A Refined Formula for the Allowable Soil Pressure Using Shear Wave Velocities

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Abstract: Based on a variety of case histories of site investigations, including extensive bore hole data, laboratory testing and geophysical prospecting at more than 550 construction sites, an empirical formulation is proposed for the rapid determination of allowable bearing pressure of shallow foundations in soils and rocks. The proposed expression corroborates consistently with the results of the classical theory and is proven to be rapid, and reliable. Plate load tests have been also carried out at three different sites, in order to further confirm the validity of the proposed method. It consists of only two soil parameters, namely, the in situ measured shear wave velocity and the unit weight. The unit weight may be also determined with sufficient accuracy, by means of other empirical expressions proposed, using P or S - wave velocities. It is indicated that once the shear and P-wave velocities are measured in situ by an appropriate geophysical survey, the allowable bearing pressure as well as the coefficient of subgrade reaction and many other elasticity parameters may be determined rapidly and reliably.

Keywords: Shear wave velocity, shallow foundations, allowable bearing pressure, dynamic technique, soils and rocks.

1. INTRODUCTION

Professor Schulze [1], a prominent historical figure in soil mechanics and foundation engineering in Germany, stated in 1943 that “For the determination of allowable bearing pressure, the geophysical methods, utilising seismic wave velocity measuring techniques with absolutely no disturbance of natural site conditions, may yield relatively more realistic results than those of the geotechnical methods, which are based primarily on bore hole data and laboratory testing of so-called undisturbed soil samples”.

Since that time, various significant contributions have been made to solving geotechnical problems by means of geophysical prospecting. The P-wave velocities, for instance, have been used to determine the unconfined compressive strengths and modulus of elasticity of soil samples by Coates [2], Hardin and Black [3], and also Hardin and Drnevich [4], based on extensive experimental data, established indispensible relations between the shear wave velocity, void ratio, and shear rigidity of soils. Similarly, Ohkubo and Terasaki [5] supplied various expressions relating the seismic wave velocities to weight density, permeability, water content, unconfined compressive strength and modulus of elasticity.

The use of geophysical methods in soil mechanics has been extensively studied for the purpose of determining the deformation properties of fine-grained soils from seismic tests. As to the in situ measurement of P and S – wave velocities, various alternate techniques are available as outlined in detail by Stokoe and Woods [23], Tezcan, et al. [24], Butcher, et al. [25], Richtar, et al. [26], Kramer [27], Santamarina, et al. [28].

2. THEORETICAL BASIS FOR THE EMPIRICAL EXPRESSION

In order to be able to arrive at a particular empirical expression for the allowable soil pressure qₓ - underneath a shallow foundation, the systematic boundary value approach used earlier by Keceli [11] will be followed. The state of
stress and the related elastic parameters of a typical soil column is shown in Fig. (1). Considering a foundation depth of $D_f$, with a unit cross-sectional area of $A = l$, the typical form of the compressive ultimate bearing capacity at the base of the foundation, nothing but only as a format, may be written approximately as:

$$q_a = \gamma D_f$$  \hspace{1cm} (2)

$$q_a = q_f / n = \gamma D_f / n$$  \hspace{1cm} (3)

where $q_f$ = ultimate bearing capacity at failure, $\gamma$ = unit weight of soil above the base of the foundation, $q_a$ = allowable bearing pressure, and $n$= factor of safety. In order to be able to incorporate the shear wave velocity $V_{s2}$ into the above expressions, the depth parameter $D_f$ will be expressed as velocity multiplied by time as:

$$D_f = V_{s2} t$$  \hspace{1cm} (4)

in which, the $V_{s2}$ is purposely selected to be the shear wave velocity measured under the foundation, $t$ = is an unknown time parameter. The time parameter $t$ is introduced herein just as a dummy parameter in order to keep consistency in appropriate units. Substituting eqn (4) into eqn (3), yields

$$q_a = \gamma V_{s2} t / n$$  \hspace{1cm} (5)

