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Asymptotic Models for the Topological Sensitivity Versus the Topological Derivative

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Abstract: An approach, based on the refined method of matched asymptotic expansions, is proposed for the construction of asymptotic models for the topological sensitivity of the energy functional with respect to the creation of a small hole in the geometrical domain. It is shown that the asymptotic model provides more information for calculations than the topological derivative.

INTRODUCTION

The shape optimization theory [1] provides well established techniques for the investigation of shape optimization problems when the topology class of the geometrical domain under consideration is supposed to be fixed. At the same time, the shape optimization methods cannot produce useful criteria whether a topological change (for instance, the creation of a hole in the interior of the geometrical domain) will lead to a decreasing value of the shape functional or not. One such criterion [2] is based on the notion of the topological derivative whose importance in the topology optimization is now widely recognized.

The present paper is devoted to analyzing the application of the topological derivative in shape and topology optimization problems which take into consideration the question of changing the topology class of the geometrical domain. More precisely, the so-called asymptotic model based on the refined asymptotic expansion is presented for the topological sensitivity of the Dirichlet integral in a special case of nucleation of a hole with the homogeneous Neumann boundary condition imposed on its boundary.

It is known that the aim of the topological sensitivity analysis (see, for example, [3,4]) is to obtain the so-called topological asymptotic expansion of a given shape functional $J(\Omega, v)$ with respect to the creation of a small opening $\omega_\varepsilon(x^0)$ of diameter $O(\varepsilon)$ with the center at a given point x^0 in the geometrical domain Ω ($0<\varepsilon$ is a small parameter).

Let $T_\omega^\varepsilon(x^0)$ be the ratio between the difference $J(\Omega_\varepsilon, u^\varepsilon) - J(\Omega, v^0)$, where u^ε is the solution of the boundary value problem defined on the singularly perturbed domain $\Omega_\varepsilon = \Omega \setminus \overline{\omega_\varepsilon(x^0)}$, and the area $\left|\omega_\varepsilon(x^0)\right|$ of the small opening $\omega_\varepsilon(x^0)$.

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We shall say that $T_\omega^\varepsilon(x^0)$ is the topological sensitivity of the shape functional $J(\Omega, v)$ with respect to the internal topological variation (i.e., creation of the small hole $\omega_\varepsilon(x^0)$ in the domain Ω). The idea of topological sensitivity (the term has been used in a number of publications [3], etc.) was introduced in [2] (the so-called characteristic function) in the framework of the bubble method for topology and shape optimization in two-dimensional elastostatic problems.

In the case of the homogeneous Neumann boundary condition imposed on the boundary $\partial\omega_\varepsilon(x^0)$ of the hole $\omega_\varepsilon(x^0)$, the topological asymptotics for the energy functional can be obtained in the form [5,3]

$$J(\Omega_\varepsilon, u^\varepsilon) = J(\Omega, v^0) + T_\omega^0(x^0)\left|\omega_\varepsilon(x^0)\right| + o(\left|\omega_\varepsilon(x^0)\right|).$$

Here, $T_\omega^0(x^0)$ is the topological derivative which determines whether a change of topology by nucleation of a small hole $\omega_\varepsilon(x^0)$ at the point x^0 in the interior of the domain Ω would result in improving the value of the shape functional $J(\Omega, v)$ or not.

The aim of the asymptotic modelling is to obtain a refined asymptotic representation for the topological sensitivity, i.e., for the increment of a given shape functional resulting from the emerging of a small opening in the interior of the domain. As a result of application of the refined method of matched asymptotic expansions [6,7], we obtain the following asymptotic formula:

$$T_\omega^\varepsilon(x^0) = S_\omega^\varepsilon(x^0) + o(\left|\omega_\varepsilon(x^0)\right|), \quad \varepsilon \to 0,$$

where $S_\omega^\varepsilon(x^0)$ is an asymptotic model for the topological sensitivity.

We stress that the so-called asymptotic model for the topological sensitivity $S_\omega^\varepsilon(x^0)$, which may be regarded as a Padé approximant [8], provides more information for calculations and they have more accuracy (see, e.g., [9,7,10]).

