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Analytic Solutions of Shear Lag on Steel-Concrete Composite T－girder under Simple Bending

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Abstract: The composite T-girders include concrete flange plates and steel beams, which are connected by shear connectors. The longitudinal stress about concrete plate is due to non-uniform distribution on cross section because of the shear lag effect. A differential equation of longitudinal forces at transverse section flange and cantilever flange is separately established according to the strain compatibility and the force equilibrium conditions about a composite T-girder. The method of separation of variables is used to solve the differential equation about the simply supported composite T-girder. The shear lag coefficient is detrmined by the ratio between stress calculated by this method and stress odetrmined by elementary beam theory. An example of such calculation is given to approve its applicability.

Keywords: Composite T-girders, longitudinal stress, differential equations, shear lag coefficient, steel-concrete.

1. INTRODUCTION

The composite T-girders are commonly used in construction, because of the mechanical characteristic of steel and concrete, namely the use of concrete compressive and steel tensile capacity. It is well known that the uneven deformation of the wider T-girder flanges can produce an uneven distribution of the longitudinal stresses [1] under symmetrical bending. The shear lag effect can result in obvious increase in the longitudinal stress near the edge of the flange and cause stress concentration. Due to shear lag effects, it can cause stress concentration in structure, leading to structural damage [2].

Guo jinqiong, Wei Lina, etal. put forward some practical theories of computation and computational methods, such as variational methods [3, 4], finite strip method [5, 6] and finite element method [6-8]. Lawrence F K, Adam S, Khaled M. Sennah *et al.* researched shear resistance of the surface of the steel-concrete composite beams with cantilever flange [9, 10].

Those methods have different characteristics, but have complex calculation and analysis to calculate composite section. The same method is used to calculate shear lag effect of composite box girders in reference [6]. It is also applied to calculate shear lag effect of the composite T-girder. Parameters of a composite T-girder are shown in Fig. (**1**).

2. DIFFERENTIAL EQUATIONS ABOUT FLANGE EQUILIBRIUM

As shown in Fig. (**1**), it is assumed that relative slip between the concrete plate and steel girder is not observed

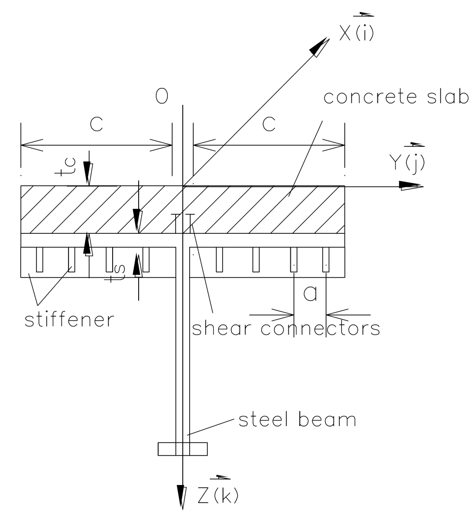

Fig. (1). A composite T-girder.

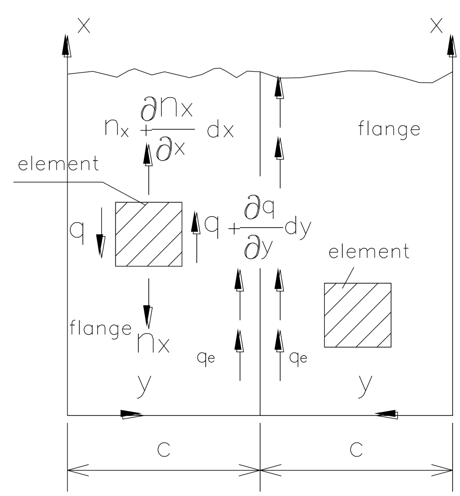

Fig. (2). A flange element of thel T-girder.

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under symmetrical bending. Both of them bear the applied loads. Any element of the flange, as shown in Fig. (**2**), bears a shear flow q_e and a normal force $n_{x,i}$ (i=1 represents one cantilever flange, i=2 represents the other). It is assumed that the shear flow q_e is resisted by the composite flange plate itself, without considering help from the stiffeners. But both the composite flange plate and stiffeners bear the normal longitudinal forces. Without special provisions, the subscript i has the same meaning as the subscript i of $n_{x,i}$ in the followings.

