The Acoustic Design of Soundproofing Doors and Windows

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Abstract: A model for manufacturing doors and windows which are capable of natural ventilating, reducing traffic noise and so on for the developing tropical countries is presented. These windows and doors combine two basic components which are ventilation unit and lighting unit. Due to the fact that the ventilation unit must have a large volume to attenuate low frequency noise, many resonance of higher-order mode wave will be generated inside the unit. In this work, a method to predict the insertion loss of rectangular ventilation unit with input and output openings at various positions is proposed by solving the wave equation, considering the resonance frequencies of higher-order mode. The results of the analysis have been confirmed by experiments.

Keywords: Soundproofing, resonance frequency, casement windows, higher-order modes.

1. INTRODUCTION

Sealing up type's doors and windows are widely used in the current houses to intercept an inside outside. Needless to say, some equipments such as air conditioners, are necessary to keep a comfortable indoor temperature while such doors and windows are closed and the using duration of such equipments is often limited because of the power consumption cost and health. The authors have been presented a concept for manufacturing windows which are capable of ventilating, regulating sunlight and reducing traffic noise for the developing tropical countries [1]. These windows combine two basic components which are ventilation unit and lighting unit. The former also serves as an import function in reducing noise.

Due to the fact that the ventilation unit must have a large volume to attenuate low-frequency noise, many resonance of higher-order mode wave will be generated inside the unit [2-5]. Consequently, it is necessary to take into consideration the selection of size and placement of input and output openings in such a way that would minimize the effects of higher-order mode in order to have a great soundproofing effect. In the previous analysis, the ventilation unit is constructed using rectangular cavity with input and output openings at both ends. Actually, an input and output can be located at various positions on the unit according to the door or window's design. Moreover, the optimum location where an insertion loss effect is effectively achieved is still an unknown problem.

In this article, a method to predict the insertion loss of rectangular cavity with input and output openings at various positions is proposed. Following the development of the theory in section 3, this is compared to experimental results in section 4, an excellent agreement is obtained.

2. DESIGN OF SOUNDPROOFING WINDOWS/DOORS AND THE VENTILATION UNIT

Casement windows and doors are consisted of one or several wooden frames that can be opened and closed at various angles and are widely used in countries with tropical climates. An example of casement door's structure is shown in Fig. (1). The windows and doors are opened during day for naturally ventilating the room and closed at night, even when closed room ventilation is still achieved because the windows and doors are constructed with a lots of ventilating slits. However, an annual increase in traffic noise level these countries have rendered these windows and doors to be useless because these ventilating slits serve as a direct pathway for traffic noise to enter the home.

![Fig. (1). The casement doors of current houses in tropical climate countries.](image-url)
Based on the previous reports, our proposed casement’s door is shown in Fig. (2). It combines two basic components: ventilation and lighting. The lighting unit can be constructed using one or two glass layers which are mounted between two ventilation components with input and output openings as shown in Fig. (3). The ventilation unit may consist of square, rectangular or more complicated shapes depending on decorative considerations. Needless to say, the unit requires a simple internal structure and large input and output to maximize ventilation, as well as preventing outside noise from entering the home. Actually, an input and output can be located at various positions on the cavity according to the door design. Therefore, these combinations are considered as follows:

![Fig. (2). The proposed casement doors.](image)

![Fig. (3). Design of the proposed casement windows.](image)

Combinations of input and output positions are shown in Fig. (4) in which (a) and (b) are the cases where they are in opposite faces while, (c) and (d) are in faces which crossed right angles. Difference between (a) and (b) is whether they are located in big or small cavity area, however it is quite similar in the theoretical analysis. Hereafter, we name (a) and (b) as Model-1.

Similarly, difference between (c) and (d) is whether they are located in big or small side area which crossed right angles. In this analysis, once we find the acoustic characteristic when the input and output are located in arbitrary face, it is easily computable for other faces by exchanging coordinates. We name (c) and (d) as Model-2. The sound propagation in (a) is the same as (c) and that in (b) is the same as (d).

