Global Attractor of a Predator-Prey System with Impulsive Effect and Control

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Abstract: In this paper, the author discusses the Global attractor of solution for the boundary value problem of the dynamic system with impulsive case. Based on the prey-Predator with impulsive Effect and HollingIII functional response is proposed and analyzed. By using the Floquet theory of impulsive equation and comparison theorem, sufficient conditions for the system to extinct and some permanence are given. Finally, the numerical simulation was introduced to support these excellent extensions of results. The authors extend dynamic behavior and the critical value to continuum more previous work [3, 7, 8, 9 and 10].

Keywords: Biological control, attractive, predator-Prey system, impulsive effect.

1. INTRODUCTION

The problem of Attractor in dynamic system is not only to examine foundation of solution in the corresponding system, and it is also to study well-pseudo-plastic with stability, branch, period, extinction and blow-up of solutions, and chaotic phenomenon premise, it is to examine the solution as branch, period, extinction and blow-up of solutions, and dependent digestion model, consider a more general situation.

2. MODEL INTRODUCED

We have a sigmoid functional response in food-dependent digestion model, consider a more general situation

\[ \begin{align*}
    x'(t) &= \alpha x(t)(r - ax(t) - by(t) - cz(t) - dx(t) - \eta(t)) \\
    y'(t) &= \eta(t)(\lambda_1 b x^k(t) / (1 + \lambda_1 b x^k(t)) - d_1) \\
    z'(t) &= \eta(t)(\lambda_2 b x^k(t) / (1 + \lambda_2 b x^k(t)) - d_2) \\
    \omega'(t) &= \rho(t)(\lambda_3 b x^k(t) / (1 + \lambda_3 b x^k(t)) - d_3) \\
    \eta'(t) &= \eta(t)(\lambda_4 b x^k(t) / (1 + \lambda_4 b x^k(t)) - d_4)
\end{align*} \quad (2.1) \]

The models of (2.1) are assumption as follow:

1. No Predator, the prey is logistic growth.
2. Four kinds of predator and a prey with relation in direct proportion to that $-b_1 x(t)y(t), -b_2 x(t)z(t), -b_3 x(t)\alpha(t), -b_4 x(t)\eta(t)$.
3. All four types of prey in the absence, resulting in the form of mortality index, which is $-d_1 y(t), -d_2 z(t), -d_3 \alpha(t), -d_4 \eta(t)$.
4. Four types of predator on prey growth rate is $\lambda_1 b x^k(t) y(t) / (1 + h_1 b x^k(t)) - \lambda_2 b x^k(t) \alpha(t) / (1 + h_2 b x^k(t))$, $\lambda_3 b x^k(t) \eta(t) / (1 + h_3 b x^k(t))$, $\lambda_4 b x^k(t) \eta(t) / (1 + h_4 b x^k(t))$, which $b_1, b_2, b_3, b_4$ for predator search rate of prey, predator $h_1, h_2, h_3, h_4$ are four kinds of prey, respectively, digestion time, $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ is the predator, respectively, the four digestion-rate of prey species.

In fact, after a simple calculation shows that the model (2.1) to the classical Lotka-Volterra model has the same line, such as damping.

We assume that:

\[ \min \left\{ \lambda_1 b_1 - d_1 b_1, \lambda_2 b_2 - d_2 b_2, \lambda_3 b_3 - d_3 b_3, \lambda_4 b_4 - d_4 b_4 \right\} > \frac{\alpha}{r} \max\{d_1, d_2, d_3, d_4\} \]

then the system (2.1) there is a balance point...
Global Attractor of a Predator-Prey System