2.1. Stress and Strain

In the elastic range, an equivalent thickness t_s of steel flanges can be obtained according to the Match-plate method.Following this, an equivalent thickness $\bar{t}$ for the composite flange can be obtained according to the equivalent conversion principle, where concrete slab thickness of flange is converted into steel equivalent thickness.

$$\bar{t} = t_1 + \frac{A_s}{a} + \frac{A_{sx}}{a_1} + t_c \frac{E_c}{E_s} = t_s + t_c \frac{E_c}{E_s} \tag{1}$$

As shown in Fig. (**1**), t_1 represents the steel plate thickness of flange, a and a_1 are the spacing of longitudinal and transverse stiffener respectively, A_s is the transverse section area of each longitudinal stiffener, A_{sx} is the transverse section area of each transverse stiffener,*and* t_c is the thickness of the concrete layer. E and E_c represent the Young's moduli of steel and concrete, respectively.

As shown in Fig. (**2**), the direct strains in the longitudinal direction are then given by,

$$\varepsilon_{x,i} = \frac{\sigma_{x,i}}{E_s} = \frac{n_{x,i}}{\bar{t}E_s} \qquad i = 1,2 \tag{2}$$

Also, it may be assumed that the transverse direct strain in the steel plate and concrete slab are the same so that

$$\varepsilon_{y,i} = -\frac{\upsilon_s n_{xs,i}}{t_s E_s} = -\frac{\upsilon_c n_{xc,i}}{t_c E_c} \qquad i = 1,2 \tag{3}$$

Where $\sigma_{x,i}$ represents the x-direction stress of different element,υs andυ$_c$ express the Poisson's ratio of the steel and concrete respectively; $n_{xs,i}$ and $n_{xc,i}$ are the sections of the normal force ($n_{x,i}$) carried by the steel and concrete components respectively, such that:

$$n_{x,i} = n_{xs,i} + n_{xc,i} \qquad i = 1,2 \tag{4}$$

According to equations (3),(4), equation (5) gives

$$n_{xs,i} = \frac{\dfrac{\upsilon_c n_{x,i}}{t_c E_c}}{\dfrac{\upsilon}{t_{s,i} E_s} + \dfrac{\upsilon_c}{t_c E_c}} \qquad i = 1,2 \tag{5}$$

By substitution from equations (3),(5), $\varepsilon_{y,i}$ can be written as

$$\varepsilon_{y,i} = -\upsilon_s \upsilon_c \frac{n_{x,i}}{\upsilon_c t_s E_s + \upsilon_s t_c E_c} \qquad i = 1,2$$

It can be simplified by using an 'equivalent' Poisson's ratio (r), with equation(6) given as.

$$\varepsilon_{y,i} = -r\frac{n_{x,i}}{\bar{t}\ E_s} = -r\varepsilon_{x,i} \qquad i = 1,2 \tag{6}$$

Where, $r = \dfrac{\upsilon_s \upsilon_c (t_s E_s + t_c E_c)}{\upsilon_c t_s E_s + \upsilon_s t_c E_c} \qquad i = 1,2$.

2.2. Shear Stress and Strain

It is assumed that no relative slippage between the shear strains(γ_i) of elements about steel and concrete slab are the same, then

$$\gamma_i = \frac{q_{s,i}}{t_1 G_s} = \frac{q_{c,i}}{t_c G_c} \qquad (i = 1,2) \tag{7}$$

