84

# **Establishment of Mathematical Model and Sensitivity Analysis of Plugging Capacity of Multi-Component Foam Flooding**

Chengli Zhang<sup>1,\*</sup>, Dezhi Liang<sup>1</sup>, Haiqing Cui<sup>1</sup>, Daiyin Yin<sup>1</sup> and Guoliang Song<sup>2</sup>

<sup>1</sup>Key Laboratory of Enhanced Oil and Gas Recovery of Ministry of Education, Northeast Petroleum University<sup>1</sup>, Daqing, 163318, China

<sup>2</sup>College of Mathematics and Statistics, Northeast Petroleum University<sup>2</sup>, Daqing, 163318, China

**Abstract:** Based on mass conservation, foam total amount balance thought and foam properties characterization model, a foam flooding mathematical model with three-phase and five components is established and numerically solved. The explicit is used to solve the saturation equation and the implicit is used to solve the pressure equation in this model. By fitting the results of foam plugging capacity experiments and numerical calculation, the correctness of the model is verified. On this basis, the conceptual model is established to simulate and evaluate the sensitivity of influencing parameters of foam flooding plugging ability. The results show that: the plugging ability of foam system increases with the increase of foaming agent concentration. When the concentration of foaming agent is more than 0.3%, the resistance coefficient tends to steady; when gas-liquid ratio is 2:1, the foam plugging ability is the strongest. The plugging ability of foam system increases with the increase of permeability showing good permeability selectivity. If the permeability in the reservoir is higher than the profile, control capability will also be stronger.

Keywords: Numerical calculation, foam composite system flooding, mathematical model, plugging capacity, sensitivity analysis.

## **1. INTRODUCTION**

Because water flooding recovery can only reach about 40%, so we can enhance oil recovery by the tertiary oil recovery. A disadvantage of the conventional surfactant flooding is that it can not achieve mobility control and easily form a cross flow phenomenon. To solve this problem, the displacing fluid resistance must be increased. If the foaming properties of the surfactant which is injected is good, then the foam flooding under the reservoir conditions can be formed [1]. It can be applied to high salinity, high temperature reservoirs and can avoid the cross flow phenomenon. The foam has a high viscosity at the same time and it can control mobility and expand swept volume. It can also reduce the oil-water interfacial tension and increase the displacement efficiency, which to be a new research direction of chemical flooding. Foam system has a unique seepage characteristic and foam for high permeability layer has strong plugging capacity [2-12], but for low permeability layer it has good swept effect. Foam has a higher apparent viscosity which can significantly improve water and oil mobility ratio. This paper established a foam flooding mathematical model with three-phase and five components, verified its correctness, simulated and evaluated the sensitivity of influencing parameters of foam flooding plugging ability.

#### **2. MATHEMATICAL MODELS**

#### 2.1. Assumed Conditions

(1) Considering three-phase: the oil, gas, water and fivecomponents: oil, nitrogen, water, foam, surfactant. Nitrogen and foam are included in the gas phase.(2) Surfactant is only in the aqueous phase, ignoring its effect on the density and the viscosity of the aqueous phase because of its small concentrations, and ignoring the dissolution of nitrogen in the water. (3) The seepage of the fluids in the reservoir and the injected fluids satisfy the generalized Darcy's law. (4) Using a modified gas phase relative permeability model to characterize the relative permeability of the foam.

## 2.2. Mass Conservation Equations

Oil, gas and water phase mass conservation equations

Oil phase:

$$\nabla \left[ \rho_o \frac{kk}{\mu_o} \nabla \left( P_o - \gamma_o Z \right) \right] + q_o = \frac{\partial \left( \phi \rho_o S_o \right)}{\partial t} \tag{1}$$

Water phase:

$$\nabla \left[ \rho_{w} \frac{kk}{\mu_{w}} \nabla \left( P_{w} - \gamma_{w} Z \right) \right] + q_{w} = \frac{\partial \left( \phi \rho_{w} S_{w} \right)}{\partial t}$$
(2)

Gas phase:

<sup>\*</sup>Address correspondence to this author at the College of Petroleum Engineering, Northeast Petroleum University, Daqing, 163318, China; Tel: +8613845948796.; Fax: +8604596503482;

