Detection of Weak Audio Signal in Ocean Based on Stochastic Resonance

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Abstract: There are plenty of audio information in ocean, whose audio feature often brings disturbance on the detection of ocean object, and some new methods are required to realize the object classification on audio frequency. Stochastic resonance (SR) can realize the detection of weak signal. However the most researches concentrate on single weak periodic signal, and less research on aperiodic audio signal. Starting from the bistable SR system, this paper adopts the combination method of theory analysis and numerical simulation, and conducts a theoretical detection on the detection of weak aperiodic audio signal under strong noise background. By using time scale transformation method, a complete detection system for weak digital signal is established, and it is applied in underwater audio signal. The analysis and simulation results show that the audio signal is successfully recovered in the condition that the signal-to-noise ratio (SNR) is -13dB. The research results possess a vast engineering significance on the further development and application of SR in the detection of weak audio signal.

Keywords: Weak audio signal detection, Stochastic resonance (SR), Time scale transformation, Aperiodicity.

1. INTRODUCTION

The weak signal detection is very important for target recognition in ocean environment. The traditional signal detection methods usually adopt linear filtering, wavelet analysis [1], etc., in order to suppress and eliminate noise, and obtain the useful information finally. However, those methods always lose effect in the condition that the strong noise background exists or the useful signal is closed to the noise frequency [2]. The theory of stochastic resonance (SR) was proposed by the physicist Benzi \textit{et al} in Italy [3], in order to explain the periodicity of earth glacier. Briefly speaking, when the weak signal submerging in strong noise background passes through a nonlinear system, and there appears a certain matching relationship among nonlinear system, signal and noise, the noise energy will occur a phenomenon of transferring to weak signal, so as to achieve the aim of suppressing noise and amplifying signal. There are plenty of SR theories, but each of them has its own defects [4]. Currently, the most studies on SR phenomenon mainly concentrate on bistable SR system, which belongs to nonlinear system and possesses noise with typical nonlinear effect. Moreover, this type of system has wide application and superior research results in the natural science areas, including physics [5], chemistry [6], etc., and the social science areas.

Due to the limitation of adiabatic approximation, the SR in engineering is suitable for the periodic signal in lower frequency. However, there is no better solution method for the detection of aperiodic signal in high frequency [7], which limits the further application of SR in practice. In audio signal, the sampling frequency can reach several kilohertz, which is far beyond the application scope of SR. Aiming at the higher frequency and aperiodicity of audio signal in practical engineering, this paper employs the bistable SR system to construct a complete detection system. Furthermore, the system feasibility is validated by means of numerical simulation and data processing.

2. SR THEORY AND ITS MODEL

2.1. The Principle of Bistable SR

Regarding the bistable system as research subject, it can be demonstrated as a stochastic differential equation, \textit{i.e.}, the nonlinear Langevin equation[8]:

\[
\frac{dx}{dt} = ax + \mu x^3 + s(t) + \xi(t)
\]  

(1)

where, \(a\) and \(\mu\) are system parameters; \(s(t)\) is input signal; \(\xi(t)\) is input noise, which is generally assumed as a Gaussian white noise with zero mean value and autocorrelation function \(\langle \xi(t)\xi(0) \rangle = 2D_\delta(t)\). where \(\langle \cdot \rangle\) represents the deduction of expectation operator and \(\delta(t)\) represents impulse function; \(D\) represents noise intensity, \textit{i.e.}, unilateral power spectrum density. From the angle of mechanics, \(s(t)\) can be considered as a overdamped particle trajectory passing in the bistable potential well-modulated by signal, whose potential function is \(U(x) = -ax^2 / 2 + \mu x^4 / 4 - s(t)x\) and the barrier height is \(\Delta U = a^2 / (4\mu).\) Due to the stochastic effect of noise on the particle and the modulation of weak signal on potential well, the system output will achieve a

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cooperative state under the certain optimal noise intensity [9].

2.2. Model Analysis and Improvement

In a shorter time, the signal $s(t)$ can be considered as a constant $h$. Subsequently, based on $y = x / \sqrt{D}$, $\bar{y} = uD$ and $\bar{h} = h / \sqrt{D}$, Eq. (1) can be further transformed, and a new bistable system model is obtained as follows:

$$\frac{dy}{dt} = ay - \bar{y}v^3 + \bar{h} + \Gamma(t)$$

(2)

Here, the system response speed is introduced to explain the generation mechanism of SR. System response speed reflects the changing ability of system along with signal, and its reciprocal is the system characteristic time, i.e., the primary time scale of system output probability density tending to asymptotic solution. Then, its equation is given as follows:

$$\lambda = \frac{\int_{-\infty}^{\infty} \left(\frac{1}{2}g(y) + \frac{1}{4}g^2(y)\right) u^2(y) + u^2(y)dy}{\int_{-\infty}^{\infty} u^2(y)dy}$$