Where, G_s and G_c express the shear moduli of the steel and concrete, respectively. $q_{s,i}$ and $q_{c,i}$ are the sections of the shear flow(q_i) carried by the steel and concrete components, respectively. The sum of $q_{s,i}$ and $q_{c,i}$ equals the shear flow(q_i). By substitution, equation (8) becomes

$$\gamma_i = \frac{q_i}{t_1 G_s + t_c G_c} \qquad i = 1,2 \tag{8}$$

That may be written as:

$$\gamma_i = \frac{q_i}{t_1^* G_s} \qquad i = 1,2 \tag{9}$$

Where the equivalent thickness of the shear action is given by:

$$t_1^* = t_1 + t_c \frac{G_c}{G_s}$$

2.3. Compatibility and Equilibrium Equations

As shown in Fig. (**2**), the equilibrium of the flange element in the longitudinal direction is given by

$$\frac{\partial n_{x,i}}{\partial x} + \frac{\partial q_i}{\partial y} = 0 \qquad i = 1,2 \tag{10}$$

As shown in Fig. (**1**), the composite flange can be regarded as a plane stress problem, by the Hooke's Law [3]. The equation controling the condition of compatibility may be written as [11]

$$\frac{\partial^2 \varepsilon_{x,i}}{\partial y^2} + \frac{\partial^2 \varepsilon_{y,i}}{\partial x^2} = \frac{\partial^2 \gamma_i}{\partial x \partial y} \qquad i = 1,2 \tag{11}$$

Substituting equations (2), (6) and (9) for calculating the strains, equation (11) becomes

$$\frac{\partial^2 n_{x,i}}{\partial y^2} - r\frac{\partial^2 n_{x,i}}{\partial x^2} = \frac{\bar{t}\ E_s}{t_1^* G_s}\frac{\partial^2 q_i}{\partial x \partial y} \qquad i = 1,2$$

and the shear flow q_i may be substituted by the normal force $n_{x,i}$ from equation (10), thus,

$$\frac{\partial^2 n_{x,i}}{\partial y^2}+\left(\frac{\bar{t}\ E_s}{t_1^* G_s}-r\right)\frac{\partial^2 n_{x,i}}{\partial x^2}=0 \qquad i=1,2 \tag{12}$$

3. SOLUTION OF DIFFERENTIAL EQUATION

For the partial differential equation, the normal force $n_{x,i}(x,y)$ can be written as the fourier series by adopting the method of separation of variables [8] under the simply supported border, such that

$$n_{x,i}(x,y)=\sum_{k=1}^{\infty} N_{k,i}(y)\sin\frac{k\pi x}{L} \qquad i=1,2 \tag{13}$$

Where, L represents the length of the simply supported span and $N_{k,i}(y)$ expresses an amplitude function which represents the variation in the cross-section direction of the normal force. Where, x represents the distance from the origin to the calculated section. k is the series term [11].

By substituting the k^{th} term of the series into equation (13), the ordinary differential equation is written as:

$$\frac{d^2 N_{k,i}}{dy^2}-(\xi_k)^2 N_{k,i}=0 \qquad i=1,2$$

Where, $\xi_k=\frac{k\pi}{L}\sqrt{\frac{\bar{t}\ E_s}{t_1^* G_s}-r}$, the common solution of the equation is written as:

$$N_{k,i}(y)=C_{i,k1}\cosh\xi_k y+C_{i,k2}\sinh\xi_k y \qquad i=1,2 \tag{14}$$

Where the coefficient $C_{i,k1}$ and $C_{i,k2}$ are constants obtained by supporting and loading conditions at the longitudinal edges of the flange. As shown in Fig. (**1**), the normal force $n_{x,i}(x,y)$ with shear lag has a symmetric distribution in the cross section under symmetrical bending. As shown in Fig. (**2**), the coefficient $C_{i,k2}$ is zero due to symmetry. The constant $C_{i,k1}$ can be determined according to the conditions of shear flow equivalent and normal force continuity. By substituting equation (14) into equation(13), the normal longitudinal force can be expressed as:

$$n_{x,i}(x,y)=\sum_{k=1}^{\infty} C_{i,k1}\cosh\xi_k y\sin\frac{k\pi x}{L} \qquad i=1,2 \tag{15}$$

The normal longitudinal force of the cut-off point about two cantilever flange must meet continuous conditions. Following this, the normal longitudinal force is written as:

$$\sum_{k=1}^{\infty} C_{1,k1}\cosh\xi_k c\sin\frac{k\pi x}{L}=\sum_{k=1}^{\infty} C_{2,k1}\cosh\xi_k c\sin\frac{k\pi x}{L} \tag{16}$$