The unknown time parameter $t$, will be determined on the basis of a calibration process. For this purpose, a typical 'hard' rock formation will be assumed to exist under the foundation, with the following parameters, as suggested earlier by Keceli [11];

$$q_a = 10 \text{ kN/m}^2, V_{s2} = 4 \text{ 000 m/sec}$$  \hspace{1cm} (6)

Substituting these numerical values into eqn (5), we obtain $t = 0.10 \text{ sec}$, thus:

$$q_a = 0.1 \gamma V_{s2} / n$$  \hspace{1cm} (7)

This is the desired empirical expression to determine the allowable bearing pressure $q_a$, in soils and rocks, once the average unit weight, $\gamma$, for the soil layer above the foundation and the in situ measured $V_{s2}$ - wave velocity for the soil layer just below the foundation base are available. The unit of $V_{s2}$ is in m/sec, the unit of $\gamma$ is in kN/m$^3$, then the resulting $q_a$ – value is in units of kPa. The unit weight values may be estimated using the empirical expressions:

$$\gamma_p = \gamma_0 + 0.002 V_{p1} \text{ and } \gamma_s = 4.3 V_{s1}^{0.25}$$  \hspace{1cm} (8a) (8b)

as proposed earlier by Tezcan et al. [21], and by Keceli [29], respectively. The second expression is especially recommended for granular soils, for which the measured $V_{s1}$ values represent appropriately the degree of water content and / or porosity. The wave velocities must be in units of m/sec. The only remaining unknown parameter is the factor of safety, $n$, which is assumed to be, after a series of calibration processes, as follows:

$$n = 1.4 \text{ (for } V_{s2} \geq 4 \text{000 m/sec)},$$  \hspace{1cm} (9)

$$n = 4.0 \text{ (for } V_{s2} \leq 750 \text{ m/sec})$$

The calibration process is based primarily on the reference $q_a$ – values determined by the conventional Terzaghi method, for all the data sets corresponding to the 550 – construction sites considered. For $V_{s2}$ values greater than 750 m/sec and smaller than 4 000 m/sec a linear interpolation is recommended. The engineering rock formations are assumed to start for $V_{s2} > 750 \text{ m/sec}$. The factors of safety, as well as the empirical allowable bearing pressure expressions, for various soil (rock) types, are given in Table 1. It is seen that three distinct ranges of values are assumed for $n$ = factor of safety. For soil types with $V_{s2} \leq 750 \text{ m/sec}$ the factor of safety is $n = 4$, for rocks with $V_{s2} \geq 4 \text{000 m/sec}$ it is $n = 1.4$ . For other intermediate values of shear wave velocity, linear interpolation is recommended. The validity of these values has been extensively checked and calibrated by the soil data at 550 construction sites. The relatively higher value of factor of safety assumed for soils is deemed to be appropriate to compensate the inaccuracies and gaps existing in the measured values of shear wave velocity. In fact, Terzaghi and Peck [30] states that “The factor of safety of the foundation with respect to breaking into the ground should not be less than about 3”.

It is determined by Terzaghi and Peck [30] also that the width of footing, $B$, has a reducing influence on the value of allowable bearing pressure for granular soils. Therefore, a correction factor $\beta$ is introduced into the formula, for sandy

Fig. (1). Soil column and related parameters.
soils only, as shown in the third line of Table I. The proposed values of this correction factor, for different foundation width $B$, are as follows:

\[
\beta = 1.00 \quad \text{for} \ (0 \leq B \leq 1.20 \text{ m})
\]
\[
\beta = 1.13 - 0.11 B \quad \text{for} \ (1.2 \leq B \leq 3.00 \text{ m}) \quad (10)
\]
\[
\beta = 0.83 - 0.01 B \quad \text{for} \ (3.0 \leq B \leq 12.0 \text{ m})
\]

