# The Kinematics and Velocity Ellipsoid Parameters of Open Star Clusters 

Elsanhoury W. H ${ }^{1, *}$, Sharaf M. A ${ }^{2}$, Nouh M. I ${ }^{3}$ and Saad A. $\mathrm{S}^{4}$<br>${ }^{1}$ Astronomy Department, National Research Institute of Astronomy and Geophysics (NRIAG), 11722 Helwan, Cairo, Egypt<br>${ }^{2}$ Astronomy Department, Faculty of Science, King Abdu-Aziz University, Jeddah, Saudi Arabia<br>${ }^{3}$ Physics Department, Faculty of Science, Northern Border University, Arar, Saudi Arabia<br>${ }^{4}$ Mathematics Department, Faculty of Science, Qassim University, Qassim, Saudi Arabia


#### Abstract

In terms of the equatorial coordinates, the galactic space velocity components were expressed in closed analytical forms. With the aid of the vectors and matrices analyses expressions for the velocity ellipsoid parameters VEPs in closed analytical forms were also represented. For the computational developments, a general computational algorithm for the basic parameters from the exact solutions of the equations involved was used to compute the basic elements of the velocity ellipsoid and demonstrate the ability of the algorithm to produce accurate results.


Keywords: Open clusters, velocity ellipsoid parameters, galactic space, analytical forms.

## 1. INTRODUCTION

The classical picture of the evolution of the velocity structure in the Galactic disk is the origin of stars within low-dispersion clusters from cool gas on near-circular orbits. These clusters evaporate, and the stellar orbit distribution is heated through the gravitational perturbations to the smooth disk potential. As the stellar population's velocity dispersion increases over time, its mean motion lags behind that of pure circular orbits at the same galactocentric radius. Thus, the velocity distribution of stars in the solar neighborhood has been characterized as an ellipsoid the centroid, size, and orientation of which vary systematically with the ages (and hence colors) of the stars under investigation [1, 2].

It is well known since a long time [3] that, in the neighborhood of the Sun, the characteristic feature of stellar motion is the fact that the peculiar velocities have an axis of greatest mobility and this characteristic is represented most conveniently on the basis of ellipsoidal law of velocity distribution. If we consider the ellipsoidal law to be associated in general and at all points with the steady state of a stellar system, the function $f$ must be expressible in the form:

$$
f=F\left(x, y, z ; a u^{2}+b v^{2}+c w^{2}+2 f v w+2 g w u+2 h u v\right)
$$

Where $a, b, c, \ldots, h$ are in general functions of $x, y$ and $z$. In this generalized form, the length and distributions of the principle axes of the velocity ellipsoid vary from point to point of the system.

[^0]The importance of the velocity ellipsoid parameters is due to their connection to the most important mathematical function of stellar astronomy, that is, the phase density function.

A relationship of the parameters of the velocity ellipsoids of F-type (about 5500) stars to their metallicity, temperature and age, which have been investigated with [4] using ubvy photometry and proper motion data, and the following results have been obtained: (1) the length of all three semiaxes of the ellipsoids increase systematically for star groups as both their temperature (at constant metallicity) and their matallicity (at constant temperature) decrease, i.e. each of the three parameters; temperature, metallicity and velocity spread - are a statistical indicator of age for F stars on the main sequence (MS); (2) with increasing age of a star group, the velocity ellipsoid becomes much more spherical, and the direction of its semimajor axis approaches the direction toward the center of the Galaxy; (3) the spread in the peculiar velocities of disk stars displays a bend in its $\sigma-[\mathrm{Fe} / \mathrm{H}]$ dependence at the point corresponding to the middle of the metallicity distribution of disk stars; (4) the angular momentum of MS stars increases with decreasing metallicity, and it decreases with decreasing temperature; and (5) the increase in the stars velocity spread with age is described well by a linear law in the disk subsystem older than $2 \times 10^{9}$ years.