By substituting equation(15) into equation(10), the equation of the first-order derivative of y on q_i is given as:

$$\frac{\partial q_i}{\partial y}=-\frac{\partial n_{x,i}}{\partial x}=-\sum_{k=1}^{\infty}\frac{k\pi}{L}C_{i,k1}\cosh\xi_k y\cos\frac{k\pi x}{L} \qquad i=1,2$$

By integrating the y and noting the value of ξ_k, the shear flow at any point may be written as:

$$q_i(x,y)=-\frac{k\pi}{\xi_k L}\sum_{k=1}^{\infty} C_{i,k1}\sinh\xi_k y\cos\frac{k\pi x}{L} \qquad i=1,2 \tag{17}$$

At the center of T-girder flange(where $y=c$) as shown in Fig. (**2**), the sum of the shear flow on two cantilever flanges equals the shear flow (q_e) transfered from the web to the flange, which can be expressed as:

$$q_e(x)=q_{e,1}(x)+q_{e,2}(x) \tag{18}$$

Where, the shear flow $q_{e,1}(x)$ and $q_{e,2}(x)$ express the shear flow transfered from the web to the flange, which can be expressed as :

$$q_e(x)=q_{e,1}(x)+q_{e,2}(x)=-\frac{k\pi}{\xi_k L}\sum_{k=1}^{\infty} C_{1,k1}\sinh\xi_k c\cos\frac{k\pi x}{L} -\frac{k\pi}{\xi_k L}\sum_{k=1}^{\infty} C_{2,k1}\sinh\xi_k c\cos\frac{k\pi x}{L} \tag{19}$$

The shear flow (q_e) can be approximately obtained from the elementary beam theory. It can be written as:

$$q_e(x)=V(x)\frac{\bar{t}be}{2I}$$

Where, $V(x)$ is the total shear force imposed on the T-girder cross section at the position x, and I is the inertial moment of the cross section, e is the distance from the neutral axial to the centroidal axis of the flange. For the simply supported span, the shear flow $q_e(x)$ can be expressed by the fourier series, such that

$$q_e(x)=\sum_{k=1}^{\infty} Q_{ek}\cos\frac{k\pi x}{L} \tag{20}$$

Where, $Q_{ek}=\frac{2}{L}\int_0^l q_e(x)\cos\frac{k\pi x}{L}dx=\frac{\bar{t}be}{IL}\int_0^L V(x)\cos\frac{k\pi x}{L}dx$.

According to the equations(16), (19) and (20), the constant $C_{i,k1}$ can be obtained, such that

$$C_{1,k1}=C_{2,k1}=-\frac{Q_{ek}L\xi_k}{2k\pi\sinh\xi_k c}$$

By substituting the constant $C_{i,,k1}$ into equation (13) and equation (14), the normal force $n_{x,i}(x,y)$ may beobtianed. Thus, the normal stresses of the steel flange and the concrete plate can respectively be given as:

$$\begin{aligned}\sigma_{sx,i}(x,y)&=\frac{n_{x,i}(x,y)}{\bar{t}} \qquad i=1,2\\ \sigma_{cx,i}(x,y)&=\sigma_{sx,i}(x,y)\frac{E_c}{E_s} \qquad i=1,2\end{aligned} \tag{21}$$

The shear stress of the steel flange and the concrete plate can be given as:

$$\begin{aligned}\tau_{sx,i}(x,y)&=\frac{q_i(x,y)}{t_1^*} \qquad i=1,2\\ \tau_{cx,i}(x,y)&=\tau_{sx,i}(x,y)\cdot\frac{G_c}{G_s} \qquad i=1,2\end{aligned} \tag{22}$$