E-mail: zhangchengli0303@163.com

$$\nabla \left[ \rho_g \frac{kk}{\mu_g} \nabla \left( P_g - \gamma_g Z \right) \right] + q_g = \frac{\partial \left( \phi \rho_g S_g \right)}{\partial t}$$
(3)

Where  $\rho_o, \rho_w, \rho_g$  - the density of oil phase, water phase and gas phase respectively;  $\phi$  - porosity; K -the absolute permeability of the rock;  $K_{ro}, K_{rw}, K_{rg}$  - the relative permeability of oil phase, water phase and gas phase respectively;  $\mu_o, \mu_w, \mu_g$  - the viscosity of the oil phase, water phase and gas phase respectively;  $P_o, P_w, P_g$  - the pressure of oil phase, water phase and gas phase respectively ;  $\gamma_o, \gamma_w, \gamma_g$  - the weight of the oil phase, water phase and gas phase respectively ;  $S_o, S_w, S_{g_-}$  the saturation of oil phase, water phase and gas phase respectively ; Z - the vertical depth of the reservoir; q -source sink term.

#### 2.3. The Foam Total Amount Balance Equation

Foam will regenerate, burst continually when it transports in porous media, since it is a dynamic equilibrium process. The basic thought of foam total amount balance: Foam increment = inflow -outflow + production-burst + source and sink term. Establish the foam total amount balance equation [5-12]:

$$\frac{\partial}{\partial t} \Big[ \phi \Big( S_f X_f n_f + S_t X_t n_t \Big) \Big] =$$

$$-\nabla \cdot \Big( n_f u_f \Big) + \phi S_g \Big( r_g - r_c \Big) + q_f$$
(4)

Where  $n_f$ ,  $n_t$  - is the average density of the flowing foam and trapped foam respectively;  $X_f$ ,  $X_t$  - represents the shunt volume of the flowing foam and trapped foam respectively;  $u_f$  - gas Darcy velocity;  $r_g$  -foam generating speed,  $r_c$  - foam burst speed;  $q_f$  -the amount of injection foam per unit volume and per unit time.

(1) Foam generating speed

Foam generation rate expression [13]

$$r_{g} = k_{1}^{0} v_{w} v_{f}^{1/3} \left[ 1 - \left( \frac{n_{f}}{n^{*}} \right)^{\omega} \right]$$
(5)

Where  $k_1^0$  - foam generating rate constant;  $v_w$  -water phase velocity;  $v_f$  - flowing foam velocity;  $n^*$  - the upper limit value of foam density;  $\omega$  - the constant is determined by experiment. (2) Foam burst speed

Foam burst speed expression:

$$r_{c} = k_{-1}^{0} \left( \frac{P_{c}}{P_{c}^{*} - P_{c}} \right)^{2} v_{f} n_{f}$$
(6)

$$P_c^* = P_{c,\max}^* \tanh\left(C_S / C_S^0\right) \tag{7}$$

Where  $k_{-1}^0$  - foam burst rate constant;  $P_c$  - capillary force;  $P_c^*$  - critical capillary force;  $P_{c,\max}^*$  -critical capillary force maximum;  $C_S$  - surfactant concentration;  $C_S^0$  reference value of surfactant concentration.

#### 2.4. Foam Properties Parameters

(1) Foam viscosity calculation

Foam system presents the characteristics of non-Newtonian fluid in porous medium. The foam effective viscosity can be characterized as [14]:

$$\mu_f = \mu_g + \frac{\alpha n_f}{v_f^{1/3}} \tag{8}$$

Where  $\mu_f$  -foam effective viscosity;  $\mu_g$  - gas phase viscosity without foam;  $\alpha$  - constant related to surfactant properties.

(2) Gas phase relative permeability

The increase of the gas phase flow resistance after generating foam is reflected by the decrease of the gas phase relative permeability. The foam molar concentration of the gas phase is a macro reflection of the foam stability. The expression of gas phase relative permeability is:

$$k_{rg} = \frac{k_{rg}^0}{1 + (R - 1)X_{gf}}$$
(9)

where  $k_{rg}$ ,  $k_{rg}^0$  represents the gas phase relative permeability with and without foam; *R* - decreasing coefficient of the gas phase relative permeability;  $X_{gf}$  - foam molar concentration of the gas phase, which characterizes the foam stability.