3. COEFFICIENT OF SUBGRADE REACTION

The shear wave velocity may be used successfully to determine $k_s$, coefficient of subgrade reaction of the soil layer just beneath the foundation base by making use of the expressions given in Fig. (1). The coefficient of subgrade reaction $k_s$, is defined, similar to the definition of spring constant in engineering mechanics, to be the necessary vertical pressure to produce a unit vertical displacement and expressed as:

\[
k_s = q_a / d \quad (11)
\]

For shallow foundations, the total vertical displacement is restricted to $1 \text{ inch} = 0.025 \text{ m}$, as prescribed by Terzaghi and Peck [30]. When, $d = 0.025 \text{ m}$ is substituted in eqn (11), the coefficient of subgrade reaction becomes in units of $kN/m^2$:

\[
k_s = 40 \frac{q_a}{d} \quad \text{or} \quad k_s = 4 \gamma V_{s2} / n \quad (12, 13)
\]

4. ELASTICITY PARAMETERS

Once, $V_{p2}$ and $V_{s2}$ seismic wave velocities are measured, by geophysical means, for the soil layer No.2 just under the foundation, several parameters of elasticity, such as $G$ = Shear modulus, $E_s$ = Constraint modulus of elasticity, $E = $ Modulus of elasticity (Young’s modulus), $E_k$ = Bulk modulus, and $\mu$ = Poisson’s ratio may be obtained easily. The Shear modulus, $G$, and the Constraint modulus, $E_s$, are related to the shear and $P$-wave velocities by the following expressions, respectively:

\[
G = \rho V_s^2 \quad \text{and} \quad E_s = \rho V_p^2 \quad (14, 15)
\]

where, $\rho$ = mass density given by $\rho = \gamma / g$. From the Theory of Elasticity, it is known that, $E = $ the Young’s modulus of elasticity is related to $E_s = $ the Constraint modulus and also to $G =$ the Shear modulus by the following expressions:

\[
E = E_s (1 + \mu) / (1 - 2\mu) \quad (16)
\]
\[
E = 2 (1 + \mu) G \quad (17)
\]

Utilising eqn (14) and (15) and also substituting $\alpha$, as

\[
\alpha = E_s / G = (V_p / V_s)^2
\]

into eqn (16) and (17), we obtain

\[
\mu = (\alpha - 2) / 2 (\alpha - 1) \quad \text{or} \quad \alpha = (2\mu - 2) (2\mu - 1) \quad (19, 20)
\]

The modulus of elasticity is directly obtained from eqn (17) as:

\[
E = (3\alpha - 4) G / (\alpha - 1) \quad (21)
\]

The Constraint modulus $E_s$, may be also obtained in terms of $\alpha$ as:

\[
E_s = (\alpha -1) E / (3\alpha - 4) \quad \text{or} \quad E_s = \gamma V_p^2 / g \quad (22, 23)
\]

The Bulk modulus $E_k$, of the soil layer, may be expressed, from the theory of elasticity, as

\[
E_k = E / 3 (1 - 2\mu) \quad \ldots \quad (24)
\]

\[
E_k = (\alpha - 1) E / 3 = \gamma (V_p^2 - 4 V_s^2 / 3) / g \quad (25)
\]

5. CASE STUDIES

The allowable bearing pressures have been also determined at more than 550 construction sites in and around the Kocaeli and Istanbul Provinces in Turkey, between the years 2005-10. At each construction site, by virtue of City by-law, appropriate number of bore holes were drilled, SPT counts conducted, undisturbed soil samples were taken for laboratory testing purposes, where shear strength -$c$, the internal angle of friction -$\phi$, unconfined compression strength -$q_u$, and unit weight -$\gamma$ were determined. Subsequently, following the classical procedure of Terzaghi and Peck [30], the ultimate capacity and also the allowable bearing pressures were determined, by assuming the factor of safety as $n=3$. For granular soils, immediate settlement calculations were also conducted, in order to determine whether the shear failure mechanism or the maximum settlement criterion would control the design.