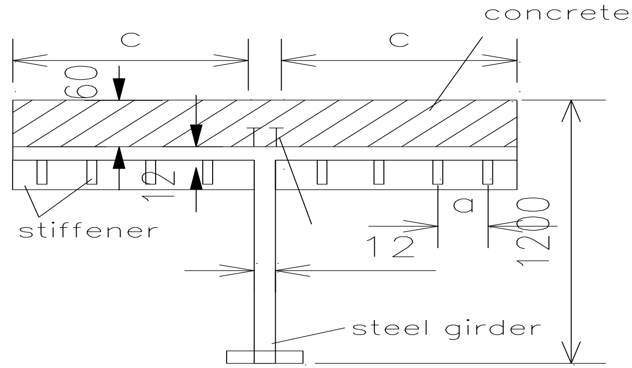
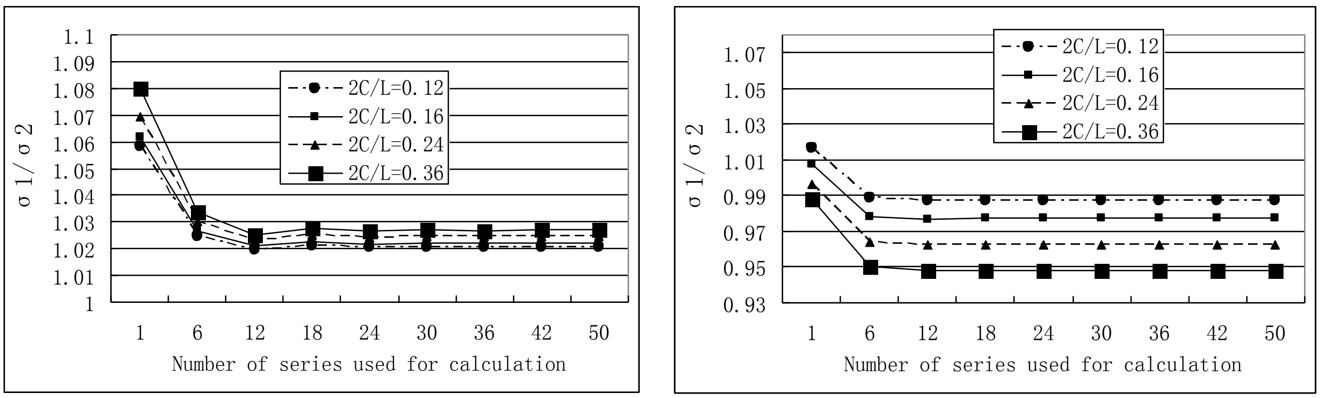


Fig. (3). Cross section of the example.



(**a**) Stress of cross point between web and flange (**b**) Stress of lateral point about flange

Fig. (4). Relationship between width-span ratio ,number of term in series and stress-ratio about cross section of midspan (σ_1-analytic solutions of this text, σ_2-Solutions of primary beam theory).

Table 1. Parameters of the Example

Parameter	Uniform Load w (N/mm)	Span L (m)	Steel Elastic Modulus E (N/mm^2)	Concrete Elastic Modulus E_c (N/mm^2)	Steel Poisson's Ratio υ	Concrete Poisson's Ratio υc
Value	50	10	210000	32500	0.27	0.2

4. CALCULATION AND CONCLUSION

According to the approximate calculation method for shear lag effect of composite T-girder, a simple manual or the program can obtain the shear lag coefficient of flange. In order to analyze the convergency, precision of the method and the parametric influence about the shear lag effect of composite T-girder should be known. The example of the transverse section is shown in Fig. (**3**). The area of the longitudinal and horizontal stiffeners is 100Square millimeters. The depth of concrete flange is 60mm. The height of the composite T-girder is 1200mm. The thickness of the steel slab is 12mm, and other parameters are illustrated in Table **1**. According to the ratio of wide-span, the flange width c is 600mm、800mm、1200mm and 1800mm.

Fig. (**4**), shows the relationship between width-span ratio , number of terms in series and the stress-ratio of flange about mid-span section, by using the method to calculate the shear lag effect, where the vertical coordinate expresses the ratio of the longitudinal stress to the concrete slab based on elementary beam theory.

As shown in Fig. (**4**), the convergency is quite fast for a uniformly distributed load. In addition, the wide-span ratios of T-girder flange have little influence on the convergency, where the convergency of the longitudinal stress in the concrete plate of mid-span is shown for different wide-span ratios and the shear lag coefficient(λ) is also defined. Shear lag coefficients of concrete slab increase or reduce with respect to its wide-span ratio.