A procedure to statistically isolate cluster members from the field stars using the total proper motion and the position angle (P.A.) of the stars in a given field. To achieve this, [5] determined the velocity ellipsoid for Hyades and $U M a$ moving groups. Thereby, the velocity ellipsoids were determined using [6, 7] data. For that purpose [2] relied on the use of statistical moments using the Hipparcos data. However [8] used a new mathematical technique, semidefinite programming, and a criterion, a difference of squares, using 246 stars
of spectral class O-B5 and luminosity class $V$ to calculate the velocity ellipsoid giving results to be considered superior to those found from the above method (i.e., method of moments).

The dependence of the velocity ellipsoid of F-G stars of the thin disk of the Galaxy on their ages and metallicites can be analyzed by [9] based on the new version of the Geneva Copenhagen Catalog. With increasing age, the velocity ellipsoid increases in size and becomes appreciably more spherical, turning toward the direction of the Galactic center, and loses angular momentum. The shape of the velocity ellipsoid remains far from equilibrium. With increasing metallicity, the velocity ellipsoid for stars of mixed age increases in size, displays a weak tendency to become more spherical, and turns toward the direction of the Galactic center (with these changes occurring substantially more rapidly in the transition through the metallicity $[\mathrm{Fe} / \mathrm{H}] \approx-0.25$ ).

Recently, [10] studied the kinematics of the G giant stars (luminosity class III) based on proper motions and parallaxes taken from van Leeuwen's new reduction of the Hipparcos catalog.

In this paper, the velocity ellipsoid parameters will be shown analytically using vectors and matrices analyses [11], while general computational algorithm for the basic parameters from the exact solutions of the equations involved was constructed here.

Finally, correlations between these kinematical properties (i.e. velocity ellipsoid parameters) with physical one (i.e. spectral types) were studied for Hyades open cluster 197 stars were used by [12] for Hipparcos main catalog. The kinametics of stars in the solar neighborhood gave fundamental information for our understanding of the structure and evolution of the Milky Way. ESA's astrometric satellite Hipparcos (ESA 1997) provided us with accurate positions and trigonometric parallaxes, as well as absolute proper motions for a large and homogeneous sample of tens of thousands of stars near the Sun. This offered the opportunity to investigate the velocity distribution in the solar neighborhood, not only for early-type stars, but also for the old population of the Galactic disc. Several studies have been performed on this topic since, e.g. [2, 13-19] and [20]. Recently, [21] studied the Milky Way (MW) thin disk with RAdial Velocity Experiment (RAVE) survey. They considered the thin and thick disks different Galactic components and presented a technique to statically disentangle the two populations.

The Hyades open star cluster provides a well known example of moving clusters, which has many features: It's total mass range from 300 to $400 \mathrm{M}_{\text {sun }}$, Age of around 600-800 $M y r$, an extension in the sky of about $20^{\circ}$. Convergent point $(A, D)$, distance $d(p c)$, velocity $V(\mathrm{~km} / \mathrm{sec})$, and center of the cluster $\left(x_{c}, y_{c}, z_{c}\right)$ was deduced [22].

- $\quad(A, D)=\left(97^{0} 54.18^{\backslash}, 6^{0} 46.39^{\}\right)$.
- The distance $d$ of the cluster could be computed from:
$d=N / \sum_{i=1}^{N} P_{i}=46.0658 \pm 0.84 p c$
where, $N$ is the total number of member stars and $P_{i}$ is the parallaxes for $N=1,2, \ldots, N$
- The velocity $V$ of the cluster is calculated from:
$V=\sum_{i=1}^{N} V_{t}^{(i)} \sin \zeta_{i} / \sum_{i=1}^{N} \sin ^{2} \zeta_{i}=46.5396 \pm 0.24 \mathrm{~km} / \mathrm{s}$
where $V_{t}$ is the tangential velocity of the $i$-star (i.e. $V_{t}=$ $4.74 \beta_{i} / P_{i}$ ), into which $\beta_{i}$ is the proper motion for $i$-star, and $\zeta_{i}$ is the spherical distance for $i$-star i.e.
$\zeta_{i}=\cos ^{-1}\left[\sin \delta_{i} \sin D+\cos \delta_{i} \cos D \cos \left(A-\alpha_{i}\right)\right]$
- The cluster center $\left(x_{c}, y_{c}, z_{c}\right)$ :
$x_{c}=\sum_{i=1}^{N} \frac{\cos \delta_{i} \cos \alpha_{i}}{P_{i}} / N=18.36$,
$y_{c}=\sum_{i=1}^{N} \frac{\cos \delta_{i} \sin \alpha_{i}}{P_{i}} / N=42.20$,
$z_{c}=\sum_{i=1}^{N} \frac{\sin \alpha_{i}}{P_{i}} / N=13.94$.
Also, Hyades played a fundamental role in astronomy as a first step on the cosmic distance ladder and as a text case for theoretical models of stellar interiors [23].