![Fig. (4). Combinations of input and output positions on rectangular cavity.](image)

### 3. METHOD OF ANALYSIS

#### 3.1. Insertion Loss

Acoustic characteristics of an acoustic element can be described by the four-pole parameters A, B, C and D as [6]

\[
\begin{bmatrix}
P_1 \\
U_1
\end{bmatrix} = \begin{bmatrix} A & B \\
C & D \end{bmatrix} \begin{bmatrix}
P_2 \\
U_2
\end{bmatrix} = \begin{bmatrix} \cos kl & jZ \sin kl \\
\frac{1}{Z} \sin kl & \cos kl \end{bmatrix} \begin{bmatrix}
P_2 \\
U_2
\end{bmatrix}
\]

(1)

where \( P_1 \) and \( U_1 \) are the sound pressure and velocity at the input, \( P_2 \) and \( U_2 \) are at the output, \( Z \) is the characteristic impedance, \( k \) is the wave number and \( l \) is the length. When this element is connected to the source, their performance can be expressed through the use of insertion loss \( IL \) defined by [6]

\[
IL = 10 \log \left( \frac{W_r}{W_0} \right) = 10 \log \left( \frac{U_1}{U_2} \right)^2
\]

(2)

Here, \( W_r \) and \( W_0 \) are the radiated power at one point in space with or without the acoustic element inserted between that point and the source. The ratio of \( U_1/U_2 \) is equal to the \( D \) parameter of Eq. (1), as far as constant velocity source is concerned.

When the acoustic element is connected in series as shown in Fig. (5), the sectional area of element 1 and 3 becomes significantly small as compared to the sectional area of element 2 and the \( D \) parameter of whole system can be described by the following approximate equation

\[
D = (\cos kl_1) (C_w) (jZ, \sin kl_1)
\]

(3)
where \( C_w \) denoted the C parameter of element 2. As shown in Eq. (2) and (3), in order to obtain a reliable IL effect, the parameter must be high enough. In other words, the design of element-2 to have a high enough parameter \( C_w \) is demanded.

![Fig. (5). Basic structure of ventilation unit.](image)

### 3.2. Computation of \( C_w \)

First of all, we find the sound pressure on the input and output of the Model-1 as shown in Fig. (6a). The input and output have a sectional area of \( S\dot{0} = 2\alpha_y \times 2\beta_y \) and \( S\dot{0} = 2\alpha_x \times 2\beta_x \) where its center is located at point \( I (x_I, y_I, 0) \) and \( O (x_0, y_0, L) \), respectively.

![Fig. (6). Model of computation.](image)

The complete wave-equation in terms of the velocity potential \( \phi \) when expressed in rectangular coordinates is given by [1]

\[
\phi = \left( A e^{\mu z} + B e^{-\mu z} \right) \left( C \sin \alpha x + D \cos \alpha x \right) + 
\left( E \sin \sqrt{s^2 - \alpha^2} y + F \cos \sqrt{s^2 - \alpha^2} y \right)
\]

where \( A, B, C, D, E \) and \( F \) are arbitrary constants determinable from the boundary conditions, \( \alpha, s \) and \( \mu \) are the constants.

Let \( V_x = -\partial \phi / \partial x \), \( V_y = -\partial \phi / \partial y \) and \( V_z = -\partial \phi / \partial z \) be the velocity components in the \( x, y \) and \( z \) directions, respectively. Assuming the walls of the cavity to be perfectly rigid and the loss at the wall can be neglected, the boundary conditions are

\[\begin{align*}
[1] &\text{ at } x = 0, \ V_x = 0 \\
[2] &\text{ at } x = a, \ V_x = 0 \\
[3] &\text{ at } y = 0, \ V_y = 0 \\
[4] &\text{ at } y = b, \ V_y = 0 \\
[5] &\text{ at } z = 0, \ V_z = V_f (x, y) \\
[6] &\text{ at } z = L, \ V_z = 0
\end{align*}\]

where \( V_f \) is the driving velocity at the input, \( F(x, y) \) is the function which has a value of 1 at the input section area and 0 at the other area, namely \( (x_I - \alpha_x \leq x \leq x_I + \alpha_x, y_I - \beta_y \leq y \leq y_I + \beta_y) \).

At first, Eq. (5) gives

\[
\frac{\partial}{\partial x} \left( C \sin \alpha x + D \cos \alpha x \right) \bigg|_{x=0} = 0 \therefore \ C = 0
\]

And from Eq. (6)

\[
\frac{\partial}{\partial x} D \cos \alpha x \bigg|_{x=0} = 0 \therefore \alpha = m \frac{\pi}{a} (m = 0, 1, 2, ...) \quad (12)
\]