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TOPOLOGICAL DERIVATIVE

Let Ω and ω be two domains on R^2 with compact closures $\overline{\Omega}$ and $\overline{\omega}$ and smooth boundaries $\partial\Omega$ and $\partial\omega$. We assume that ω contains the origin O. By ε we denote a small positive parameter. For small ε it is possible for any fixed point $x^0 \in \Omega$ to remove the set $\omega_\varepsilon(x^0) = \{x = (x_1, x_2) \,|\, \varepsilon^{-1}(x - x^0) \in \omega\}$ from Ω, obtaining the singularly perturbed domain Ω_ε with the boundary $\partial\Omega_\varepsilon = \partial\Omega \cup \partial\omega_\varepsilon(x^0)$. In such a domain we consider the following mixed boundary value problem for the Poisson equation:

$$-\Delta u^\varepsilon(x) = f(x), x \in \Omega_\varepsilon; \quad u^\varepsilon(x) = 0, x \in \partial\Omega; \tag{1}$$

$$\partial_n u^\varepsilon(x) = 0, x \in \partial\omega_\varepsilon(x^0). \tag{2}$$

Here, ∂_n stands for the derivative in the direction of the outward (with respect to Ω_ε) normal vector n.

In this paper the asymptotic behavior of the solution $u^\varepsilon(x)$ is considered and the leading terms of asymptotic expansions are constructed. The inclusion $f \in C^1(\overline{\Omega})$ is sufficient for our purposes.

As $\varepsilon \to 0$, the hole $\omega_\varepsilon(x^0)$ is collapsed to the point x^0, the boundary condition (2) disappears, and relations (1) form the limit problem

$$-\Delta v^0(x) = f(x), x \in \Omega; \quad v^0(x) = 0, x \in \partial\Omega; \tag{3}$$

We consider the case of the energy functional

$$J(\Omega, v^0) = \frac{1}{2}\int_\Omega \left|\nabla_x v^0(x)\right|^2 dx. \tag{4}$$

The topological derivative $T_\omega^0(x^0)$ of the functional $J(\Omega, v^0)$ at the point $x^0 \in \Omega$ is defined by [5,4]

$$T_\omega^0(x^0) = \lim_{\varepsilon \to 0+} \frac{J(\Omega_\varepsilon, u^\varepsilon) - J(\Omega, v^0)}{\left|\omega_\varepsilon(x^0)\right|}. \tag{5}$$

By definition, we put

$$T_\omega^\varepsilon(x^0) = \frac{J(\Omega_\varepsilon, u^\varepsilon) - J(\Omega, v^0)}{\left|\omega_\varepsilon(x^0)\right|}, \tag{6}$$

where $T_\omega^\varepsilon(x^0)$ is the topological sensitivity.

The topological derivative (5) in the case (2) under the condition of a hole $\omega_\varepsilon(x^0)$ being a ball in R^2 was introduced in [5] with a reference to the original idea by Schumacher (1995), suggested for the special case of the energy functional in linear elasticity in the framework of the bubble method [2]. The topological derivative concept was latter generalized for nucleation of cavities of arbitrary shape [3,11] as well, as for the cases of different types of boundary conditions imposed on $\partial\omega_\varepsilon(x^0)$ and different state differential equations defined on $\Omega_\varepsilon \in R^n$ ($n = 1,2$) [4,12]. Also, different approaches were suggested for calculation of the topological derivative [3,13,4]. Let us emphasize [14] that the topological derivatives can be obtained in a different way, but the forms of such derivatives are equivalent.

The methods of topology optimization based on the bubble method are used for the topology optimization in structural mechanics [2,15]. Numerical results obtained by help of the topological derivative can be found in [3,12]. We refer to [16,17] for applications in inverse problems. The topological derivative was incorporated [18] into the level set method [19]. Note that asymptotic models can be connected, in particular, with the primal-dual active set method for crack problems with non-penetration [20] as a criteria for the kinking of a crack. Note also that asymptotic models for dilute and densely packed composites [21] and asymptotic solutions for periodic problems obtained in [22] could be used in the homogenization method for shape and topology optimization [23].