(3) Score calculation of the trapped foam

The trapped foam fraction can be characterized as:

$$X_t = X_{t,\max}\left(\frac{\beta n_t}{1+\beta n_t}\right) \tag{10}$$

Where  $X_{t,\max}$  - maximum trapped foam fraction;  $\beta$  - empirical constants.

Injection Pore Volume /PV	Experimental $\Delta P$ /MPa	Numerical Calculation $\Delta P$ /MPa	<b>Relative Error/%</b>
0	0	0	0
0.2	0.131	0.126	3.82
0.4	0.252	0.237	5.95
0.6	0.340	0.328	3.53
0.8	0.415	0.412	0.72
1	0.490	0.499	1.84
1.2	0.542	0.551	1.66
1.4	0.623	0.601	3.53
1.6	0.648	0.662	2.16
1.8	0.747	0.723	3.21
2	0.798	0.782	2.02
2.2	0.802	0.782	2.49
2.4	0.790	0.782	1.01
2.6	0.785	0.782	0.38
2.8	0.783	0.782	0.13
verage error /%		-	1.19

Table 1. Pressure difference fitting results of foam plugging experiment.

#### 2.5. Auxiliary Equation

(1) Saturation constraint equation

$$S_o + S_W + S_f = 1 \tag{11}$$

(2) Capillary pressure equation

$$P_{cow} = P_o - P_w \tag{12}$$

$$P_{cgo} = P_g - P_o \tag{13}$$

## **3. MODEL VALIDATION**

IMPES method (the implicit pressure-explicit-its saturation method) is used to calculate the three-phase and fivecomponent foam flooding mathematical model. It is implicit pressure explicit method, in which implicit is used to solve the pressure equation and explicit is used to solve saturation equation.

The results of numerical calculation of established model are used in the foam plugging experiment, and then the correctness of the model can be verified. An orthogonal grid is established for simulation of nitrogen foam flooding plugging experiment, which is 45m×1m×1m and grid spacing is 1m×5cm×5cm. The simulation parameters are as follows:  $\varphi = 0.28$ ,  $K = 1200 \times 10^{-3} \mu m^2$ ,  $\mu_w = 0.6 m Pa \cdot s$ ,  $\mu_g = 2.15 \times 10^{-2} m Pa \cdot s$ ,  $k_1^0 = 2.03 \times 10^{15} s^{1/3} m^{-13/3}$ ,  $k_{-1}^0 = 9.5 m^{-1}$ ,  $P_{c,max}^* = 27 k Pa$ ,  $X_{t,max} = 0.81$ ,  $\alpha = 8.2 \times 10^{-18} Pa \cdot s^{2/3} m^{10/3}$ ,  $C_S^0 = 0.075 w t\%$ ,  $C_S = 0.48 w t\%$ ,  $n^* = 1 \times 10^{12} m m^{-3}$ ,  $\omega = 2.9$ . The fitting results are shown in Table 1. It can be inferred that simulation results are basically consistent with the experimental results, and the fitting effect of experiment is good. Hence, the rationality of established model and the correctness of the numerical calculation method are verified.

# 4. THE SENSITIVITY ANALYSIS OF MODEL PA-RAMETERS

The plugging ability of foam is an important indicator to evaluate its performance [15-17], and there are many factors influencing the plugging ability of foam. Besides the stability of foam [18], the concentration of foam, gas-liquid ratio of foam system, and rock permeability are all important factors. Foam resistance coefficient is used to describe the ability of foam plugging. It is the ratio of pressure difference at both ends of core and straight water flooding. Its value is the key index to evaluate the quality of foam system. The sensitivity analysis of parameters, which affects the plugging ability of foam, is conducted through the research on the numerical



Fig. (1). Relationship between the concentration of surfactant and the resistance coefficient / residual resistance factor.



Fig. (2). Resistance coefficient / residual resistance factor under different gas-liquid ratio.

simulation. The basic model parameters are mentioned before.