Fig. (**5**), expresses the normal longitudinal distribution of concrete plate stress, where the finite elements method (*FEM*) refers to *ANSYS* program. The calculated values with FEM are larger than that with the proposed method because of stress concentration at the intersection with the flange of the Web.

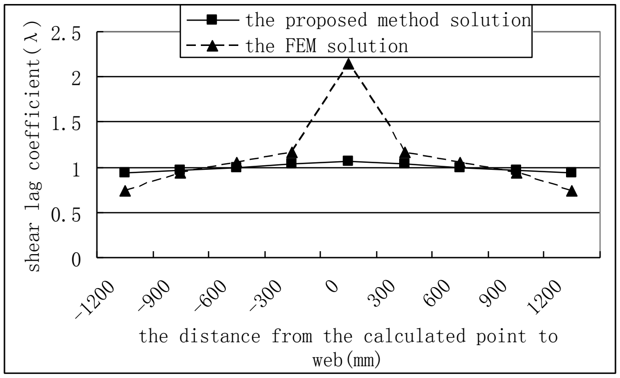


Fig. (5). Transverse distribution of concrete plate stress about mid-span.

CONCLUSION

The proposed method for solving the shear lag effect of the composite T-girders has fast convergency from simple calculations. The precision of result is sufficient for engineering.This has great effect on the shear lag effect of the composite T-girder. And the shear lag coefficient (λ) increases or decreases with the change in the wide-span ratios. Thus, more attention should be paid to the wider T-girders. The method can also be applied for multi-span girders which may be divided into the simply supported and cantilevered girders .

CONFLICT OF INTEREST

The author confirm that this article content has no conflict of interest.

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REFERENCES

[1] X. Cheng, "*Bridges theory and computation*," People Communication Publisher, P. R. China, Beijing, pp. 208-213, 1990.

[2] S.D. Zhang, X.H. Deng, and W.Z. Wang, "*Shear Lag Effect of Box Thin Wall Girders,*" People Communication Publisher, P.R. China, Beijing, pp.20-40, 1997.

[3] J. Guo, Z. Fang, and X. Luo, "Analysis of shear lag effect of box girder bridge," *Journal of Civil Engineering*, vol. 16, no. 1, pp. 1-4, Jan. 1983.

[4] F.L. Wei, and T.Y. Fang, "An approximate method for calculating shear lag in variable cross-section box girders," *Journal of Civil Engineering*, vol. 30, no. 1, pp. 65-68, Jan. 1997.

[5] J. y. Huang, and Y. B. Yang, "Analysis of shear lag effect of multi-cell box girder with generalized finite stripe method," *Bridge Construction*, vol. 4, pp. 40-43, Aug.1995.

[6] V. Kristek, "A shear lag analysis for composite box girder," *Journal of Construction, Steel Research*, vol.16, no.1, pp.15-17, Jan.1990.

[7] J.Y. Huang, Y.B. Yang, and T. Qian, "River second bridge of variable cross-section continuous box girder with shear lag effect of finite element analysis," *Bridge Construction*, vol. 1, pp. 70-73, Jan.1994.

[8] Y. Jiang, S. Xu, and L. Huang, "Nonlinear FEM analysis of steel fiber reinforced concrete composite beam and the computer simulation of the entire failing process," *Journal of Sichuan University (Engineering Sciences)*, vol. 42, no. 1, pp. 80-86, Jan. 2010.

[9] K.F. Lawrence, and S. Adam, "Interface shear in high strength composite T-beams," *PCI Journal*, vol. 49, no. 4, pp. 102-110, Aug. 2004.

[10] K.M. Sennah, X. Zhang, and J.B. Kennedy, Discussion of "Impact Factors for Horizontally Curved Composite Box Girder Bridges," *Journal of Bridge Engineering*, vol. 11, no. 3, pp. 378-379, Jun. 2006.

[11] Z.L. Xu, "*Elastic mechanics,*" Higher Education Press, P.R. China, Beijing, pp. 12-30, 2006.

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