In [14] the case of a finite number of circular holes was treated by means of the so-called topological gradient which contains the topological derivatives evaluated at the centers of holes. In [24] two new approaches were proposed for the modelling of so-called internal multiple topological variations. The first approach is developed in the framework of the self adjoint extensions of differential operators, the second is based on the variational formulation with point asymptotic conditions in a functional space with separated asymptotics. In the present paper the third approach is proposed for the asymptotic modeling of the internal topological variations. This approach is based on the refined method of matched asymptotic expansions [6,7]. Namely, the refined method of matched expansions in the form [7] is applied to construction of asymptotic models for the topological sensitivity of the energy functional.

To this end, Sections 1 and 2 briefly recall the notion of topological derivative, evaluation of the topological derivative by help of the method of matched asymptotic expansions and its refined modification [7]. The performed asymptotic analysis is formal. Estimates for the proposed approximations in weighted Hölder spaces [4] and weighted Sobolev spaces [25] were derived in the context of shape optimization.

The asymptotics of the solution to the singularly perturbed problem (1), (2) is known (see, [26]). We use the method of matched asymptotic expansions [27,28]. This method implies the following structure for the inner asymptotic expansion:

$$u^\varepsilon(x) = w^0(\xi) + \varepsilon w^1(\xi) + \ldots, \tag{7}$$

which is valid in a small neighborhood of $\omega_\varepsilon(x^0)$. In (7), we introduced the so-called stretched variables

$$\xi = (\xi_1, \xi_2), \quad \xi_i = \varepsilon^{-1}(x_i - x_i^0). \tag{8}$$

Changing to the stretched coordinates (8) in (1) and (2), after passage to the limit $\varepsilon = 0$, we conclude that $w^q(\xi)$ ($q = 0,1$) must satisfy the following relations:

$$\Delta_\xi w^q(\xi) = 0, \xi \in R^2 \setminus \overline{\omega}; \quad \partial_\nu w^q(\xi) = 0, \xi \in \partial\omega. \tag{9}$$

The Dirichlet boundary condition (1) is replaced with an asymptotic condition restricting the behavior of $w^q(\xi)$ as $\xi \to \infty$. That asymptotic condition results from matching with the solution $v^0(x)$ of the limit problem (3). The matching procedure [27,28] implies that the leading terms in the asymptotic expansions for $v^0(x)$ and $w^0(\xi) + \varepsilon w^1(\xi)$ formally coincide as $x \to 0$ and $\xi \to \infty$, respectively.

Consequently, in view of the Taylor formula

$$v^0(x) = v^0(x^0) + (x - x^0)\nabla v^0(x^0) + O(|x - x^0|^2), \tag{10}$$

we add to the second limit problem (9), $q = 1$, the asymptotic condition $w^1(\xi) = \xi\nabla v^0(x^0) + O(|\xi|^{-1})$ as $\xi \to \infty$. Hence, $w^1(\xi)$ can be represented as

$$w^1(\xi) = \sum_{l=1,2} \frac{\partial v^0}{\partial x_l}(x^0) Y^l(\xi). \tag{11}$$

Here, $Y^L(\xi) = \xi_L + Y_0^L(\xi)$ ($l = 1,2$) are the special solutions to the exterior Neumann problem (9) that admit the following asymptotic expansions:

$$Y_0^l(\xi) = \frac{1}{2\pi|\xi|^2} \sum_{k=1,2} m_{kl}(\omega)\xi_k + O(|\xi|^{-2}). \tag{12}$$

The outer asymptotic expansion for the solution $u^\varepsilon(x)$ to our problem (1), (2) has the form

$$u^\varepsilon(x) = v^0(x) + \varepsilon^2 v^1(x) + \dots. \tag{13}$$

In view of (11) and (12), the function $v^1(x)$ must satisfy the following asymptotic condition ($x \to x^0$):

$$v^1(x) = \sum_{l,k=1}^{2} m_{kl}(\omega) \frac{\partial v^0}{\partial x_l}(x^0) \frac{x_k - x_k^0}{2\pi\left|x - x^0\right|^2} + O(1). \tag{14}$$

We denote by $G^{(k)}(x^0, x)$ ($k = 1,2$) singular solutions to the homogeneous problem (1) that admit the following representation:

$$G^{(k)}(x^0, x) = \frac{x_k - x_k^0}{2\pi\left|x - x^0\right|^2} + g^{(k)}(x^0, x). \tag{15}$$

In accordance with (14), we obtain

$$v^1(x) = \sum_{l,k=1}^{2} m_{kl}(\omega) \frac{\partial v^0}{\partial x_l}(x^0) G^{(k)}(x^0, x), \tag{16}$$

where $m(\omega)$ is the matrix consisting of the coefficients from the asymptotic formula (12).

