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Design of a Pressurized Rectangular Conduit With Triangular Bottom Using The Rough Model Method

Bachir ACHOUR*

Research Laboratory in Subterranean and Surface Hydraulics (LARHYSS) University of Biskra, PO Box 145 RP 07000, Biskra, Algeria

Abstract: A new theory is presented to design pressurized conduits, particularly rectangular shaped conduit with triangular bottom. This theory is based on a referential rough conduit model characterized by an arbitrarily assigned relative roughness taken in the rough turbulent flow regime. Thus, the geometric elements of the chosen model are well defined. In particular, the relative height of the rough model, which is expressed by an implicit relationship, has been resolved by an appropriate limited development. The obtained relationship is used to calculate the almost exact value of the relative height of the model depending solely on the given side slope. The absolute height of the rough model is given by an explicit relationship based on measurable data in practice, such as side slope, discharge and energy slope. The required linear dimensions of the conduit are obtained by multiplying the homologues linear dimensions of the rough model by a non-dimensional correction factor which depends on the known rough model characteristics. Practical examples are presented to explain the procedure of calculation and to better understand the advocated method.

Keywords: Discharge, energy slope, pressurized conduit, rectangular conduit, rectangular-triangular bottom, rough model method, turbulent flow.

1. INTRODUCTION

The design of conduits or channel is one of the three great categories of problems encountered in hydraulic engineering practice. This consists in determining the linear dimensions of the conduit for given parameters such as discharge Q, energy slope J, absolute roughness ε which characterizes the state of the inner wall of the conduit and kinematic viscosity ν [1]. Taking for example the case of circular pipe, the diameter D is related to these parameters by the following functional relationship [1-3]: $D = \phi(Q, J, \varepsilon, \nu)$. The absolute roughness and the kinematic viscosity are measured in practice and rarely cause any particular problem. It is particularly important to consider the kinematic viscosity to make the solution of the problem of design covering the entire domain of turbulent flow. When the conduit is characterized by more than one linear dimension, one of them must be among the given data of the problem, such is the case of the pressurized vaulted rectangular conduit which is characterized by two linear dimensions, namely the height of the conduit and the diameter of the vault [4]. Design of conduits problem is often solved using the three basic relationships of turbulent flow, namely Darcy-Weisbach, Colebrook-White and Reynolds number [5-8]. The difficulty lies in assessing the friction factor since the Colebrook-White relationship is implicit. Moreover, the three basic equations of turbulent flow do not allow expressing the geometric element of the conduit in an explicit form. The solution involves many trials and tedious computations or laborious graphical solutions. The implicitness of the solution is found in the entirety of the geometric profiles known in practice. For pipe-flow problem, some authors have proposed approximate relation or graphical solution for the diameter of the pipe [8, 9]. Referring to the literature, one can find studies on the circular pipe but no study is published on pressurized rectangular shaped conduit with triangular bottom despite its extensive use in practice as water supply lines, sanitary sewers, culverts and storm drains or penstocks as well. In addition to its practical relevance, it is one of the rare geometric profiles whose design problem does not require the knowledge of one of the three linear dimensions that characterize it. Moreover, the study will show that this type of geometric profile induced interesting mathematical equations and remarkable hydraulic conclusions. This study was initiated to enrich the literature by studying this type of conduit under pressurized condition of the flow. The design problem is tackled with a new theoretical approach known as the referential Rough Model Method (RMM) which has been proven in the recent past [1, 4, 10-15]. Our attention is focussed on the computation of the three linear dimensions of the conduit using the strict minimum of data. The three basic equations of turbulent flow are applied to a rough

*Address correspondence to this author at the Research Laboratory in Subterranean and Surface Hydraulics, Biskra University, PO Box 145 RP 07000 Biskra Algeria; Tel: 00213557263947; Fax:0021333742481; E-mail: bachir.achour@larhyss.net

conduit model characterized by an arbitrarily assigned relative roughness value. Firstly, the method is applied to a rough model of the same shape in order to establish the equations governing its geometric and hydraulic characteristics. These equations are then secondly used to easily deduce the required linear dimensions of the current conduit by introducing a non-dimensional correction factor. Resulting RMM equations are not only explicit but also cover the entire domain of Moody diagram [6], corresponding to Reynolds number $R \geq 2300$ and relative roughness varying in the wide range [0; 0.05].