The numerical values of the allowable bearing pressures, $q_a$, determined in accordance with the conventional Terzaghi theory, are shown by a triangular ($\Delta$) symbol, in Fig. (2), where the three digit numbers refer to the data base file numbers of specific construction sites. Parallel to these classical soil investigations, the $P$- and $S$-wave velocities have been measured in situ, right at the foundation level for the purpose of determining the allowable bearing pressures, $q_a$, which are shown by means of a circle ($o$), in Fig. (2). Two separate linear regression lines were also shown in Fig. (2), for the purpose of indicating the average values of allowable bearing pressures determined by ‘dynamic’ and ‘conventional’ methods. In order to obtain an idea about the relative conservatism of the two methods, the ratios of allowable bearing pressures $(r = q_a / q_a^c)$, as determined by the ‘dynamic’ and ‘conventional’ methods, have been plotted against the $V_s$ – values in Fig. (3).
It is seen that the linear regression line indicates for \( V_s \) – values smaller than 400 m/sec a narrow band of \( r = 1.03 \) to \( r = 1.12 \), which should be regarded as quite acceptable. The ‘dynamic’ method proposed herein yields allowable bearing pressures slightly (on the order of 3 to 10 percent) greater than those of the ‘conventional’ method for \( V_s \) – values smaller than 400 m/sec. In fact, the ‘conventional’ method fails to produce reliable and consistent results for relatively strong soils and soft rocks, because it is difficult to determine the appropriate soil parameters \( c \) and \( \phi \) for use in the ‘conventional’ method. At construction site Nos: 133, 134, 138, 139, 206, 207, 214, 215, 219, 502, 507 and 544, where the soil conditions have been mostly weathered andesite, granodiorite arena, greywacke, limestone, etc did not allow for the measurement of \( c \) and \( \phi \) – values. Therefore, the use of ‘dynamic’ method becomes inevitable for such strong soils with \( V_s > 400 \) m/sec.

The list of soil parameters determined by in situ and also by laboratory testing through geotechnical prospecting, as well as the in situ measured \( V_p \) and \( V_s \) – velocities at each of the 550 construction sites, are too voluminous to be included herein. Those researchers interested to have access to these particular data base, may inquire from internet <tezokan@superonline.com>, <www.tezokan.com>.

6. SEISMIC WAVE VELOCITIES

The seismic wave velocities have been measured using \( P \) – and \( S \) – geophones by means of a 24 – Channel Geometrics Abem – Pasi seismic instrument, capable of noise filtering. The \( P \) – waves have been generated by hitting 6 – blows vertically, with a 0.15 kN hammer, onto a 250 x 250 x 16 mm size steel plate placed horizontally on ground. For the purpose of generating \( S \) – waves however, an open ditch of size 1.4 x 1.4 x 1.4 m was excavated and then two steel plates...
were placed on opposite vertical faces of this ditch parallel to the centerline of the geophones. Using the same 0.15 kN hammer, 6 heavy horizontal blows were applied onto each of these vertical steel plates. The necessary polarity of the $S$ - waves was achieved by hitting these vertical steel plates horizontally in opposite directions, nonconcurrently.

7. PLATE LOAD TESTING

For purposes of correlating the allowable bearing pressures determined by various methods, plate loading tests have been carried out at three particular construction Sites Nos: 335, 502 and 544. The soil parameters $c$, $q_u$, and $\gamma$ as determined by laboratory testing, as well as the $P$ and $S$ - wave velocities measured at site by geophysical prospecting are all shown in Table 3. A thick steel bearing plate of 316.2 mm x 316.2 mm = 0.10 square meter in size is used under the test platform of size 1.50 m by 1.50 m. The tests are carried out right at the bottom elevations of foundations. One half of the bearing pressure $\sigma_0$, which produced a settlement of $s = 12.7$ mm was selected as the allowable pressure $q_a$ as shown in Fig. (4). It is seen clearly in Table 2 that the results of the proposed ‘dynamic’ method using $P$ and $S$ – wave velocities are in very close agreement with those of the plate load test.