Note that the obtained asimptotics (13), (16) can be derived by the method of compound asymptotic expansions [26] or the asymptotic method [29] based on layer potential techniques (see, formula (4.16)).

An asymptotic approximation of the functional $J(\Omega_\varepsilon, u^\varepsilon)$ can be derived from formulas (4) and (11), (16). However, using Green's formula, we obtain

$$J(\Omega_\varepsilon, u^\varepsilon) = \frac{1}{2} \int_{\Omega_\varepsilon} f(x) u^\varepsilon(x)\, dx. \tag{17}$$

Here, the homogeneous Dirichlet boundary condition (1) on $\partial\Omega$ and the Neumann boundary condition (2) on $\partial\omega_\varepsilon(x^0)$ were taken into account.

The following two-term asymptotic expansion of the energy functional is valid (see, e.g., [26,31,32]):

$$\begin{aligned} J(\Omega_\varepsilon, u^\varepsilon) = J(\Omega, v^0) - \frac{\varepsilon^2}{2} v^0(x^0) f^0(x^0) |\omega| \\ + \frac{\varepsilon^2}{2} \nabla v^0(x^0)^{\mathrm{T}} m(\omega) \nabla v^0(x^0) + o(\varepsilon^{2+\sigma}), \end{aligned} \tag{18}$$

where $\sigma > 0$ is small.

Hence, since $|\omega_\varepsilon(x^0)| = \varepsilon^2 |\omega|$, the following value of the topological derivative is obtained:

$$\begin{aligned} T_\omega(x^0) = -\frac{1}{2} v^0(x^0) f^0(x^0) \\ + \frac{1}{2|\omega|} \nabla v^0(x^0)^{\mathrm{T}} m(\omega) \nabla v^0(x^0). \end{aligned} \tag{19}$$

Here, $m(\omega)$ is a symmetric positive definite 2×2 matrix associated with the virtual mass tensor [30]. Following [4,22], we call $m(\omega)$ the polarization matrix.

ASYMPTOTIC MODEL

We use the refined method of matched asymptotic expansions in the form [7]. In view of (13) and (16) we take the sum

$$V^\varepsilon(x) = v^0(x) + \varepsilon^2 \sum_{k=1}^{2} C_k G^{(k)}(x^0, x) \tag{20}$$

as the outer asymptotic representation of the solution $u^\varepsilon(x)$ to the singularly perturbed problem (1), (2).

By formulas (10) and (15) we have

$$\begin{aligned} V^\varepsilon(x) = v^0(x^0) + \varepsilon^2 \sum_{k=1,2} C_k g_0^{(k)} + \Delta x \nabla v_0^0 \\ + \varepsilon^2 \sum_{k=1}^{2} C_k \left\{ \frac{\Delta x_k}{2\pi\left|\Delta x\right|^2} + \Delta x \nabla g_0^{(k)} \right\} + O(\left|\Delta x\right|^2); \end{aligned} \tag{21}$$

$\nabla g_0^{(k)} = \nabla g^{(k)}(x^0, x^0)$, $\nabla v_0^0 = \nabla v^0(x^0)$, $\Delta x = x - x^0$.

Substituting the stretched coordinates (8) in (21), we obtain

$$V^{\varepsilon}(x) = v^0(x^0) + \varepsilon^2 \sum_{k=1,2} C_k g_0^{(k)} + \varepsilon\xi\nabla v_0^0 + \varepsilon^2 \sum_{k=1}^{2} C_k \left\{ \frac{\xi_k}{\varepsilon 2\pi|\xi|^2} + \varepsilon\xi\nabla g_0^{(k)} \right\} + O(\varepsilon^2|\xi|^2). \tag{22}$$

Following [7], we derive from (22) the refined matching asymptotic condition

$$W^{\varepsilon}(\xi) = v^0(x^0) + \varepsilon^2 \sum_{k=1,2} C_k g_0^{(k)} + \varepsilon\xi\left(\nabla v_0^0 + \varepsilon^2 \sum_{k=1}^{2} C_k \nabla g_0^{(k)} \right) + O(|\xi|^{-1}). \tag{23}$$