## 2. BASIC FORMULATIONS

In this section, the basic equations governing the determination of the velocity ellipsoid will be derived by using the vectors and matrices analysis.

The components $U, V$ and $W$ (i.e. the system of galactic space coordinates) can be computed by the transformation formulae [24]. The direction to the galactic pole in the new $J 2000.0$ equatorial system is $\alpha_{G}=12^{h} 51^{m} 26^{s} .2755 ; \delta_{G}=$ $27^{\circ} 7^{\prime} 41^{\prime \prime} .704$.

$$
\left.\begin{array}{l}
U=-0.054875539 X-0.873437105 Y-0.483834992 Z \\
V=0.494109454 X-0.444829594 Y+0.746982249 Z  \tag{1}\\
W=-0.867666136 X-0.198076390 Y+0.455983795 Z
\end{array}\right\}
$$

where $X, Y$ and $Z$ are the components of the space velocity along $x, y$ and $z$ axes of a coordinate system whose center is the Sun, such that the $x$-axis points towards the point ( $\alpha=$ $0^{h}, \delta=0^{0}$ ), the $y$-axis is oriented towards the point ( $\alpha=6^{h}$, $\delta=0^{0}$,) and the $z$-axis towards the north celestial pole at definite epoch.

According to the well known formulae [25], we have:

$$
\left.\begin{array}{l}
X=-4.738 d \beta_{\alpha} \cos \delta \sin \alpha-4.738 d \beta_{\delta} \sin \delta \cos \alpha+V_{r} \cos \delta \cos \alpha  \tag{2}\\
Y=4.738 d \beta_{\alpha} \cos \delta \cos \alpha-4.738 d \beta_{\delta} \sin \delta \sin \alpha+V_{r} \cos \delta \sin \alpha \\
Z=\quad+4.738 d \beta_{\delta} \cos \delta+V_{r} \sin \delta
\end{array}\right\}
$$

From equations (1) and (2) it is clear that the components of the space velocity $U_{i}, V_{\mathrm{i}}$ and $W_{i}$ of the $i^{\text {th }}$. stars of a group could be obtained from observed quantities (i.e. $\alpha, \delta, \mu_{\alpha}$, $\mu_{\beta}, \ldots$ etc.)

Now, the coordinates of the $i^{\text {th }}$ star with respect to axes parallel to the original axes, but shifted to the center of the distribution, i.e. to the point $U, V$ and $W$, will be $\left(U_{i}-\bar{U}\right) ; \quad\left(V_{i}-\bar{V}\right) ; \quad\left(W_{i}-\bar{W}\right), \quad$ where the components $\bar{U}, \bar{V}$ and $\bar{W}$ of the mean velocity are defined as:
$\bar{U}=\frac{1}{N} \sum_{i=1}^{N} U_{i} ; \quad \bar{V}=\frac{1}{N} \sum_{i=1}^{N} V_{i} ; \quad \bar{W}=\frac{1}{N} \sum_{i=1}^{N} W_{i}$
With $N$ being the total number of the stars.
In order to study the distribution of the residual velocities $\left(U_{i}-\bar{U}\right) ; \quad\left(V_{i}-\bar{V}\right) ; \quad\left(W_{i}-\bar{W}\right) ; i=1,2,3, \ldots, N$ of the group of stars, let us take an arbitrary axis $\xi$ (say) drawn through the center of the distribution and let its zero point coincide with the center of the distribution. Let, further, $l, m$ and $n$ be the direction cosines of the $\xi$ axis with respect to the shifted ones. Then, the coordinates $Q_{i}$ of the point $i$, with respect to the $\xi$ - axis are given by:

$$
\begin{equation*}
Q_{i}=l\left(U_{i}-\bar{U}\right)+m\left(V_{i}-\bar{V}\right)+n\left(W_{i}-\bar{W}\right) \tag{4}
\end{equation*}
$$

The scatter components $Q_{i}$ as a generalization of the mean square deviation can be defined by
$\sigma^{2}=\frac{1}{N} \sum_{i=1}^{N} Q_{i}^{2}$
From Equations (3), (4) and (5), we deduce after some calculations that
$\sigma^{2}=\underline{x}^{T} B \underline{x}$
where $x$ is the $(3 \times 1)$ direction cosines vector and $B$ is $(3 \times$ 3) symmetric matrix $\mu_{i j}$.
where
$\mu_{11}=\frac{1}{N} \sum_{i=1}^{N} U_{i}^{2}-(\bar{U})^{2} ; \quad \mu_{12}=\frac{1}{N} \sum_{i=1}^{N} U_{i} V_{i}-\bar{U} \bar{V} ;$
$\mu_{13}=\frac{1}{N} \sum_{i=1}^{N} U_{i} W_{i}-\bar{U} \bar{W} ; \mu_{22}=\frac{1}{N} \sum_{i=1}^{N} V_{i}^{2}-(\bar{V})^{2} ;$
$\mu_{23}=\frac{1}{N} \sum_{i=1}^{N} V_{i} W_{i}-\bar{V} \bar{W} ; \quad \mu_{33}=\frac{1}{N} \sum_{i=1}^{N} W_{i}^{2}-(\bar{W})^{2}$.
$\mu_{i j}$ are the matrix elements. The necessary conditions for an extremum are now
$(B-\lambda I) \underline{x}=0$
This is called the eigenvalue problem for the velocity ellipsoid. There are three homogenous equations in three unknowns having a nontrivial solution if and only if
$D(\lambda)=|B-\lambda I|=0$
Where $\lambda$ is eigenvalue of the above equations, and $x$ and $B$ are given as:
$\underline{x}=\left[\begin{array}{l}l \\ m \\ n\end{array}\right]$
and
$B=\left|\begin{array}{lll}\mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{12} & \mu_{22} & \mu_{23} \\ \mu_{13} & \mu_{23} & \mu_{33}\end{array}\right|$
Equation (9) is the characteristic equation for the matrix $B$. Then the required roots (i.e. eigenvalues)
$\lambda_{1}=2 \rho^{\frac{1}{3}} \cos \frac{\phi}{3}-\frac{k_{1}}{3} ;$
$\lambda_{2}=-\rho^{\frac{1}{3}}\left\{\cos \frac{\phi}{3}+\sqrt{3} \sin \frac{\phi}{3}\right\}-\frac{k_{1}}{3} ;$
$\lambda_{3}=-\rho^{\frac{1}{3}}\left\{\cos \frac{\phi}{3}-\sqrt{3} \sin \frac{\phi}{3}\right\}-\frac{k_{1}}{3}$.
where

$$
\left.\begin{array}{l}
k_{1}=-\left(\mu_{11}+\mu_{22}+\mu_{33}\right) \\
k_{2}=\mu_{11} \mu_{22}+\mu_{11} \mu_{33}+\mu_{22} \mu_{33}-\left(\mu_{12}^{2}+\mu_{13}^{2}+\mu_{23}^{2}\right)  \tag{11}\\
k_{3}=\mu_{12}^{2} \mu_{33}+\mu_{13}^{2} \mu_{22}+\mu_{23}^{2} \mu_{11}-\mu_{11} \mu_{22} \mu_{33}-2 \mu_{12} \mu_{13} \mu_{23}
\end{array}\right\}
$$