(3)

Note that, the system response speed proposed here is quite different from the traditional linear response speed. The variational method, used in the calculation of system response speed, can convert Eq. (3) into $(K - \lambda M)d = 0$, where $K$ is a semi-positive definite symmetric matrix and $M$ is a positive definite symmetric matrix. Then, we calculate the generalized eigenvalues, which are all greater than or equal to 0. As a result, the minimum value must be 0 and we select the second small positive as the system response speed. Fig. (1) shows the relationship between the system response speed and parameter $u'$.

3. THE DETECTION OF WEAK AUDIO SIGNAL

In audio detection system, the SR system is regarded as a signal receiver[10], which is used for processing the transmission of binary baseband signal. We transmit some binary data, which is assumed to be alternated between +1 and -1. The variance of input noise is 16. Then, we observe the output of SR system and adjust the system parameter $u$, see Fig. (2).

![Fig. (2). Digital Sequence and Noise-Added Sequence.](image)

As seen from Fig. (3), we can get that the output waveforms of system are not the same when system parameters $u$ are different. This phenomenon can be explained by means of system response speed. Concretely speaking, when parameter $u$ is smaller, the system response speed is also

![Fig. (3). The SR Output with Adjusting System Parameter $u$.](image)
The Relationship Between System Parameter $u$ and BER.

Fig. (4). The Relationship Between System Parameter $u$ and BER. smaller, which results in that the system cannot keep up with the changes of signal and then the signal output is disordered. In contrary, when parameter $u$ is overlarge, the system response speed increases so greatly that it can not only follow the changes of signal, but also follow the changes of noise. At this time, the output signal is also disordered. When SR is applied in the detection of periodic signal, the output signal-to-noise ratio (SNR) is usually used as an index for measuring the resonance degree. However, with regard to the non-periodic signal, being continued to use the SNR is not suitable obviously. As a result, we will use bit error rate (BER) as the index to measure the resonance degree of non-periodic signals. In the transmission of digital signal, there must exist a range of the optimal system parameter $u$. Among this range, we can use BER as the performance index to generate the stochastic resonance. Assume that the sampling frequency is $f_s = 1kHz$, and the noise variance is 30. Then, we gradually increase the value of system parameter $u$. As seen from Fig. (4), the BER decreases in first and then increases, which is a kind of stochastic resonance phenomenon obviously.

4. THE APPLICATION IN THE DETECTION OF UNDERWATER AUDIO SIGNAL

In the detection of audio signal, because the code rate of data is 64kps, the system response speed must be greater than 64K. According to the response speed formula shown in former section, we can get that the system parameter $u$ is very large at this time. However, the overlarge $u$ can result in that the mean value of system output is very weak [11], meanwhile the BER of system is also greatly enhanced. Therefore, we use a time scale transformation method in order that the lower BER is maintained in the condition of

Fig. (5). The Block Diagram of Weak Audio Signal Detection System.

Fig. (6). The Block Diagram of SR Simulation System.
un-reduced code rate. The system diagram is shown in Fig. (5), where the BER

![Signal Diagram](image)

**Fig. (7).** The Detection and Recovery of Audio Signal.

detection module is used to adjust system parameter of bistable SR system according to the current BER.

Furthermore, the SR simulation system is shown in Fig. (6).

In audio signal, the sampling rate is 8K and the code rate is 64kps. After the expansion by using time scale, the code rate decreases to 1kps equally. The variance of added noise is 16, and the input SNR is -13db. According to the traditional treatment method, in such a low SNR condition, we will not obtain any useful information, which has been submerged by noise. In contrary, if the SR treatment method is used, we will get the clear audio signal. At this time, the BER of system is $2.61 \times 10^{-10}$, which fully meets the requirement of actual detection. From Fig. (7), we can see that the digital audio signal treated by using SR does not need timing, and the system structure and realization algorithm are very simple, which has important significance for the design of low-power wireless communication devices.

**CONCLUSIONS**

This paper studies the application of SR in the detection of weak audio signal, improves the bistable system, and analyzes the influence of system response speed on the detection of weak audio signal. Moreover, it further analyzes the SR phenomenon of digital signal, and gives theoretical explanations on the related phenomenon. Finally, combined with the time scale transformation method, the bistable stochastic resonance system is successfully used in the detection of audio signal. In the condition that the input SNR is -13dB, the audio signal is recovered and the BER is low, which possesses very important significance in engineering application.

**CONFLICT OF INTEREST**

The authors confirm that this article contains no conflict of interest.

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