Hence, the inner asymptotic representation, which satisfies the second limit problem (9), (23), has the form $W^{\varepsilon}(\xi) = w^0(\xi) + \varepsilon w^1(\xi)$, where

$$w^1(\xi) = \sum_{l=1}^{2} \left(\frac{\partial v^0}{\partial x_l}(x^0) + \varepsilon^2 \sum_{k=1}^{2} C_k \frac{\partial g^{(k)}}{\partial x_l}(x^0) \right) Y^l(\xi). \tag{24}$$

Introducing the column of solutions $Y = (Y^1, Y^2)^{\mathrm{T}}$, where T denotes the transpose, and the symmetric 2×2 matrix $\nabla g_0^{(\bullet)}$ with the elements $\partial_{x_l} g^{(k)}(x^0, x^0)$ ($k, l = 1,2$), we rewrite (24) in the form

$$w^1(\xi) = \left(\nabla v_0^0 + \varepsilon^2 \nabla g_0^{(\bullet)} C \right)^{\mathrm{T}} Y(\xi), \tag{25}$$

where $C = (C_1, C_2)^{\mathrm{T}}$.

Making use of the relations (12) we obtain

$$w^1(\xi) \cong \left(\xi + \frac{1}{2\pi|\xi|^2}\xi m(\omega) \right)\left(\nabla v_0^0 + \varepsilon^2 \nabla g_0^{(\bullet)} C \right), \tag{26}$$

On the other hand, from (22) we derive

$$V^{\varepsilon}(x) - w^0 \cong \varepsilon\xi\left(\nabla v_0^0 + \varepsilon^2 \nabla g_0^{(\bullet)} C \right) + \frac{\varepsilon}{2\pi|\xi|^2}\xi C. \tag{27}$$

Comparing (26) and (27), we obtain the equation

$$C = m(\omega)\left(\nabla v_0^0 + \varepsilon^2 \nabla g_0^{(\bullet)} C \right). \tag{28}$$

From (28) we readily find

$$C = \left(I - \varepsilon^2 m(\omega) \nabla g_0^{(\bullet)} \right)^{-1} m(\omega) \nabla v_0^0, \tag{29}$$

where $I = \mathrm{diag}\{1,1\}$.

Note that, first introduced in [6], the specific matrix notation used in [4] for asymptotic analysis of various shape functionals also provides asymptotic representations essentially similar to (25) and (20), (29) as it has been comprehensively shown in [33]. The refined method of matched asymptotic expansions [6,7] was applied for construction of asymptotic models in contact mechanics [33,34], in the theory of cracks [35], and in the theory of acoustic diffraction [36].

In order to evaluate an asymptotic approximation of the energy functional (4), we construct the uniformly suitable asymptotic representation

$$U^{\varepsilon}(x) = v^0(x) + \varepsilon^2 \sum_{k=1,2} C_k G^{(k)}(x^0, x) + \varepsilon \sum_{l=1,2} \left\{ \frac{\partial v^0}{\partial x_l}(x^0) + \varepsilon^2 \sum_{k=1}^{2} C_k \frac{\partial g^{(k)}}{\partial x_l}(x^0) \right\} \times \left(Y_0^l\left(\frac{x - x^0}{\varepsilon} \right) - \varepsilon \sum_{k=1,2} m_{kl}(\omega) \frac{x_l - x_l^0}{2\pi|x - x^0|^2} \right), \tag{30}$$

where the coefficients C_k are defined by (29).

Thus, replacing the function $u^{\varepsilon}(x)$ with its asymptotics (30) in the integral (17), we obtain

$$J(\Omega_{\varepsilon}, u^{\varepsilon}) = J(\Omega, v^0) + S_{\omega}^{\varepsilon}(x^0)\left| \omega_{\varepsilon}(x^0) \right| + o(\varepsilon^{2+\sigma}), \tag{31}$$

where $\sigma > 0$ is small.