$$
\begin{equation*}
q=\frac{1}{3} k_{2}-\frac{1}{9} k_{1}^{2} \quad ; \quad r=\frac{1}{6}\left(k_{1} k_{2}-3 k_{3}\right)-\frac{1}{27} k_{1}^{3} \tag{12}
\end{equation*}
$$

$$
\begin{equation*}
\rho=\sqrt{-q^{3}} \tag{13}
\end{equation*}
$$

$$
\begin{equation*}
x=\rho^{2}-r^{2} \tag{14}
\end{equation*}
$$

and
$\phi=\tan ^{-1}\left(\frac{\sqrt{x}}{r}\right)$

## 3. VELOCITY ELLIPSOID PARAMETERS (VEPS)

Depending on the matrix that controls the eigenvalue problem [Equation (8)] for the velocity ellipsoid, [11] established analytical expressions of some parameters for the correlation studies in terms of the matrix elements $\mu_{i j}$ of the eigenvalue problem for the velocity ellipsoid. The importance of these expressions in terms of matrix elements is due to fact that the parameters become by means of Equations (1), (2), (5) and (6), measurable variables from direct observational quantities. This result saves a great deal of analytical efforts usually needed for the solution of the parameters.

If there exist correlations between the physical and kinematical properties (e.g. $\alpha, \delta, \beta_{\infty} \beta_{\delta}$,..etc.) of a given group of stars, then these parameters which are functions of the matrix elements (of these functions are, for example VEPs) of the eigenvalue problem for the velocity ellipsoid disclose definite trends with the physical properties. In what follows, the parameters will be treated separately.

Table 1. VEPs for F-Type ( $6750 K^{\boldsymbol{o}}$ ) Hyades Stars

| VEPs |  |  |  |
| :---: | :---: | :---: | :---: |
| $(\bar{U}, \bar{V}, \bar{W})$ | $-41.8998703 \pm 2.33119312$ | $-19.4813257 \pm 0.799149427$ | $-0.94087471 \pm 1.68027343$ |
| $\left(\sigma_{1}, \sigma_{2}, \sigma_{3}\right)$ | $45.9042 \pm 0.0391$ | $1.15246 \pm 0.00032$ | $1.649545 \pm 0.000355$ |
| $\left(\lambda_{1}, \lambda_{2}, \lambda_{3}\right)$ | $2107.2 \pm 3.59$ | $1.328165 \pm 0.000735$ | $2.720995 \pm 0.001175$ |
| $\left(l_{1}, l_{2}, l_{3}\right)$ | $0.906921 \pm 0.000026$ | $0.410995 \pm 0.0002565$ | $-0.09607415 \pm 0.00085025$ |
| $\left(m_{1}, m_{2}, m_{3}\right)$ | $0.420775 \pm 0.000084$ | $-0.893312 \pm 0.000062$ | $0.15793 \pm 0.000575$ |
| $\left(n_{1}, n_{2}, n_{3}\right)$ | $0.0210415 \pm 0.000558$ | $0.1836555 \pm 0.0008755$ | $0.9827645 \pm 0.0001755$ |
| $\left(L_{1}, L_{2}, L_{3}\right)$ | $24.8894 \pm 0.005$ | $65.3359 \pm 0.0121$ | $57.6871 \pm 0.1324$ |
| $\left(B_{1}, B_{2}, B_{3}\right)$ | $1.20568 \pm 0.03198$ | $10.58275 \pm 0.05105$ | $79.34715 \pm 0.05445$ |
| $(M, H, E)$ | $7615.225 \pm 5.465$ | $2111.25 \pm 3.59$ | $365.5365 \pm 0.1315$ |