Unlike to current methods, the application of the MMR for designing conduits or channels does not require the introduction of the friction coefficient as defined by Colebrook-White neither Chezy's coefficient nor Manning's roughness coefficient which are determined with great difficulty. This is the major advantage of the method. The three basic equations of turbulent flow are then very easy to handle.

2. BASIC EQUATIONS

The relationships on which the study is based are simple well known hydraulic equations namely, Darcy-Weisbach equation, Colebrook-White equation and Reynolds number formula. The Colebrook-White relationship was established for the circular pipe but its application can be extended to all other geometric profiles [5]. The energy slope of a conduit or channel is given by the Darcy-Weisbach relationship as:

$$J = \frac{f}{D_h} \frac{Q^2}{2gA^2} \tag{1}$$

where Q is the discharge, g is the acceleration due to gravity, A is the wetted area, D_h is the hydraulic diameter and f is the friction factor given by the well known Colebrook-White formula as:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon / D_h}{3.7} + \frac{2.51}{R\sqrt{f}}\right) \tag{2}$$

where ε is the absolute roughness and R is the Reynolds number which can be expressed as :

$$R = \frac{4Q}{P\nu} \tag{3}$$

where ν is the kinematic viscosity and P is the wetted perimeter.

3. REFERENTIAL ROUGH MODEL

3.1. Relationship of the Height $\overline{Y}$

The application of the rough model method (RMM) is based, for the sake of calculation, on a referential rough model shown in Fig. **(1)**.

The rough model is characterized by three linear dimensions, namely the horizontal dimension $\overline{a}$ and the vertical dimensions $\overline{y}$ and $\overline{Y}$. The triangular section at the bottom of the conduit is characterized by a side slope m horizontal to 1 vertical. The rough model we consider is a pressurized rectangular conduit with triangular bottom characterized by $\overline{\varepsilon} / \overline{D_h} = 0.037$ as the arbitrarily assigned relative roughness value. The chosen relative roughness value is so large that the prevailed flow regime is fully rough. Thus, the friction factor is $\overline{f} = 1/16$ according to Eq. (2) for $R = \overline{R}$ tending to infinitely large value. Applying Eq. (1) to the rough model leads to:

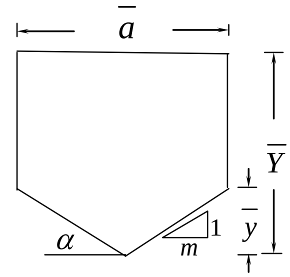

Fig. (1). Rough model of a pressurized rectangular shaped conduit with triangular bottom.

$$\overline{J} = \frac{\overline{f}}{\overline{D_h}} \frac{\overline{Q}^2}{2g\overline{A}^2} \tag{4}$$

Bearing in mind that $\overline{D_h} = 4\overline{A} / \overline{P}$ and $\overline{f} = 1/16$, Eq. (4) can be rewritten as:

$$\overline{J} = \frac{1}{128g} \frac{\overline{P}}{\overline{A}^3} \overline{Q}^2 \tag{5}$$

The wetted perimeter $\overline{P}$ and the water area $\overline{A}$ are expressed in different forms, depending on the chosen linear dimension. If one selected the linear dimension "$\overline{y}$", then $\overline{P}$ and $\overline{A}$ are respectively:

$$\overline{P} = 2\overline{y}\left(\frac{\overline{Y}}{\overline{y}} + m + \sqrt{1+m^2} - 1\right) \tag{6}$$

$$\overline{A} = m\overline{y}^2 \left(2\frac{\overline{Y}}{\overline{y}} - 1\right) \tag{7}$$