Taking into account (29) and (31), we find

$$S_{\omega}^{\varepsilon}(x^0) = -\frac{1}{2} v^0(x^0) f(x^0) + \frac{1}{2|\omega|}\left(\nabla v_0^0 \right)^{\mathrm{T}} \left(I - \varepsilon^2 m(\omega) \nabla g_0^{(\bullet)} \right)^{-1} m(\omega) \nabla v_0^0. \tag{32}$$

Note that, since $\left(I - \varepsilon^2 m(\omega) \nabla g_0^{(\bullet)} \right)^{-1} = I + O(\varepsilon^2)$, the asymptotic expansion (18) follows from (31). The proof uses the same tools as for the case (18). Note also that $S_{\omega}^{\varepsilon}(x^0) = T_{\omega}^0(x^0) + O(\varepsilon^2)$ as $\varepsilon \to 0$.

We emphasize that the formulas (18) and (31) are known to possess the same asymptotic accuracy, since both inner asymptotic representations (7) and (25) do not eliminate the discrepancy in the boundary condition (2) on $\partial\omega_{\varepsilon}(x^0)$ left by the term $O(|x - x^0|^2)$ from the expansion (10). However, the refined asymptotic representation (31) gives in fact better results because its construction is obtained by summation of asymptotic terms corresponding to the term $O(|x - x^0|)$.

The following asymptotic formula for the energy increment due to appearing of the cavity $\omega_{\varepsilon}(x^0)$ in the domain Ω is obtained (see, formula (31)):

$$\frac{J(\Omega_{\varepsilon}, u^{\varepsilon}) - J(\Omega, v^0)}{\left| \omega_{\varepsilon}(x^0) \right|} \cong S_{\omega}^{\varepsilon}(x^0), \quad \varepsilon \to 0. \tag{33}$$

Thus, we have $T_{\omega}^{\varepsilon}(x^0) \cong S_{\omega}^{\varepsilon}(x^0)$ as $\varepsilon \to 0$ (see, (6)), where $T_{\omega}^{\varepsilon}(x^0)$ is the topological sensitivity and $S_{\omega}^{\varepsilon}(x^0)$ is its asymptotic model of the first order.

CONCLUSIONS

Let us point out that the difference between the asymptotic model (32) for the topological sensitivity $T_{\omega}^{\varepsilon}(x^0)$ and the topological derivative $T_\omega^0(x^0)$ is substantial. First, the asymptotic model (33), (32) provides more information, since it takes into account the influence of a hole $\omega_\varepsilon(x^0)$ on the solution $u^\varepsilon(x)$ of the perturbed problem (1), (2), whereas the topological derivative (5), (19) depends only on the solution $v^0(x)$ of the non-perturbed problem (3). This additional information is contained in the matrix $\nabla g_0^{(\bullet)}$ whose components are defined by the geometry of the domain Ω and depend on the location of the point x^0. In other words, unlike the topological derivative, the asymptotic model for the topological sensitivity $S_\omega^\varepsilon(x^0)$ provides essential information of the response caused by the creation of a cavity $\omega_\varepsilon(x^0)$ in the geometrical domain Ω.

Second, the asymptotic model (32) for the topological sensitivity $S_\omega^\varepsilon(x^0)$ defined by (33) may be regarded as a Padé approximant (see, e.g., [8]). A relation between the Padé approximation and the refined method of matched asymptotic expansions was established in [7]. Namely, this relation explains a surprising increasing of accuracy of asymptotic representations such as (32) for $\varepsilon \in (0, \varepsilon_0)$ [7,10].

Third, the asymptotic model based topological sensitivity is more sensitive tool in obtaining the optimal topology in the problem under consideration. In fact, using the asymptotic model (33), (32), it is possible to consider differences between points with the same value of the topological derivative. On the other hand, the topological derivative is a less sensitive tool because its value is obtained evaluating the limit $T_\omega^0(x^0) = \lim_{\varepsilon\to 0+} S_\omega^\varepsilon(x^0)$. We stress that the topological derivative gives a possibility for insertion of an infinitesimally small hole, whereas the bubble method [2,15] requires insertion of a hole of small but finite size.

Fourth, one of the main advantages of the asymptotic model for the topological sensitivity is that it provides a more accurate tool for identification of cracks and cavities in the inverse problems (see, e.g., [16,17]). At the same time, the asymptotic model based topological sensitivity $S_\omega^\varepsilon(x^0)$ defined by (32) is computed using only information of the non-damaged domain Ω.

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