## - The $\boldsymbol{H}$ and $\boldsymbol{M}$ Parameters

The $H$ parameter is defined as the sum of the eigen values $\lambda_{i} ; i=1,2,3$ of the eigen value problem. According to the theory of matrices, the $H$ parameter is then the trace of the matrix, i.e.
$H=\mu_{11}+\mu_{22}+\mu_{33}$
while, $M$ parameter is defined as the product of the eigenvalues $\lambda_{i} ; i=1,2,3$ of the eigenvalue problem to the well known relation between the product of the eigenvalues and the constants term of the characteristic equation, i.e.

$$
\begin{equation*}
M=2 \mu_{12} \mu_{13} \mu_{23}+\mu_{11} \mu_{22} \mu_{33}-\mu_{23}^{2} \mu_{11}-\mu_{13}^{2} \mu_{22}-\mu_{12}^{2} \mu_{33} \tag{17}
\end{equation*}
$$

## - The $\sigma_{i} ; \boldsymbol{i}=1,2,3$ Parameters

The $\sigma_{i} ; i=1,2,3$ parameters are defined as
$\sigma_{i}=\sqrt{\lambda_{i}}$

## - The $\boldsymbol{l}_{\boldsymbol{i}}, \boldsymbol{m}_{\boldsymbol{i}}$ and $\boldsymbol{n}_{\boldsymbol{i}}$ Parameters

The $l_{i}, m_{i}$ and $n_{i}$ are the direction cosines for eigenvalue problem. We then have the following expressions for $l_{i}, m_{i}$ and $n_{i}$ as

$$
\begin{align*}
& l_{i}=\left[\mu_{22} \mu_{33}-\sigma_{i}^{2}\left(\mu_{22}+\mu_{33}-\sigma_{i}^{2}\right)-\mu_{23}^{2}\right] / D_{i} ; i=1,2,3  \tag{19}\\
& m_{i}=\left[\mu_{23} \mu_{13}-\mu_{12} \mu_{33}+\sigma_{i}^{2} \mu_{12}\right] / D_{i} ; i=1,2,3  \tag{20}\\
& n_{i}=\left[\mu_{12} \mu_{23}-\mu_{13} \mu_{22}+\sigma_{i}^{2} \mu_{13}\right] / D_{i} ; i=1,2,3 \tag{21}
\end{align*}
$$

where

$$
\begin{align*}
D_{i}^{2}= & \left(\mu_{22} \mu_{33}-\mu_{23}^{2}\right)^{2}+\left(\mu_{23} \mu_{13}-\mu_{12} \mu_{33}\right)^{2}+\left(\mu_{12} \mu_{23}-\mu_{13} \mu_{22}\right)^{2} \\
& +2\left[\left(\mu_{22}+\mu_{33}\left(\mu_{23}^{23}-\mu_{22} \mu_{33}\right)+\mu_{12}\left(\mu_{23} \mu_{13}-\mu_{12} \mu_{33}\right)+\mu_{13}\left(\mu_{12} \mu_{23}-\mu_{13} \mu_{22}\right)\right] \sigma_{i}^{2}\right.  \tag{22}\\
& +\left(\mu_{33}^{2}+4 \mu_{22} \mu_{33}+\mu_{22}^{2}-2 \mu_{23}^{2}+\mu_{12}^{2}+\mu_{13}^{2}\right) \sigma_{i}^{4}-2\left(\mu_{22}+\mu_{33}\right) \sigma_{i}^{6}+\sigma_{i}^{8} .
\end{align*}
$$

## - The $L_{i}$ and $B_{i}$ Parameters

Let $L_{i}$ and $B_{i} ; i=1,2,3$ be the galactic longitude and the galactic latitude of the directions which correspond to the extreme values of the dispersion, then
$L_{i}=\tan ^{-1}\left(-m_{i} / l_{i}\right) ; i=1,2,3$
$B_{i}=\sin ^{-1}\left(n_{i}\right) ; i=1,2,3$

## - The $E$ Parameter

This represents the volume of the ellipsoid, i.e.
$E=\frac{4}{3} \pi \sigma_{1} \sigma_{2} \sigma_{3}$

## 4. COMPUTATIONAL ALGORITHM

## - Purpose

1. To compute the components of the galactic space velocity, $U, V$ and $W$.
2. To compute mean galactic space velocity, $U, V$ and $W$.
3. To compute the matrix elements, $\mu_{i j}$.
4. To compute the VEPs (i.e. $\sigma_{i}, \lambda_{i}, l_{i}, m_{i}, n_{i}, L_{i}$ and $\left.B_{i}\right) \forall i=1,2,3$.
5. To compute the VEPs, (i.e. $M, H$ and $E$ ).