Inserting Eq. (6) and Eq. (7) into Eq. (5) leads to:

$$\left(\frac{z + \chi_1}{z^3 \chi_2}\right) \frac{\overline{Q}^{*2}}{128} = 1 \tag{8}$$

where :

$$\chi_1 = 2m + 2\sqrt{1+m^2} - 1 \tag{9}$$

$$\chi_2 = m^3 \tag{10}$$

$$z=\left(2\bar{\lambda}-1\right) \tag{11}$$

$$\bar{\lambda}=\frac{\bar{Y}}{\bar{y}} \tag{12}$$

$$\bar{Q}^{*}=\frac{\bar{Q}}{\sqrt{g\bar{J}\bar{y}^{5}}} \tag{13}$$

Eq. (12) and Eq. (13) express respectively the relative height of the rough model and the relative conductivity related to the vertical linear dimension $\bar{y}$. Note that for $z=1$, the relative height is $\bar{\lambda}=1$ according to Eq. (11). As a result, Eq. (12) indicates that $\bar{y}=\bar{Y}$. This configuration corresponds to a rough triangular shaped model whose height is equal to that of the rough model shown in Fig. (**1**). Accordingly, Eq. (8) becomes:

$$\left(\frac{1+\chi_1}{\chi_2}\right)\frac{\bar{Q}^{*2}}{128}=1 \tag{14}$$

where the relative conductivity $\bar{Q}^{*}$ can be written as:

$$\bar{Q}^{*}=\frac{\bar{Q}}{\sqrt{g\bar{J}\bar{y}^{5}}}=\frac{\bar{Q}}{\sqrt{g\bar{J}\bar{Y}^{5}}} \tag{15}$$

According to Eq. (14) and Eq. (15), one can deduce:

$$\bar{Y}=\left(\frac{1+\chi_1}{128\chi_2}\right)^{1/5}\left(\frac{\bar{Q}^{2}}{g\bar{J}}\right)^{1/5} \tag{16}$$

Assuming $\bar{Q}=Q$ and $\bar{J}=J$, Eq. (16) is rewritten as:

$$\bar{Y}=\left(\frac{1+\chi_1}{128\chi_2}\right)^{1/5}\left(\frac{Q^{2}}{gJ}\right)^{1/5} \tag{17}$$

For the given data *m*, *Q* and *J*, Eq. (17) allows to compute explicitly the height $\bar{Y}$ of the rough model shown in Fig. **(1)**.

3.2. Relationship of the relative height $\bar{\lambda}$

If one selected the linear dimension" $\bar{Y}$ ", the wetted perimeter $\bar{P}$ and the water area $\bar{A}$ of the rough model are respectively:

$$\bar{P}=\bar{Y}\left[2+\bar{\lambda}^{-1}\left(\chi_1-1\right)\right] \tag{18}$$

$$\bar{A}=m\bar{Y}^{2}\bar{\lambda}^{-1}\left(2-\bar{\lambda}^{-1}\right) \tag{19}$$

Inserting Eq. (18) and Eq. (19) into Eq. (5), one can write:

$$\bar{Q}^{*2}=\frac{128\chi_2\left(1-z\right)^{3}}{\left(1+\chi_1\right)\left[1-\left(\frac{\chi_1-1}{\chi_1+1}\right)\sqrt{z}\right]} \tag{20}$$

where :

$$\bar{Q}^{*}=\frac{\bar{Q}}{\sqrt{g\bar{J}\bar{Y}^{5}}} \tag{21}$$

$$z=\left(1-\bar{\lambda}^{-1}\right)^{2} \tag{22}$$

Referring to Eq. (14), Eq. (20) is reduced to:

$$\frac{\left(1-z\right)^{3}}{\left[1-\left(\frac{\chi_1-1}{\chi_1+1}\right)\sqrt{z}\right]}=1 \tag{23}$$

Eq. (23) is implicit with respect to the variable *z*. It particularly shows that *z*, and therefore $\bar{\lambda}$, depends solely on χ_1 and thus of *m*. From the only known value of *m*, it is then possible to calculate the relative height $\bar{\lambda}$ of the rough model. Some methods for solving the implicit Eq. (23) are presented in what follows.