## 4.1. Concentration of Foaming Agent

Through establishing a conceptual model, numerical simulation of foam flooding is studied by changing the concentration of surfactant in the injecting model to calculate the resistance coefficient and residual resistance coefficient, as shown in Fig. (1). As can be seen from the calculation, both resistance coefficient and residual resistance coefficient increase with the increase of concentration of foaming agent. But resistance coefficient and residual resistance coefficient keep constant, and tend to be gentle, when the concentration of foaming agent is more than 0.3%.

## 4.2. Gas-Liquid Ratio (GLR)

Gas-liquid ratio of foam system is an important indicator of the foam flooding. The best gas-liquid ratio is optimized by the studying of foam plugging ability through gas-liquid ratio, so the plugging ability of foam system can be stronger. Through establishing a conceptual model, numerical simulation of foam flooding is studied by changing the gas-liquid ratio of foam system in the injecting model to calculate the resistance coefficient and residual resistance coefficient, as shown in Fig. (2).

It is evident from the figure that the relationship between the gas-liquid ratio and the resistance coefficient / residual resistance coefficient exhibits the foam that is low at two ends and high in the middle. Foam plugging ability enhances with the increase of the gas-liquid ratio when the gas-liquid ratio is low. The resistance coefficient / residual resistance coefficient reaches the maximum of 45.52, when the gas liquid ratio is 2:1. The resistance coefficient / residual resistance coefficient of the foam system decreases with the increase of the gas-liquid ratio, when the gas-liquid ratio is more than 2:1. So the foam plugging ability is strongest when the gas-liquid ratio is 2:1.

#### 4.3. The Reservoir Permeability

Through establishing conceptual model of different permeability, numerical simulation of foam flooding is studied



Fig. (3). Relationship between permeability and resistance coefficient/residual resistance factor.

by setting foaming agent concentration to 0.3% and gasliquid ratio to 2:1 to calculate the relationship between the resistance coefficient, residual resistance coefficient and permeability as shown in Fig. (3).

The relationship between the foam resistance coefficient and permeability is the distinctive feature of foam system. Plugging ability of foam system increases with the increase of permeability and presents good linear relationship showing excellent selective permeability. The characteristic of foam is the result of the foam apparent viscosity which is reflected by additional pressure difference caused by elastic deformation of foam in the pores, when foam system transports in the porous media. When the larger volume of foam migrates in the larger pore, there is stronger ability of profile control, as well as the larger deformation amount. In terms of reservoir, the higher the permeability is, the stronger the ability of profile control is.

# CONCLUSION

(1) Based on mass conservation, foam total amount balance thought and foam properties characterization model, a foam flooding mathematical model with three-phase and five components is established and numerically solved.

(2) The numerical calculation results show that the resistance coefficient / residual resistance coefficient increases with the increase of the foaming agent concentration, but when the foaming agent concentration is greater than 0.3%, resistance coefficient/residual resistance coefficient remains the same, and tends to be steady.

(3) The relationship between the gas-liquid ratio and the resistance coefficient / residual resistance coefficient exhibits the foam that is low at two ends and high in the middle. Foam plugging ability enhances with the increase of the gas-liquid ratio when the gas-liquid ratio is low. The resistance coefficient / residual resistance coefficient reaches the maximum, when the gas liquid ratio is 2:1; The resistance coefficient / residual resistance coefficient of the foam

system decreases with the increase of the gas-liquid ratio, i.e. when the gas-liquid ratio is more than 2:1. So the foam plugging ability is strongest when the gas-liquid ratio is 2:1.

(4) Plugging ability of foam system increases with the increase of permeability and presents good linear relationship. In terms of reservoir, the higher the permeability, the stronger the ability of profile control.

# **CONFLICT OF INTEREST**

The authors confirm that this article content has no conflict of interest.

## **ACKNOWLEDGEMENTS**

This work is financially supported by the Natural Science Foundation of China under Grant No. 51474071.

