to the time for which the pulsar is accelerating particles of the needed energy. The results are for helium.

erg. To this should be added the later energy - which is the energy 'seen' after the obscuration has ceased, but here we ignore this component which should not be large. The value can be compared with the total energy released in a typical SN: about 10^{51} erg, of which perhaps 3×10^{49} erg appears in visible light [16]. Thus, the energetics are acceptable.

5.4. The Products of the Nucleus-Photon Interactions

If the whole of the CR spectrum were due to the PSNR mechanism then the overall change of slope of the primary CR spectrum would correspond to a transfer of energy from CR particles to gamma rays, via the electron pairs generated in the particle-SNR photon interaction. Inspection of the CR spectrum shows that the energy density of Galactic gamma rays would be about 2×10^{-5} eV cm^{-3}, a value comparable with that detected in the diffuse gamma radiation, the latter usually being attributable to CR particles interacting with gas in the ISM. Such a situation would be 'dangerous' for the model but probably not catastrophic. In the more likely

situation (in our view) that it is only the Single Source that produces the knee then copious gamma ray fluxes will have been produced soon after the interaction (some few 10^5 y ago?). However, it is likely that these gamma rays will have 'been and gone' by now.

The conclusion is that the non-observation of the gamma rays associated with the P-SNR interactions does not pose a threat to the model.

5.5. The Characteristics Needed for the Pulsar

The first characteristic is that the pulsar should be able to accelerate particles to rigidities of some 10 PeV. Ref. [24] give the following expression for the maximum achievable rigidity.

$$R_{max} = 6.6 \times 10^{12} B_{12} \ P^2$$

here, B_{12} is the magnetic field in 10^{12} Gauss, P is the Period in seconds and the rigidity R is in Volts. If we need $R_{max} = 10^{16}$ V then, for $B_{12} = 3$ (the mean, from 'model A' of Ref. [25] a period of 23 ms is required. Interestingly, this is close to the birth period in the same model, which has P = 22 ms.

The problem here is the likely age of pulsars of such characteristics. Using the work [10] again, this is given by

$$T = \frac{P^2}{2 \ B_{12}^2} \times 10^{15} = 3 \times 10^{10} \text{ s}$$

Such a time (1000 y) would be far too long to satify the requirements of Fig. (**3**) where the 'cooling time' needed is unlikely to be longer than ~ 100 d.

Pulsars of much shorter period are required, for example, $P \sim 1$ ms and, say, $B_{12} = 3$, for which $T \sim 500$ days.

'Normal' pulsars [25] have a standard deviation about the mean of $\sigma(\log B) \sim 0.3$ and will have a similar standard deviation for period [26], thus they will not satisfy the bill, even to the extent of 3 standard deviations from the mean.

Magnetars [23] would, at first sight, appear to satisfy the requirements. They have extremely high magnetic fields, typically 6×10^{14} G but, when measured, their periods are of order seconds. In Ref. [18] a theory is developed for the evolution of magnetars, in which the important birth period is in the range 10-740 ms, with a mean of 161ms. However, the mean field is given by $\langle \log B \rangle \simeq 11.9$. Clearly, the T-value will be, again, too long. Another factor militating against a magnetar as being the 'local' pulsar responsible for the knee is the fact that their birth rate is quoted as only $\sim 10^{-3}$ y^{-1} for the Galaxy as a whole; furthermore, none is in our vicinity; the McGill catalogue, [23], gives the nearest as being some 3.5kpc away. In fact the most likely nearby SNR for the single source is a conventional one: Monogem [16].

6. CONCLUSIONS

That the suggested PSNR mechanism is very unlikely to work for even a single, rare 'object' is provided by the following facts.

• The losses of energy by the pulsar-accelerated particles in collision with the very early SNR ejecta is expected to be very great. In our calculations we assumed that the ejecta was clumpy enough to allow such passage but this appears highly unlikely.

• For both models considered here the 'source activity time' is far too long for any known type of pulsar to be responsible; CR particles emitted after the SNR illumination has ceased will dilute the spectral sharpness to an unacceptable degree. This is intrinsic in both Models and can be seen directly in Fig. (**7**); for a typical cooling time of 100 d the pulsar must be active for less than a year.

• Even if an ultra-rare pulsar of the required characteristics was proposed, the probability of its being close enough (and its contemporary presence undetected) is remote in the extreme.

• The mass composition of the ambient CR in general and the Single Source in particular are quite different from what would be expected from a pulsar.

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CONFLICT OF INTEREST

None Declared.

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