The first method consists in applying to Eq. (23) the Lagrange's theorem [16] to derive an exact analytical expression for *z*. Before applying this theorem, write Eq. (23) as follows:

$$z=1-\left(1-\sigma\sqrt{z}\right)^{1/3} \tag{24}$$

where:

$$\sigma=\left(\frac{\chi_1-1}{\chi_1+1}\right)=\left(1+m-\sqrt{1+m^{2}}\right) \tag{25}$$

Eq. (24) is in the following form:

$$z=\ a_0+\theta\,\phi(z) \tag{26}$$

where:

$a_0 = 1$, $\theta = -1$, $\phi(z) = \left(1 - \sigma\sqrt{z}\right)^{1/3}$

According to Lagrange's theorem, the function $f(z)$ is given in terms of an infinite series as:

$$f(z) = f(a_0) + \sum_{i=1}^{\infty} \frac{\theta^i}{\Gamma(i+1)} \left\{ \frac{d^{i-1}}{dx^{i-1}} [f'(x)\phi^i(x)] \right\}_{x=a_0} \quad (27)$$

The function Γ is defined as $\Gamma(i+1) = i!$

According to Eq. (22) one can write:

$$f(z) = 1 - \sqrt{z} = \overline{\lambda}^{-1} \quad (28)$$

As a result, we can deduce that:

$$f(a_0) = 1 - \sqrt{1} = 0 \ , \ f'(x) = -\frac{1}{2\sqrt{x}}$$

This then allows us to write that:

$$\overline{\lambda}^{-1} = \sum_{i=1}^{\infty} \frac{(-1)^i}{\Gamma(i+1)} \left\{ \frac{d^{i-1}}{dx^{i-1}} \left(\frac{-\left[1 - \sigma\sqrt{x}\right]^{i/3}}{2\sqrt{x}} \right) \right\}_{x=1} \quad (29)$$

Eq. (29) is the exact solution of the parameter $\overline{\lambda}$. However, its convergence must be studied to answer the question at what order i should we truncate the series to obtain an acceptable value.

The second method we suggest is a numerical method which consists in approaching successively the solution of Eq. (23). The calculation process is iterative and operates on Eq. (24) after selecting a first value of z. Assume that the first value of z is z_0. According to Eq. (24), we obtain the next values of z such that:

$$z_1 = 1 - \left(1 - \sigma\sqrt{z_0}\right)^{1/3}$$

$$z_2 = 1 - \left(1 - \sigma\sqrt{z_1}\right)^{1/3}$$ …and so on.

The calculation process stops when z_i and z_{i+1} are sufficiently close. It is obvious that the speed of convergence of the described iterative process depends strongly on the value of z_0 initially selected. We suggest calculating the value of z_0 instead of choosing it with an arbitrary manner. The calculation showed that almost exact value of z is obtained, in the worst case, at the end of the eighth step of calculating for z_0 such that:

$$z_0 \approx \left(\frac{3\sigma}{9 - \sigma^2} \right)^2 \quad (30)$$

Remember that σ is calculated using Eq. (25) for the given value of *m*. Once the final value of z is calculated, the relative height $\overline{\lambda}$ is derived from Eq. (22).

The third method is to apply to the implicit Eq. (24) a limited development. The calculation showed that for all values of σ, the quantity $\sigma\sqrt{z}$ contained in the right member of the Eq. (24), is less than unity. This suggests applying a limited development whose the most appropriate order is 3. Thus, Eq. (24) can be written as:

$$z = 1 - \left(1 - \sigma\sqrt{z}\right)^{1/3} \approx 1 - \left(1 - \frac{\sigma}{3}\sqrt{z} - \frac{\sigma^2}{9}z - \frac{5\sigma^3}{81}z^{3/2}\right) \quad (31)$$

Eq. (31) can be rewritten as:

$$z - \frac{9(9 - \sigma^2)}{5\sigma^3}\sqrt{z} + \frac{27}{5\sigma^2} = 0 \quad (32)$$

By adopting the following change of variables:

$$\sqrt{z} = X \quad (33)$$

Eq. (32) becomes:

$$X^2 - \frac{9(9 - \sigma^2)}{5\sigma^3}X + \frac{27}{5\sigma^2} = 0 \quad (34)$$

We thus obtain a second order equation whose real solution is:

$$X = \frac{3}{10\sigma^3}\left(27 - \sqrt{729 - 162\sigma^2 - 51\sigma^4} - 3\sigma^2\right) \quad (35)$$

Taking into account Eq. (22) and Eq. (33), the relative height $\overline{\lambda}$ is expressed as:

$$\overline{\lambda}^{-1} = 1 - \frac{3}{10\sigma^3}\left(27 - \sqrt{729 - 162\sigma^2 - 51\sigma^4} - 3\sigma^2\right) \quad (36)$$

Eq. (36) allows computing the almost exact value of the relative height $\overline{\lambda}$ using the only known value of *m*.