## - Input Data

For Hyades open clusters, we can compute the components of the galactic space velocities $\left(U_{i}, V_{i}, W_{i}\right) ; i=1,2, \ldots$, $N ;(N=197)$. The stars of this cluster have spectral classes ( $A, F, G, K$ and $M$ ). We ignore stars of spectral types $A$ and $M$, since they are few (20 and 7, respectively) to be statistically treated.

## - Numerical Results

For Hyades open cluster, the VEPs (i.e. $\sigma_{i}, \lambda_{I}, l_{i}, m_{i}, n_{i}$ ...etc.) were computed for spectral types ( $F=64$ stars, $G=47$ stars, and $K=52$ stars). The results are shown in the Tables (1-3). All of these calculations were carried out by constructing a special program with the aid of Mathematica software version 5.1.

Table 2. VEPs for G-Type ( $5500 K^{\boldsymbol{o}}$ ) Hyades Stars

| VEPs |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(\bar{U}, \bar{V}, \bar{W})$ | $-42.397 \pm 1.7723$ | $-19.2734 \pm 1.7372$ | $-1.5082 \pm 1.7981$ |  |  |  |
| $\left(\sigma_{1}, \sigma_{2}, \sigma_{3}\right)$ | $46.12765 \pm 0.04815$ | $0.982334 \pm 2.19 \times 10^{-4}$ | $2.065615 \pm 2.5 \times 10^{-5}$ |  |  |  |
| $\left(\lambda_{1}, \lambda_{2}, \lambda_{3}\right)$ | $2127.76 \pm 4.44$ | $0.9649805 \pm 4.305 \times 10^{-4}$ | $4.26677 \pm 1.1 \times 10^{-4}$ |  |  |  |
| $\left(l_{1}, l_{2}, l_{3}\right)$ | $0.909973 \pm 1.24 \times 10^{-4}$ | $0.311689 \pm 3.16 \times 10^{-4}$ | $-0.2734935 \pm 7.735 \times 10^{-4}$ |  |  |  |
| $\left(m_{1}, m_{2}, m_{3}\right)$ | $0.4133835 \pm 2.095 \times 10^{-4}$ | $-0.733728 \pm 2.4 \times 10^{-5}$ | $0.539219 \pm 1.93 \times 10^{-4}$ |  |  |  |
| $\left(n_{1}, n_{2}, n_{3}\right)$ | $0.03260125 \pm 8.0485 \times 10^{-4}$ | $0.6037325 \pm 1.345 \times 10^{-4}$ | $0.7965195 \pm 1.345 \times 10^{-4}$ |  |  |  |
| $\left(L_{1}, L_{2}, L_{3}\right)$ | $-24.4314 \pm 0.0139$ | $66.9841 \pm 0.0216$ | $63.10575 \pm 0.07365$ |  |  |  |
| $\left(B_{1}, B_{2}, B_{3}\right)$ | $1.868245 \pm 0.046145$ | $37.13765 \pm 9.65 \times 10^{-3}$ | $52.79905 \pm 0.01275$ |  |  |  |
| $(M, H, E)$ | $8760.725 \pm 14.145$ | $2132.99 \pm 4.44$ | $392.0655 \pm 0.3165$ |  |  |  |