Substituting Eqs. (11) and (12) into Eq. (4) gives

\[
\phi = \sum_{m=0}^{\infty} \left( A e^{\mu_{e_{m}}} + B e^{-\mu_{e_{m}}} \right) \left( E \sin \sqrt{s^2 - \alpha^2} y + F \cos \sqrt{s^2 - \alpha^2} y \right)
\]

from Eq. (7)

\[
\frac{\partial}{\partial y} \left( E \sin \sqrt{s^2 - \alpha^2} y + F \cos \sqrt{s^2 - \alpha^2} y \right) \bigg|_{y=0} = 0 \therefore \ E = 0
\]

and from Eq. (8)

\[
\frac{\partial}{\partial y} \left( F \cos \sqrt{s^2 - \alpha^2} y \right) \bigg|_{y=0} = 0 \therefore \sqrt{s^2 - \alpha^2} = n \frac{\pi}{b} (n = 0, 1, 2, ...)
\]

Substituting Eqs. (14) and (15) into Eq. (13), so \( \phi \) becomes

\[
\phi = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left( A e^{\mu_{e_{m,n}}} + B e^{-\mu_{e_{m,n}}} \right) \cos \left( \frac{m\pi x}{a} \right) \cos \left( \frac{n\pi y}{b} \right)
\]

where \( \mu_{e_{m,n}} = \sqrt{\left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 - k^2} \) (17)

Next, from Eq. (10)

\[
\frac{\partial}{\partial c} \left( A e^{\mu_{e_{m,n}}} + B e^{-\mu_{e_{m,n}}} \right) \bigg|_{z=L} = 0 \therefore \ B = A e^{2\mu_{e_{m,n}}}
\]

Substituting \( B \) into Eq. (16) yields

\[
\phi = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} e^{2\mu_{e_{m,n}}} \cosh \left( \mu_{e_{m,n}} (z-L) \right) \cos \left( \frac{m\pi x}{a} \right) \cos \left( \frac{n\pi y}{b} \right)
\]

from Eq. (9)
\[- \frac{\partial}{\partial z} \sum_{m,n=0}^{\infty} e^{\omega_{m,n} z} \cosh(\mu_{m,n}(z - L)) C_{m,n} \cos \left( \frac{m \pi x}{a} \right) \cos \left( \frac{n \pi y}{b} \right) \]
\[
\cos \left( \frac{n \pi y}{b} \right) \bigg|_{z=0} = V_i F_i(x, y)
\]

By multiplying both sides of Eq. (20) by \(\cos(m \pi x / a)\cos(n \pi y / b)\) and integrating with respect to \(x\) from 0 to \(a\) and with respect to \(y\) from 0 to \(b\), \(C_{m,n}\) can be determined.

Substitution of \(C_{m,n}\) to (19) yields
\[
\phi = \sum_{m,n=0}^{\infty} \cosh(\mu_{m,n}(z - L)) \mu_{m,n} \sinh(\mu_{m,n} L) U_i D_{m,n} \cos \left( \frac{m \pi x}{a} \right) \cos \left( \frac{n \pi y}{b} \right)
\]

where
\[
D_{m,n} = \frac{16}{m n \pi S_i} \sum_{\substack{\sigma \in \Sigma \\sigma = 0}} \cosh(\mu_{m,n}(z - L)) \mu_{m,n} \sinh(\mu_{m,n} L) U_i
\]

Moreover, expanding Eq. (28) corresponding to the variation of \(m\) and \(n\), we have
\[
\bar{P}_0 = j Z_n U_1 \left\{ - \frac{1}{\sin(kL)} + \sum_{\sigma = 1}^{\infty} \frac{\bar{Q}_{m,0}}{\mu_{m,0} \sinh(\mu_{m,0} L)} + \sum_{\sigma = 0}^{\infty} \frac{\bar{Q}_{m,0}}{\mu_{m,0} \sinh(\mu_{m,0} L)} \right\}
\]

where
\[
\bar{Q}_{m,0} = \frac{S_n S_i}{S_0} \sum_{\sigma = 0}^{\infty} \frac{\bar{Q}_{m,0}}{\mu_{m,0} \sinh(\mu_{m,0} L)}
\]

and the average sound pressure on the output becomes
\[
\bar{P}_0 = \frac{1}{S_0} \int_{y_0}^{y_0 + \beta_0} \int_{y_0}^{y_0 + \beta_0} P(x, y, L) dx dy
\]

\[
= j k Z_n U_0 \sum_{m,n=0}^{\infty} \frac{\bar{Q}_{m,n}}{\mu_{m,n} \sinh(\mu_{m,n} L)}
\]

The \(C_{m,n}\) can be obtain by \(C_{m,n} = U_i / \bar{P}_0\) under the condition of the output velocity of the volume, where \(U_0 = V_i S_i\) is equal to zero. Note that, this condition has already been considered in Eq. (10).