Among the three methods we have just described, only the third one seems to be the most appropriate and most convenient.

4. NON-DIMENSIONAL CORRECTION FACTOR OF LINEAR DIMENSION

The RMM states that any linear dimension L of a conduit and the linear dimension $\overline{L}$ of its rough model are related by the following equation, applicable to the whole domain of the turbulent flow:

$$L = \psi \overline{L} \tag{37}$$

where ψ is a non-dimensional correction factor of linear dimension, less than unity, which is governed by the following relationship [1, 10] :

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon / \overline{D_h}}{4.75} + \frac{8.5}{\overline{R}}\right)\right]^{-2/5} \tag{38}$$

where $\overline{R}$ is the Reynolds number in the rough model given by:

$$\overline{R} = \frac{4Q}{\overline{P}\nu} \tag{39}$$

5. COMPUTATION STEPS OF LINEAR DIMENSIONS

To design the studied geometric profile shown in Fig. **(1)**, only the following data must be given:

Q, J, $m = \text{cotg}(\alpha)$, ε and ν. Note that no linear dimension is required as a given data of the problem. The following steps are recommended to compute the three linear dimensions of the conduit:

1. With the known value of *m*, compute χ_1, χ_2 and σ using Eq. (9), Eq. (10) and Eq. (25) respectively.
2. According to Eq. (17), compute the height $\overline{Y}$ of the rough model.
3. Using Eq. (36), compute the relative height $\overline{\lambda}$ and deduce the linear dimension $\overline{y}$ according to Eq. (12).
4. Compute the wetted perimeter $\overline{P}$ and the water area $\overline{A}$ of the rough model by the use of Eq. (18) and Eq. (19) respectively. Deduce the hydraulic diameter $\overline{D_h} = 4\overline{A}/\overline{P}$ and the Reynolds number $\overline{R}$ according to Eq. (39).
5. Using Eq. (38), the non-dimensional correction factor of linear dimension ψ is then worked out.
6. Finally, the vertical linear dimensions of the conduit are given by the fundamental Eq. (36) as:

$$Y = \psi \overline{Y}$$

$$y = \psi \overline{y}$$

From geometrical considerations operated on Fig. **(1)**, the horizontal linear dimension *a* is:

$$a = 2my$$

6. PRACTICAL EXAMPLE 1

Compute the linear dimensions of the pressurized rectangular conduit with triangular bottom shown in Fig. **(1)** for the following data:

$Q = 3.46\ m^3/s$, $J = 2\times 10^{-4}$, $\varepsilon = 10^{-3} m$

$\alpha = 30°$ $(m = 1.732050808)$, $\nu = 10^{-6} m^2/s$

Using Eq. (9), Eq. (10) and Eq. (25), the parameters χ_1, χ_2 and σ are respectively:

$$\chi_1 = 2m + 2\sqrt{1+m^2} - 1$$
$$= 2\times 1.732050808 + 2\times\sqrt{1+1.732050808^2} - 1$$
$$= 6.46410162$$

$$\chi_2 = m^3 = 1.732050808^3 = 5.19615242$$

$$\sigma = 1 + m - \sqrt{1+m^2}$$
$$= 1 + 1.732050808 - \sqrt{1+1.732050808^2}$$
$$= 0.73205081$$

According to Eq. (17), the height $\overline{Y}$ of the rough model is:

$$\overline{Y} = \left(\frac{1+\chi_1}{128\chi_2}\right)^{1/5}\left(\frac{Q^2}{gJ}\right)^{1/5}$$

$$= \left(\frac{1+6.4641016}{128\times 5.19615242}\right)^{1/5}\left(\frac{3.46^2}{9.81\times 2\times 10^{-4}}\right)^{1/5}$$

$$= 2.32866908\ m$$

According to Eq. (36), the relative height $\overline{\lambda}^{-1}$ is:

$$\overline{\lambda}^{-1} = \frac{\overline{y}}{\overline{Y}} = 0.73877621$$

This leads to:

$$\overline{y} = 2.32866908\times 0.73877621 = 1.72036533\ m$$

Using Eq. (18) and Eq. (19), the wetted perimeter $\overline{P}$ and the water area A of the rough model are respectively:

$$\overline{P} = \overline{Y}\left[2 + \overline{\lambda}^{-1}(\chi_1 - 1)\right]$$
$$= 2.32866908\times\left[2 + 0.73877621\times(6.4641016 - 1)\right]$$
$$= 14.0575891\,m$$

$$\overline{A} = m\overline{Y}^2\overline{\lambda}^{-1}\left(2 - \overline{\lambda}^{-1}\right)$$
$$= 1.732050808\times 2.32866908^2$$
$$\times 0.73877621\times(2 - 0.73877621) = 8.75147464\ m^2$$