![Fig. (4). Load test results at Sites No: 335, 502, and 544.](image)

Table 2. Comparative Evaluation of Allowable Pressures

<table>
<thead>
<tr>
<th>Site No</th>
<th>Owner Lot Nos (soil type)</th>
<th>Various Soil Parameters ($\phi = 0$)</th>
<th>$q_a = $ Allowable Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$q_u$</td>
<td>$D_t$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>kPa</td>
<td>m</td>
</tr>
<tr>
<td>335</td>
<td>Suleyman Turan 8 Paft/A930 Pars. (silty clay)</td>
<td>172</td>
<td>1.50</td>
</tr>
<tr>
<td>544</td>
<td>Ayhan Dede G22B / 574 / 11 (weathered diorite)</td>
<td>190</td>
<td>1.50</td>
</tr>
<tr>
<td>502</td>
<td>Ebru Çınar 30 L1C / 440 / 8 (clay stone)</td>
<td>147</td>
<td>1.00</td>
</tr>
</tbody>
</table>

$^{(1)} q_u = $ unconfined compressive strength.

$^{(2)}$ Terzaghi and Peck (1976).

$^{(3)} q_a = 0.025 \gamma \sqrt{V_p}$ (Eq.7), n = 4.
ing. The allowable bearing pressures \( q_a \), in accordance with the conventional theory assuming \( N_c = 5.14 \) and \( N_q = 1 \), are also calculated using

\[
q_a = \left( c N_c + \gamma D_l N_q \right) / 3.0
\]

\[ (26) \]

### 8. NUMERICAL EXAMPLE

For purposes of illustration, a soft clayey soil layer of \( H = 15 \text{ m} \) beneath a shallow strip footing of depth \( D_l = 2.90 \text{ m} \), width \( B = 1.30 \text{ m} \), is considered. The in situ measured seismic wave velocities are determined to be \( V_p2 = 700 \text{ m/sec} \) and \( V_s2 = 200 \text{ m/sec} \), within the soil layer just below the foundation base. By coincidence, the \( P \) – wave velocity within the soil layer above the foundation base is also measured to be \( V_p1 = 700 \text{ m/sec} \). A comprehensive set of classiﬁcal soil investigations, including a number of bore hole data and laboratory testing exist for this particular site, together with the numerical values of various soil parameters \( c = 52 \text{ kPa} \), and \( \phi = 0 \), including the bearing pressure capacity determined to be \( q_f = 322 \text{ kPa} \) by the conventional method of Terzaghi and Peck [30]. Therefore, the validity and the reliability of the proposed empirical formulae have been rigorously veriﬁed. Calculation of some elasticity parameters, using the empirical expressions presented herein, is summarised in Table 3.

### 9. DISCUSSION ON THE DEGREES OF ACCURACY

The degrees of accuracy of the proposed ‘dynamic’ method are quite satisfactory and consistent as attested rigorously at more than 550 construction sites. The conventional approach however, depends heavily on the degrees of accuracy of in situ and laboratory determined soil parameters. In fact, the allowable bearing pressure calculations are very sensitive to the values of \( c \), and \( \phi \), determined in the laboratory using so-called ‘undisturbed’ soil samples, which may not necessarily represent the true in situ conditions. This may explain the reason why at a number of construction sites, some inconsistent and erratic results for \( q_a \) are obtained using the classical theory, as already depicted in Fig. (2), because the laboratory measured \( c \), and \( \phi \) – values differed considerably from one soil sample into the other. The ‘Point Load’ tests of rock samples have been carried out for \( V_s2 \) – values greater than 400 m/sec as recommended by Hunt [31].