The introduction of periodic pulse injected to kill pests and natural enemies to control pest populations to achieve balance. To this end digestion of food-dependent model (2.1) to improve the time for the pulse to give with impulsive differential equation

\[ \begin{align*}
\dot{x}(t) &= x(t)(r - ax^2(t) - by(t) - cz(t) - d\omega(t) - e\eta(t)) \\
\dot{y}(t) &= y(t)(\lambda h x^3(t)/[1 + h b x^3(t)] - d) \\
\dot{z}(t) &= z(t)(\lambda h x^3(t)/[1 + h b x^3(t)]) \\
\dot{\omega}(t) &= \omega(t)(\lambda h x^3(t)/[1 + h b x^3(t)]) - d) \\
\dot{\eta}(t) &= \eta(t)(\lambda h x^3(t)/[1 + h b x^3(t)]) - d)
\end{align*} \]

(2.2)

where

\[ \begin{align*}
\Delta x(t) &= -p e^{\theta(t)} \\
\Delta y(t) &= -p y(t) + \tau_i \\
\Delta z(t) &= -p z(t) + \tau_i \\
\Delta \omega(t) &= -p \omega(t) + \tau_i \\
\Delta \eta(t) &= -p \eta(t) + \tau_i
\end{align*} \]

Lemma 2.2 Set \( \sigma(t) \) is solution of the initial value of the system (2.2) with \( \sigma(0^+) \geq 0 \), then if for arbitrary \( t \geq 0 \), we have \( \sigma(t) \geq 0 \). Thus we obtain \( \sigma(t) > 0 \), for the \( \sigma(0^+) > 0 \), here \( \sigma(t) = (x(t), y(t), z(t), \omega(t), \eta(t)) \).

Proof: easy to have

\[ \begin{align*}
x(t) &= x(0^+)[1 - \theta(1 + R\theta)] e^{\alpha(t)} \\
\mathcal{X}(\sigma) &= \exp \left\{ \int_{0^+}^{t} (r - ax^2(s) - by(s) - cz(s) - d\omega(s) - e\eta(s))ds \right\} \\
\mathcal{Y}(\sigma) &= \exp \left\{ \int_{0^+}^{t} \lambda h x^3(s)/[1 + h b x^3(s)] - d, ds \right\}
\end{align*} \]

which the \( m \) is in the interval \([0, t]\) of the pulse frequency, \( t \in [0, +\infty) \) is arbitrary, in the same way, for \( z(t), \omega(t), \eta(t) \), by above same calculating method that we obtain similar result.

Definition 2.3 If \((x(t), y(t), z(t), \omega(t), \eta(t))\) is solution of system (2.2) satisfy \( x(0^+), y(0^+), z(0^+), \omega(0^+) \) and \( \eta(0^+) > 0 \) when there is constants \( M \geq m > 0 \) such that \( m \leq x(t) \leq M, m \leq y(t) \leq M, m \leq \eta(t) \leq M \), (2.3)

which \( m \leq x(t) \leq M, m \leq y(t) \leq M, m \leq \eta(t) \leq M \), (2.3)

then the system (2.2) is called uniformly persistence.

We consider the following system of the nature of extinction,

\[ \begin{align*}
d\sigma / dt &= -p\sigma(t), t \neq nT, \\
\Delta \sigma(t) &= -p\sigma(t) + \tau, t = nT.
\end{align*} \]

Lemma 2.3 System (2.4) there is a global asymptotic stability of positive periodic solution:

\[ \begin{align*}
\sigma(t) &= \sigma e^{x(t)(-t)} / \{1 - (1 - P) e^{-pt}\}, t \in [nT, (n + 1)T], n \in N^*
\end{align*} \]

where, the initial value of \( \sigma(0^+) = \sigma / \{1 - (1 - P) e^{-pt}\} \).

Form lemma 2.3, we may get below lemma.