In the case of Model-2 as shown in Fig. (6b), the input and output have a sectional area of \(S_i = 2 \alpha_i \times 2 \beta_i\) and \(S_o = 2 \alpha_o \times 2 \beta_o\) where its center is located at point \(I (x_i, y_i, 0)\) and \(O (x_o, b, z_o)\), respectively.

After the similar computation procedure is performed from Eq. (4) to Eq. (23), the sound pressure on the side where the output located can be derived as
\[
P_0 = P(x, y, z) = j k Z_n U_0 \sum_{m,n=0}^{\infty} \frac{\cosh(\mu_{m,n}(z - L))}{\mu_{m,n} \sinh(\mu_{m,n} L)} D_{m,n} \cos \left( \frac{m \pi x}{a} \right) \cos \left( \frac{n \pi y}{b} \right)
\]
Therefore, the average output sound pressure becomes

$$P_0 = \frac{1}{S_0} \int_{z_0-a_0}^{z_0+a_0} \int_{\varphi_0}^{\varphi_0+b} p(x, b, z) \, dz \, dy$$

$$= jk Z_n U_j \left\{ \frac{-\overline{R}_{0,0}}{k \sin(kL)} + \sum_{m=1}^{\infty} \frac{\overline{R}_{m,0}}{\mu_{m,0} \sinh(\mu_{m,0} L)} \right\}$$

$$+ \sum_{n=1}^{\infty} \frac{\overline{R}_{m,n}}{\mu_{m,n} \sinh(\mu_{m,n} L)}$$

(38)

where

$$\overline{R}_{0,0} = -\frac{1}{S_0} \int_{z_0-a_0}^{z_0+a_0} \int_{\varphi_0}^{\varphi_0+b} \cos(k(z - L)) \, dz \, dx$$

$$= -\frac{1}{S_0} \frac{2 \alpha_{\varphi}}{k} \left\{ \sin(k(\beta_0 + L - z_0)) + \sin(k(\beta_0 - L + z_0)) \right\}$$

(39)

Finally, $C_w$ can be derived as

$$C_w = U_j \frac{P_0}{P_0}$$

(40)

4. RESULT AND DISCUSSION

$C_w$ is defined by Eq. (40) including the average output sound pressure $\overline{P}_0$ at its denominator, where $\overline{P}_0$ is given in Eq. (30) and Eq. (38) for Model-1 and Model-2, respectively. Note that both models have the same average input sound pressure $\overline{P}$ defined by Eq. (25). In order to obtain an IL effectively $C_w$ must be at least, in other words, low level of $\overline{P}_0$ is preferable. Referring to Eq. (30), $\overline{P}_0$ becomes great when its denominator $\sin(kL)$ and $\mu_{m,n} \sinh(\mu_{m,n} L)$ are zero, namely, at the following resonance frequencies of

$$\sin(kL) = 0 \quad \therefore \quad f_0 = \eta \frac{c}{2L} \quad (\eta = 1, 2, 3, \ldots)$$

$$\mu_{m,n} \sinh(\mu_{m,n} L) = 0 \quad \therefore \quad f_{m,n} = \frac{c}{2\eta} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{\eta\pi}{L}\right)^2}$$

(42)

The $f_0$ represents the resonance frequencies of the plane wave and $f_{m,n}$ represents those of traverse wave. The first resonance frequencies of $f_{m,n}$ when $\eta=0$ occur sequentially, are shown in Table 1, when the dimensions of the cavity are $a=0.48m$, $b=0.075m$ and $L=0.29m$, respectively. Generally, at the frequency range where the traverse waves are generated, the sound pressure $\overline{P}_0$ will increase and when the resonance frequencies of other modes co-occur, $C_w$ will be small and the IL can not be expected to be as great. Generation mechanism of these frequencies can be understood according to the calculation example as shown in Fig. (7a) with (2,0) and (4,0) modes. Dimensions of the cavity used in calculation are the same as in Table 1.

Table 1. The First Resonance Frequency of Some (m,n) Mode Occur in the Cavity when $a=0.48m$, $b=0.075m$ and $L=0.29m$

<table>
<thead>
<tr>
<th>Mode</th>
<th>1st Resonance Frequency (Hz)</th>
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<tbody>
<tr>
<td>m</td>
<td>n</td>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
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<td>4</td>
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<td>8</td>
<td>0</td>
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</table>

Sound pressure level of those wave components in dB are shown in Fig. (7b). They also have many resonance frequencies which occur corresponding to the increasing of $\eta$ in Eq. (42). Therefore, it is clear that when we eliminate an arbitrary higher-order wave mode by any method, we will not only avoid many resonances generated by this mode but also obtain the low level of the entire output sound pressure.
The theoretical results of Eqs. (30) and (31) show that the resonance frequencies appear in the cavity when the denominators become zero. Therefore, the resonance frequencies will differ from those of the measured results shown in Fig. 8b. In order to show the difference among the resonance frequencies, the measured result at position E, which had been described above, was added to Fig. 8a.