For ‘hard soil’ formations, corresponding to shear wave velocities, greater than \( V_s2 > 400 \text{ m/sec} \), the ‘conventional’ method is unable to yield any reliable \( q_a \) – allowable soil pressure, since neither \( c \) nor \( \phi \) - values may not be determined in the laboratory. Any approximate approach however, using either, \( q_u \) - unconfined compressive strength or, \( RQD \) ratios etc, will not be accurate enough. The ‘dynamic’ method in such cases produced consistently the same results as those obtained from the ‘Point Load’ tests. It is a fact that, the orientation of joints within a rock formation plays an important role in the in situ measured \( V_s \) – values. The aver-

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**Table 3. Results of Numerical Example (H=15 m, \( V_p2 = 700 \text{ m/sec} \), \( V_s2=200 \text{ m/sec} \) c = 52 kPa, \( \phi =0 \), \( V_p1 = 700 \text{ m/sec} \)**

<table>
<thead>
<tr>
<th>Formula</th>
<th>Equation</th>
<th>Numerical Calculations</th>
<th>Result</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_p = \gamma_0 + 0.002 V_p1 )</td>
<td>eqn (8a)</td>
<td>( \gamma_p = 16 + 0.002 ) (700)</td>
<td>17.4(1)</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Laboratory</td>
<td></td>
<td>( \gamma_p = 16 + 0.002 ) (700)</td>
<td>17.2</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( n = 4 )</td>
<td></td>
<td>( V_s2 = 750 \text{ m/sec} )</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>( q_i = c N_c + \gamma D_l N_q )</td>
<td>eqn (26)</td>
<td>( q_i = 52 (5.14) + 17.2 (2.9) ) 1</td>
<td>318</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( q_i = 0.1 \gamma V_s )</td>
<td>eqn (7)</td>
<td>( q_i = 0.1 (17.4) 200 )</td>
<td>348</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( q_i = q_i / n )</td>
<td>eqn (3)</td>
<td>( q_i = 348 / 4 )</td>
<td>87</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( k_s = 40 q_i = 4 \gamma V_s / n )</td>
<td>eqn (12)</td>
<td>( k_s = 40 (87) )</td>
<td>3480</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( G = \gamma V_s^2 / g )</td>
<td>eqn (14)</td>
<td>( G = 17.4 (200)^2 / 9.81 )</td>
<td>70948</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( \alpha = (V_p2 / V_s2)^2 )</td>
<td>eqn (18)</td>
<td>( \alpha = (700 / 200)^2 )</td>
<td>12.25</td>
<td></td>
</tr>
<tr>
<td>( \mu = (\alpha - 2) / (2 (\alpha - 1)) )</td>
<td>eqn (19)</td>
<td>( \mu = (12.25 - 2) / (2(11.25)) )</td>
<td>0.456</td>
<td></td>
</tr>
<tr>
<td>( E = 2 (1+\mu) G )</td>
<td>eqn (17)</td>
<td>( E = 2 (1.456) 70948 )</td>
<td>206537</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( E_c = \gamma V_s^2 / g )</td>
<td>eqn (15)</td>
<td>( E_c = 17.4 (700)^2 / 9.81 )</td>
<td>870000</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( E_b = E / 3 (1-2\mu) )</td>
<td>eqn (24)</td>
<td>( E_b = 206537 / 3 (1-2\mu) )</td>
<td>774417</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( E_k = E (\alpha - 1) / 3 )</td>
<td>eqn (25)</td>
<td>( E_k = 206537 (12.25 - 1) / 3 )</td>
<td>774514</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( d = \text{displacement} )</td>
<td>eqn (11)</td>
<td>( d = q_i / k_s = 87 / 3480 )</td>
<td>0.025</td>
<td>m</td>
</tr>
</tbody>
</table>

**Note:** Result of eqn (8a), \( \gamma = 17.4 \text{kN/m}² \) is used in all subsequent expressions.
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