Lemma 2.4 System (2.2) there is a predator eradication periodic solution, that is

\[ \begin{align*}
\{0, y^*(t), z^*(t), \omega^*(t), \eta^*(t)\}
\end{align*} \]

Definition 2.1 Set \( V \subseteq V_k \), Then

\( (t, \sigma) \in (nT, (n + 1)T) \times R^k \), on systems (2.2) is defined as the upper right derivative

\[ D^+ V(t, \sigma) = \limsup_{h \to 0^+} \left\{ V(t + h, \sigma + h\sigma) - V(t, \sigma) \right\} / h^* \]
We use the literature [2, Theorem 3.1.1] the methods and results are given the following lemma

**Lemma 2.5** [3] (comparison theorem) Let $V \in V_u$ to satisfy the following inequality

\[
\begin{align*}
D^+ V(t, \sigma) &\leq h(t, V(t, \sigma)), \quad t \neq nT, \\
V(t, \sigma(t)) &\leq \psi_o(t, V(t, \sigma)), \quad t = nT,
\end{align*}
\]

Which $h : R^+ \rightarrow R$ satisfy in the Theorem 3.1.1 [2]. Assumption that $h : R \times R^+ \rightarrow R$, in $(nT,(n+1)T] \times R^+$ continuous, and the $\sigma \in R^+$, $n \in N$, there exists $h(t,u) \rightarrow h(nT^*, \sigma)$, as

\[(t,u) \rightarrow (nT^*, \sigma).\]

Here $\psi_o : R_+ \rightarrow R_+$ is non-decrease function. Let $r(t)$ is the maximal solution on $[0, +\infty)$ in the following scalar impulsive differential equation.

\[
\begin{align*}
u' &= h(t, u(t)), \quad t \neq nT, \\
u(t^+) &= \psi_o(u(t)), \quad t = nT, \\
u(0^+) &= u_0.
\end{align*}
\]

Then $V(0^+, \sigma) \leq u_0$. So $V(t, \sigma(t)) \leq r(t), t \geq 0$. $\sigma(t)$ is the solution of (2.2).

We give the fundamental nature of subsystems of the (2.2)

\[
\begin{align*}
y' &= -d_1 y(t), t \neq nT, \\
y(t^+) &= (1 - p_2) y(t) + \tau_1, \\
y(0^+) &= y_0, \\
z' &= -d_2 z(t), t \neq nT, \\
z(t^+) &= (1 - p_2) z(t) + \tau_2, \\
z(0^+) &= z_0. \\
\end{align*}
\]

(2.7) of (2.2) has a positive periodic solution $y^*(t), z^*(t), \omega^*(t), \eta^*(t)$, and the any other solution

\[
y(t), z(t), \omega(t), \eta(t)\]

of the subsystems (2.7), we have

\[
y(t) - y^*(t) \rightarrow 0, z(t) - z^*(t) \rightarrow 0, \omega(t) - \omega^*(t) \rightarrow 0, \eta(t) - \eta^*(t) \rightarrow 0
\]
as when $t \rightarrow \infty$, where

\[
y'(t) = \tau_1 e^{-d_1(t-nT)} / [1 - (1 - p_2)e^{-d_1T}], t \in (nT,(n+1)T], n \in N, \\
y'(0^+) = \tau_1 / [1 - (1 - p_2)e^{-d_1T}], \\
z'(t) = \tau_2 e^{-d_2(t-nT)} / [1 - (1 - p_2)e^{-d_2T}], t \in (nT,(n+1)T], n \in N, \\
z'(0^+) = \tau_2 / [1 - (1 - p_2)e^{-d_2T}], \\
\]

\[\eta'(t) = \tau_1 e^{-d_1(t-nT)} / [1 - (1 - p_2)e^{-d_1T}], t \in (nT,(n+1)T], n \in N, \\
\eta'(0^+) = \tau_1 / [1 - (1 - p_2)e^{-d_1T}].
\]

**3. MAIN RESULTS**

We will give control of the conditions of Prey $x(t)$ extinction.

**Theorem 3.1** The system set $\{x(t), y(t), z(t), \omega(t), \eta(t)\}$ (2.2) of any one solution, if

\[T < r^{-1} \ln \left(1/(1 - \theta p_1)\right) + r^{-1} \sum_{i=1}^{N} (b_i \tau_i / r - (1 - P_i)) \Delta = T_{\text{max}}\]

established, Periodic Solutions of the predator extinction $(0, y^*(t), z^*(t), \omega^*(t), \eta^*(t))$ is globally asymptotically stable.