Table 3. VEPs for K-Type ( $4250 K^{\boldsymbol{o}}$ ) Hyades Stars

| VEPs |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(\bar{U}, \bar{V}, \bar{W})$ | $-42.887 \pm 1.4156$ | $-19.353 \pm 0.6964$ | $-1.463 \pm 1.151$ |  |  |
| $\left(\sigma_{1}, \sigma_{2}, \sigma_{3}\right)$ | $46.6337 \pm 0.0313$ | $1.012845 \pm 9.5 \times 10^{-5}$ | $1.140725 \pm 3.75 \times 10^{-4}$ |  |  |
| $\left(\lambda_{1}, \lambda_{2}, \lambda_{3}\right)$ | $2174.7095 \pm 2.9105$ | $1.025855 \pm 1.85 \times 10^{-4}$ | $1.302145 \pm 4.5 \times 10^{-5}$ |  |  |
| $\left(l_{1}, l_{2}, l_{3}\right)$ | $0.9112115 \pm 2.25 \times 10^{-5}$ | $0.364682 \pm 1.27 \times 10^{-4}$ | $-0.191573 \pm 1.36 \times 10^{-4}$ |  |  |
| $\left(m_{1}, m_{2}, m_{3}\right)$ | $0.4107375 \pm 1.45 \times 10^{-5}$ | $-0.76885 \pm 3.87 \times 10^{-4}$ | $0.4900645 \pm 5.955 \times 10^{-4}$ |  |  |
| $\left(n_{1}, n_{2}, n_{3}\right)$ | $0.0314265 \pm 4.581 \times 10^{-4}$ | $-0.5252385 \pm 4.785 \times 10^{-4}$ | $-0.850374 \pm 3.12 \times 10^{-4}$ |  |  |
| $\left(L_{1}, L_{2}, L_{3}\right)$ | $-24.26395 \pm 1.25 \times 10^{-3}$ | $64.624 \pm 0.0189$ | $68.6487 \pm 0.0374$ |  |  |
| $\left(B_{1}, B_{2}, B_{3}\right)$ | $1.8009 \pm 0.02626$ | $-31.6843 \pm 0.0322$ | $-58.2524 \pm 0.034$ |  |  |
| $(M, H, E)$ | $2902.99 \pm 2.52$ | $2177.03 \pm 2.92$ | $225.6895 \pm 0.0975$ |  |  |

The Hyades mean velocity i.e. $(\bar{U}, \bar{V}, \bar{W})=$ $(-20,-12,-2) \mathrm{km} \mathrm{s}^{-1}$ was computed by [20]. All of these calculations were carried out relative to the Local Standard of Rest (LSR), adopting the standard solar motion $\left(U_{\text {sun }}, V_{\text {sun }}, W_{\text {sun }}\right)=(10,5.25,7.17) \mathrm{km} \mathrm{s}^{-1}$ from [2]. An accurate estimate of the LSR, i.e. $\left(U_{\text {sun }}, V_{\text {sun }}, W_{\text {sun }}\right)=(7.5 \pm 1.0,13.5 \pm 0.3,6.8 \pm 0.1) \mathrm{km} \mathrm{s}^{-1}$ was computed by. Recently [21], derived two of the solar motion components as $\left(U_{\text {sun }}, W_{\text {sun }}\right)=(10.9 \pm 1.0,7.2 \pm 1.3) \mathrm{km} \mathrm{s}^{-1}$.

## 5. CONCLUSION

Summarizingly, in the present paper we expressed the galactic space velocity components (in closed analytical forms) in terms of the equatorial coordinates. Expressions for the velocity ellipsoid parameters VEPs in closed analytical forms with aid of the vectors and matrices analyses were also represented. We developed a general computational algorithm for the basic parameters from the exact solutions of the equations. Various calculations of the ellipsoid parameters with the effective temperature (spectral type) were presented, which indicate small variation in these parameters. So, we recommend, using more astrometric observations for many
open clusters, which will help our understanding of these variations. This will be done in the near future.

## CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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[^0]:    *Address correspondence to this author at the Astronomy Department, National Research Institute of Astronomy and Geophysics (NRIAG), 11722 Helwan, Cairo, Egypt; Tel: +202 25560046; Fax: +202 25548020;
    E-mail: welsanhoury@gmail.com