## REFERENCES

- Q. Wang, P. Guo, Z. Wang, and X. Li, "Foam fluid flow characteristics in porous media," *Oil Drilling & Production Technology*, vol.30, no.2, pp. 90-92, 2008.
- [2] B. Li, Z. Li, and Z. Liu, "Experiment on profile control and flooding by multiphase foam system," *Journal of China University* of *Petroleum (Edition of Natural Science)*, vol. 34, no. 4, pp. 93-98, 2010.
- [3] S. Li, Z. Li, and R. Lin, "Mathematical model building for foam diversion acidizing and its application," *Journal of China University of Petroleum (Edition of Natural Science)*, vol. 32, no. 5, pp. 77 – 82, 2008.
- [4] Z. Li, M. Sun, and R. Lin, "Laboratory study on foam plugging and selective divided-flow," *Acta Petrolei Sinica*, vol.28, no. 4, pp. 115-118, 2007.
- [5] A. R. Kovscek, C. J. Radke, "Fundamentals of Foam Transport in Porous Media," *Foams: Fundamentals and Applications in the Petroleum Industry*," Washington, DC: American Chemical Society, pp.115-163, 1994.
- [6] G. Chen, G. Liao, J. Niu, "Equivalent numerical simulation model for foam flowing in porous medium," *Petroleum Geology & Oilfield Development in Daging*, pp. 72-75, 2001.
- [7] H. Cheng, and Z. Lang, "Capillary Crossflow in Foam Flood and Its Numerical Simulation," *Journal of Chongqing University*, vol. 23, pp. 161-165, 2003.

### Establishment of Mathematical Model and Sensitivity Analysis

#### The Open Chemical Engineering Journal, 2015, Volume 9 89

- [8] A. R. Kovscek, T. W. Patzek, and C. J. Radke, "Simulation of Foam Transport in Porous Media," SPE26402, pp. 309-318, 1993.
- [9] W. R. Roseen, S. C. Zeilinger, J. Shi, and M. T. Lim, "Mechanistic Simulation of Foam Processes in Porous Media," SPE, 28940, pp. 493-500, 1994.
- [10] A. H. Palls, G. J. Hirasaki, T. W. Patzek, D. A. Gaualltz, D. D. Miller, T. Ratulowsk, "Development of a Mechanistic Foam Simulator: The Population Balance and Generation by Snap-Off," *SPE14961*, pp. 884-892, 1988.
- [11] T. W. Patzek, "Description of Foam Flow in Porous Media by the Population Balance Method," *Society of Petroleum Engineers*, pp. 1-9, 1986.
- [12] T. Lu, Z. Li, J. Li, R. Li, "A Mathematical Model of Foam Flooding Based on Foam Microscopic Seepage Characteristics," *Chinese Journal of Computational Physics*, vol. 29, no. 4, pp. 519-524, 2012.
- [13] A. H. Falls, and P. A. Gauglitz, and G. J. Hirasaki, "Development of a mechanistic foam simulator: The population balance and generation by snap-off," SPE 14961, 1986.

Received: June 16, 2015

Revised: July 23, 2015

Accepted: August 16, 2015

© Zhang et al.; Licensee Bentham Open

This is an open access article licensed under the terms of the (https://creativecommons.org/licenses/by/4.0/legalcode), which permits unrestricted, non-commercial use, distribution and reproduction in any medium, provided the work is properly cited.

- [14] Z. I. Khatlb, G. J. Hirasaki, and A. H. Falls, "Effects of Capillary Pressure on Coalescence and Phase Mobilities in Foams Flowing Through Porous Media," *SPE15442*, pp. 919-926, 1988.
  [15] Q. Wang, G. Zhou, P. Guo, and X. Li, "Experimental study of
- [15] Q. Wang, G. Zhou, P. Guo, and X. Li, "Experimental study of foam sealing ability," *Southwest Petroleum Technology*, vol. 25, no.6, pp. 40-42, 2003.
- [16] A. R. Kovscek, and H. J. Bertin, "Foam Mobility in Heterogeneous Porous Media (I: Scaling Concepts)," *Transport in Porous Media*, vol. 52, no.1, pp.17-35, 2003.
- [17] Y. Liu, H. Liu, and Z. Pang, "Numerical simulation of nitrogen foam injection to control water cresting by bilateral horizontal well in bottom water reservoir," J. Open Fuels and Energy Science Journal, vol. 5, no.1, pp. 53-60, 2012.
- [18] H. Mei, H. Dong, H. Gu, M. Ren, and L. Chen, "Frother stable performance evaluation experiments," *Xinjiang Petroleum Geolo*gy, vol. 25, no.6, pp. 644-646, 2004.