Moreover, expanding the above equation with mode (0,0) we have

\[
P_A = jkZ_uS_w \left[ \frac{1}{2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{\mu_{m,n} \sinh(\mu_{m,n}L/2)} D_{m,n} \cos \left( \frac{m\pi}{2} \right) \cos(n\pi) \right]
\]

As a result, the denominators of \( P_A \) become \( \sin(kL/2) \) and \( \sinh(\mu_{m,n}L/2) \), therefore, the resonance frequencies will differ from those of Fig. 8b. Calculation examples of mode(0,0) and mode(4,0) are shown in Fig. 8b. The frequencies at which \( \sin(kL/2) \) and \( \sinh(\mu_{m,n}L/2) \) become zero correspond well to those of the measured results shown in Fig. 8b. In order to show the difference among the resonance frequencies, the measured result at position E, which had been described above, was added to Fig. 8a.

Fig. (10) shows the measured result at point C (a, b/2, L/2) that locates in the right side of the cavity. Because the resonance frequencies are the same as point A as described above, the effectiveness of our calculation method has been proven.
Resonance frequencies appear in the cavity when the output is located at position A and E. (a) computed result of the denominator of Eq. (44) (b) measured result of $C_w$.

Fig. (10). Measured results at position A and C.

Fig. (11) shows the theoretical and measured results of Model-1 at point A ($a/2$, $b$, $L/2$) which locates on the top of the cavity when the input located on the opposite face. In this case, the first resonance frequencies of $f_{m,n}$ that occur sequentially are shown in Table 2. Because the input was located on the big cavity area, resonance frequencies of higher-order mode generally appear in low frequency range.

Table 2. The First Resonance Frequency of Some (m,n) Mode Occur in the Cavity when $a=0.48m$, $b=0.29m$ and $L=0.075m$

<table>
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5. CONCLUSIONS

The characteristic of sound propagation in the rectangular ventilation having an input and output located at various positions, has been presented by solving the wave equation, considering the higher-order mode effects. Based on the obtained results, the cause and mechanism of resonance frequencies of parameter C are discussed in detail. To prove the theory, experiments with various positions were carried out and excellent agreement is obtained. The formulas derived from the present study enable the account of the insertion loss of the ventilation in practical applications. Also, it will be suitable for further study on the determination of optimized positions of input and output. This technology will be presented in an upcoming report.

APPENDIX

A block diagram of the whole experimental apparatus to measure the parameter $C_w$ is shown in Fig. (12) [7]. Two microphones were located at both sides of the ventilation unit to measure the sound pressures $P_A$ and $P_B$. Relationship between $P_A$ and $P_B$ is given by

$$
\begin{bmatrix}
P_A \\
U_A
\end{bmatrix} = \begin{bmatrix}
\cos kl_0 & jZ_0 \sin kl_0 \\
j\frac{1}{Z_0} \sin kl_0 & \cos kl_0
\end{bmatrix} \begin{bmatrix}
A_w \\
B_w \\
C_w \\
D_w
\end{bmatrix} \begin{bmatrix}
P_B \\
U_B
\end{bmatrix}
$$

(A-1)

where the first term and the second term represent the four-pole parameters of the input pipe which has length $l_0$ and the ventilation unit, respectively. Symbols $U_A$, $U_B$ and $Z_0$ represent velocity of the volume and acoustic impedance of the input pipe. By installing the microphone 2 on the output piston, $U_B$ will become zero, therefore Eq. (A-1) can be written as

$$P_A = (A_w \cos kl_0 + C_w Z_0 \sin kl_0) P_B$$

(A-2)

Furthermore, since the first term can be disregarded as compared to the second term, Eq. (A-2) can be written in terms of dB as follows

$$20 \log_{10} |C_w| = 20 \log_{10} \left| \frac{P_A}{P_B} \right| - 20 \log_{10} |Z_0 \sin kl_0|$$

(A-3)

This means $C_w$ can be obtained by subtracting the acoustic characteristic of the input pipe $Z_0 \sin kl_0$ from the measured sound pressures $P_A$ and $P_B$.

![Fig. (12). Block diagram of the experimental apparatus.](image)

REFERENCES