**Proof**: First, we consider the small perturbation solution of periodic solutions identify the local stability. Our definition of $x(t) = u_1(t), y(t) = y^*(t) + u_2(t), \cdots, \eta(t) = \eta^*(t) + u_3(t)$, then we have

\[
\begin{pmatrix}
u_1(t) \\
u_2(t) \\
u_3(t)
\end{pmatrix} = \Phi(t)
\begin{pmatrix}
u_1(0) \\
u_2(0) \\
u_3(0)
\end{pmatrix}, 0 \leq t < T',
\]

where \(\Phi\) to satisfy the following equation

\[
\frac{d\Phi(t)}{dt} = \begin{pmatrix} r - by^*(t) - cz^*(t) - d\omega^*(t) - e\eta^*(t) & 0 & L & 0 \\
0 & -d_1 & 0 & M \\
0 & 0 & 0 & -d_2 \\
b_1 \eta^*(t) & 0 & 0 & -d_3 \\
\end{pmatrix} \Phi(t)
\]

$\Phi(0) = I$ is a unit matrix. From the above equation can be easily find

\[
\Phi(t) = \begin{pmatrix}\exp\left(\int_0^t (r - by(s) - cz(s) - d\omega(s) - e\eta(s)) ds\right) & 0 & L & 0 \\
\Delta & e^{-d_1t} & L & 0 \\
M & \Delta & 0 & 0 \\
\Delta & \Delta & \Delta & e^{-d_3t}
\end{pmatrix}
\]

This brings me to sign "\(\Delta\)" items to determine the form of the following does not affect our analysis, not calculated.
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By for $t = nT$ case of system (2.2) reduce into the linear equation as

$$
\begin{align*}
\begin{pmatrix}
    u_1(nT^+) \\
    u_2(nT^+) \\
    \vdots \\
    u_m(nT^+)^T
\end{pmatrix}
&=egin{pmatrix}
    1-\theta(1+R\theta) p_1 & 0 & \cdots & 0 \\
    0 & 1-p_2 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & 1-p_m
\end{pmatrix}
\begin{pmatrix}
    u_1(nT) \\
    u_2(nT) \\
    \vdots \\
    u_m(nT)
\end{pmatrix}
\end{align*}
$$

We are $T$ periodic pulse from the linear differential equations by Floquet theory [1] that $(0, y^*(t), z^*(t), \omega^*(t), \eta^*(t))$ is the stability of periodic solutions given by the following single-valued matrix $M$ defined by eigenvalues.

Therefore, if the matrix

$$
M = \begin{pmatrix}
    1-\theta(1+R\theta) p_1 & 0 & \cdots & 0 \\
    0 & 1-p_2 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & 1-p_m
\end{pmatrix}
$$

Where in above matrix that the diagonal elements; $\mu_i = m_i, (i = 1, 2, 3, 5)$. If the five eigenvalues of modulus less than 1, then the cycle is a partial solution $(0, y^*(t), z^*(t), \omega^*(t), \eta^*(t))$ stable. In fact, the five Floquet multipliers are

$$
\mu_i = (1-\theta(1+R\theta) p_1) \exp \int_0^t \begin{pmatrix}
    r-b y(s)-c z(s)-d \omega(s)-e \eta(s)ds
\end{pmatrix},
$$

and $\mu_5 = (1-p_5)e^{-d_5 t}, j = 2, 3, 4, 5$, according to Floquet theory, if $|\mu_i| < 1, i = 1, 2, 3, 4, 5$, that is,

$$
T < r^{-1}\ln \left(1/\{1-\theta p_1\}\right) + r^{-1} \sum_{i=1}^5 \left(b_i\tau_i/d_i(1-P_{i+1})\right)
$$

System (2.2) Periodic Solutions $(0, y^*(t), z^*(t), \omega^*(t), \eta^*(t))$ is a local stable.