Thus, the hydraulic diameter $\overline{D_h} = 4\overline{A} / \overline{P}$ is:

$$\overline{D_h} = 4 \times 8.75147464 / 14.0575891 = 2.49017795m$$

Using Eq. (39), the Reynolds number $\overline{R}$ is:

$$\overline{R} = \frac{4Q}{\overline{P}\nu} = \frac{4 \times 3.46}{14.0575891 \times 10^{-6}} = 984521.59$$

According to Eq. (38), the non-dimensional correction factor of linear dimension ψ is:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon / \overline{D_h}}{4.75} + \frac{8.5}{\overline{R}}\right)\right]^{-2/5} = 0.77300413$$

The required linear dimensions of the conduit are then:

$$Y = \psi\overline{Y} = 0.77300413 \times 2.32866908 \approx 1.8\ m$$

$$y = \psi\overline{y} = 0.77300413 \times 1.72036533$$

$$= 1.32984951 \approx 1.33\ m$$

$$a = 2my = 2 \times 1.732050808 \times 1.32984951$$

$$= 4.6073384\ m \approx 4.607\ m$$

This step aims to verify the validity of the calculations. To do this, determine from Eq. (5) the energy slope $\overline{J}$ which must be equal to J for $\overline{Q} = Q$. Hence:

$$\overline{J} = \frac{1}{128g}\frac{\overline{P}}{\overline{A}^3}Q^2 = \frac{14.0575891 \times 3.46^2}{128 \times 9.81 \times 8.75147464^3}$$

$$= 0.00019996 \approx 2 \times 10^{-4}$$

It is indeed the value of J given in the problem statement.

7. PRACTICAL EXAMPLE 2

This second example is proposed to show that the rough model method is also applicable in the turbulent smooth regime corresponding to a absolute roughness tending to zero. Data, different from those of Example 1, are considered in order to check once again the validity of the calculations resulting from the advocated method.

Compute the linear dimensions of the pressurized rectangular conduit with triangular bottom shown in Fig. (**1**) for the following data:

$Q = 5\ m^3 / s$, $J = 10^{-4}$, $\varepsilon \to 0$, $\alpha = 45°$ $(m = 1)$, $\nu = 10^{-6} m^2 / s$

1. Using Eq. (9), Eq. (10) and Eq. (25), the parameters χ_1, χ_2 and σ are respectively:

$$\chi_1 = 2m + 2\sqrt{1+m^2} - 1 = 2 \times 1 + 2 \times \sqrt{1+1^2} - 1$$

$$= 3.82842712$$

$$\chi_2 = m^3 = 1^3 = 1$$

$$\sigma = 1 + m - \sqrt{1+m^2} = 1 + 1 - \sqrt{1+1^2}$$

$$= 0.58578644$$

2. According to Eq. (17), the height $\overline{Y}$ of the rough model is:

$$\overline{Y} = \left(\frac{1+\chi_1}{128\chi_2}\right)^{1/5}\left(\frac{Q^2}{gJ}\right)^{1/5}$$

$$= \left(\frac{1+3.82842712}{128 \times 1}\right)^{1/5}\left(\frac{5^2}{9.81 \times 10^{-4}}\right)^{1/5}$$

$$= 3.94978397\ m$$

3. According to Eq. (36), the relative height $\overline{\lambda}^{-1}$ is:

$$\overline{\lambda}^{-1} = \frac{\overline{y}}{\overline{Y}} = 0.79646353$$

This leads to:

$$\overline{y} = 3.94978397 \times 0.79646353 = 3.14585887\ m$$

Using Eq. (18, 19), the wetted perimeter $\overline{P}$ and the water area A of the rough model are respectively:

$$\overline{P} = \overline{Y}\left[2 + \overline{\lambda}^{-1}(\chi_1 - 1)\right]$$

$$= 3.94978397 \times \left[2 + 0.79646353 \times (3.82842712 - 1)\right]$$

$$= 16.7974005\ m$$

$$\overline{A} = m\overline{Y}^2\overline{\lambda}^{-1}\left(2 - \overline{\lambda}^{-1}\right)$$

$$= 1 \times 3.94978397^2 \times 0.79646353 \times (2 - 0.79646353)$$

$$= 14.9544978\ m^2$$

Using Eq. (39), the Reynolds number $\overline{R}$ is:

$$\overline{R} = \frac{4Q}{\overline{P}\nu} = \frac{4 \times 5}{16.7974005 \times 10^{-6}} = 1190660.42$$

4. According to Eq. (38) for $\varepsilon \to 0$, the non-dimensional correction factor of linear dimension ψ is:

$$\psi = 1.35\left[-\log\left(\frac{8.5}{\overline{R}}\right)\right]^{-2/5} = 0.70102483$$

5. The required linear dimensions of the conduit are then:

$$Y = \psi \overline{Y} = 0.70102483 \times 3.94978397 \approx 2.769\ m$$

$$y = \psi \overline{y} = 0.70102483 \times 3.14585887$$

$$= 2.20532518 \approx 2.205\ m$$

$$a = 2my = 2 \times 1 \times 2.20532518$$

$$= 4.41065035\ m \approx 4.410\ m$$

6. This step aims to verify the validity of the calculations. To do this, determine from Eq. (5) the energy slope $\overline{J}$ which must be equal to J for $\overline{Q} = Q$. Hence:

$$\overline{J} = \frac{1}{128g} \frac{\overline{P}}{\overline{A}^3} Q^2 = \frac{14.0575891 \times 5^2}{128 \times 9.81 \times 14.9544978^3}$$

$$= 0.000099997 \approx 10^{-4}$$

It is indeed the value of J given in the problem statement.

8. RESULTS

The three basic equations of turbulent flow are easy to handle when they are applied to the rough model, particularly the Colebrook-White relationship which induces a known and constant friction factor. Mathematical transformations have made them explicit and simple. The most significant result is the fact that the relative vertical height of the rough model depends solely on the known parameter m. Moreover, the calculation of the three linear dimensions of the studied conduit is not linked to the knowledge of one of these. This calculation depends solely on the discharge Q, the energy slope J, the side slope m, the absolute roughness ε and the kinematic viscosity ν. The required linear dimensions of the conduit are obtained by multiplying the homologues linear dimensions of the rough model by a non-dimensional correction factor.

CONCLUSION

The rough model method (RMM) was successfully applied to design a pressurized rectangular shaped conduit with triangular bottom. The method was firstly applied to a rough model of the same shape characterized by well known parameters. This led to the establishment of explicit equations relating the characteristics of the flow in the rough domain. The explicit form of these equations was made possible through mathematical transformations. By introducing the non-dimensional correction factor of linear dimensions, these equations were secondly used to derive the required linear dimensions of the studied conduit. It emerged from this study that the design of such a conduit does not require any linear dimension as a given data of the problem. The problem of the design was completely solved with the bare minimum of data measurable in practice. All established relationships are valid in the entire domain of the turbulent flow.

The theoretical development as well as the calculation example we proposed show no restriction in the application of the rough model method. However, it should be applied to other shapes of geometric profiles to observe its scalability and performance. Its application to open channels should also be investigated to solve the problem of the normal depth which remains relevant.

NOTATION

A	=	Water area
a	=	Horizontal linear dimension
D_h	=	Hydraulic diameter
f	=	Friction factor
g	=	Acceleration due to gravity
J	=	Energy slope
L	=	Linear dimension
m	=	Side slope
P	=	Wetted perimeter
Q	=	Discharge
Q^*	=	Relative conductivity
R	=	Reynolds number
R_h	=	Hydraulic radius
y	=	Vertical linear dimension
Y	=	Vertical linear dimension
ε	=	Absolute roughness
ψ	=	Non-dimensional correction factor
λ	=	Relative vertical height
ν	=	kinematic viscosity

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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Decleared none.

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