Next, proof the $(0, y^*(t), z^*(t), \omega^*(t), \eta^*(t))$ is global attractor.

We consider the following impulsive differential equation

$$
\begin{align*}
\begin{cases}
    y' = -dv(t), t \neq nT, \\
    \Delta y(t) = y(t^+) - y(t) = -pv(t) + \tau, t = nT,
\end{cases}
\end{align*}
$$

By Lemma 2.2 and Lemma 2.3, to be $y(t), z(t), \omega(t), \eta(t) \geq u(t)$, and $v(t) \to y^*(t)$, as $t \to \infty$.

So $y^*(t) - v(t) \leq y(t)$, then for $z(t), \omega(t), \eta(t)$ by above same method for all $t \geq 0$ that similar conclusion.

From the system (2.2) available

$$
\begin{align*}
\begin{cases}
    t \leq T \leq (t + nT) \Rightarrow \{r-b y(t) - c z(t) - d \omega(t) - e \eta(t)\}, \\
    x(t') = (1-\theta p_1)x(t), t = nT.
\end{cases}
\end{align*}
$$

Use Lemma 2.3, easy access to

$$
x(nT) \leq x(nT)\exp\int_0^{nT} \left[r-b y(t) - c z(t) - d \omega(t) - e \eta(t)\right]dt
$$

As a result of $\exp\int_0^{nT} \left[r-b y(t) - c z(t) - d \omega(t) - e \eta(t)\right]dt \to 0$. Because $0 < x(t) \leq x(nT)(1-\theta p_1)e^{\tau T}$

for $nT < t \leq (n+1)T$. So $x(t) \to 0$ when $n \to \infty$.

**Theorem 3.2** Assume that constant $M > 0$, such that for any a solution of system (2.1) with $\sigma(t) = (x(t), y(t), z(t), \omega(t), \eta(t))$, as time $t$ enough large, that

$$
\max \{x(t), y(t), z(t), \omega(t), \eta(t)\} \leq M
$$

**Proof:** Let $\sigma(t) = (x(t), y(t), z(t), \omega(t), \eta(t))$ for with any a solution of system (2.1).

Assume

$$
V(t) = \bar{\lambda} x(t)/2\sqrt{\bar{h_1} b_1} + y(t) + z(t) + \omega(t) + \eta(t),
$$

then $V(t) \in V_0 = V(t_0)$ and imply

$$
D^+ V(t) + CV(t)
$$

$$
= \bar{\lambda} x(t)r - ax \omega(t) - by(t) - cz(t) - d \omega(t) - e \eta(t)
$$

$$
+ c \left(\bar{\lambda} x(t)/2\sqrt{\bar{h_1} b_1} + y(t) + z(t) + \omega(t) + \eta(t)\right)
$$

$$
\leq \bar{\lambda} (r+c)x(t)/2\sqrt{\bar{h_1} b_1} - \bar{\lambda} ax \omega(t)/2\sqrt{\bar{h_1} b_1} - \bar{\lambda} bx y(t)/2\sqrt{\bar{h_1} b_1} - \bar{\lambda} cx z(t)/2\sqrt{\bar{h_1} b_1}
$$

$$
- \bar{\lambda} dx(t)\omega(t)/2\sqrt{\bar{h_1} b_1} - \bar{\lambda} cx \omega(t)/2\sqrt{\bar{h_1} b_1} - (d-c) y(t)-(d-c) z(t)
$$

$$
- (d-c) \omega(t)-(d-c) \eta(t), t \neq nT,
$$

$$
V(nT) = \bar{\lambda} x(nT)/2\sqrt{\bar{h_1} b_1} + y(nT) + z(nT) + \omega(nT) + \eta(nT) + \tau
$$

Clearly, when $0 < c < \min \{d, d_2, d_3, d_4\}$, the first inequality (4.4) is bounded, and choose these constants $c_0, m_0, \tau = (\tau_1, \tau_2, \tau_3, \tau_4)$. So, the (4.4) reduces

$$
\{DV(t) \leq -c_0 V(t) + M_0, t \neq nT,
$$

$$
V(nT^+) \leq V(nT) + \tau, t = nT.
$$
By lemma (3.4) in [3], we get
\[ V(t) \leq V(0) - M \sigma / \alpha \left( 1 - e^{-\sigma T} \right) + r \left( 1 - e^{-\sigma T} \right) e^{-\sigma (T-t)} + M \sigma / \alpha, \]
where \( t \in \left( \frac{nT}{n+1} \right) \). Thus, \( V(t) \) is essentially bounded, and there is constant \( M > 0 \), such that any a solution of (2.1) with \( \sigma(t) = (x(t), y(t), z(t), \alpha(t), \eta(t)) \), if \( t \) enough large, it is also hold \( \max \{ x(t), y(t), z(t), \alpha(t), \eta(t) \} \leq M \).

**Remark:** The further we can along with the way in [3], the system (2.2) can be the persistence by
\[ rT - \sum_{i=1}^{d} \left( a_i \tau_i / \sigma_i \right) > \ln \left( 1 - \theta p_i \right)^{-1}. \]

### 4. Numerical Computation

The Numerical simulations have been carried out to substantiate our analytical findings and investigate the global dynamical behavior of the nonlinear system (2.1). In the previous sections, the qualitative analyses of system are presented in Predator-Prey systems. Now, to see dynamical behavior of the system by Fig 1(a), (b), (c), and Fig. 2 (a), (b), (c).

For convenience, only consider three species with impulsive case for us as the following set of parameters: \( \theta = 0, r = 3 \), \( k = 1, a = 0.2, b_1 = 0.75, b_2 = 0.9, c_1 = 0.9, c_2 = 0.85, \)
\( d_1 = 0.2, d_2 = 0.18, h_1 = 0.8, h_2 = 0.9 \). Assume that initial value \( \sigma(t) = (2, 0.1, 0.1) \), by condition of above theorem 3.1 and theorem 3.2, and computing that critical value
\( T_{\text{max}} = 4.000 \). When impulsive periodic \( T < T_{\text{max}} = 4.000 \) for action that the prey can be eradicated (see Fig. 1 (a) (b) (c)).

If it is with impulsive periodic \( T > T_{\text{max}} = 4.000 \), that occur complex dynamic action of the periodic coexistence, and strange attractor. etc. (see Fig. 2 (a) (b) (c).

![Fig. 1(a). Figure description case for pest species with parameters r=3, k=1, a=0.2, b1=0.75, b2=0.9, c1=0.9, c2=0.85, d1=0.2, d2=0.18, h1=0.8, h2=0.9. When the impulsive periodic T=3.5, the prey species x(t) with decrease case for changing time.](image1a)

![Fig. 1(b). Figure description implies change of nature enemies species y(t) with increase for changing time.](image1b)

![Fig. 1(c). Figure description the change of nature enemies species z(t) with decrease for changing time.](image1c)

![Fig. 2A. The system (2.2) has a chaos attractor, parameters with r=3, k=1, a=0.2, b1=0.75, b2=0.9, c1=0.9, c2=0.85, d1=0.2, d2=0.18, h1=0.8, h2=0.9. When the impulsive periodic T=5.5, the prey species x(t) with decrease case for changing time.](image2a)

![Fig. 2b. Implies change of nature enemies species y(t) along time.](image2b)
CONCLUSION

This article mainly studies the general dynamics behaviour of Burger-equation of blasting and extinguishing phenomenon, to discuss these cases through certain parameters values. Such equations always exhibit a rich phenomenology attracts many attention in engineering mechanics, material mechanics and fluid mechanics with application value. The authors extend dynamic behaviour and the critical value to continuum more previous work [3, 8, 10] for in-depth achieve for apply value.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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