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Perfectively Deducing Bessel Mean Square Error Formula

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Abstract: In the survey teaching materials of China, deducing Bessel mean square error formula is all based on survey values with same mathematical expectation. These methods aren't perfect. So, based on survey values without same mathematics expectation to prove Bessel mean square error formula is very necessary. Therefore, considering different mathematical expectation, it is meaningful that this paper has perfectively deduced Bessel mean square error formula.

Keywords: Bessel mean square error formula, survey values with different mathematics expectation, deducing.

1. INTRODUCTION

Bessel mean square error formula

$$m=\pm\sqrt{\frac{[vv]}{n-1}} \tag{1}$$

is a sort of basic formula about survey error. Where, m is mean square error, n is quantity of survey values, v_i $(i=1,2,\cdots,n)$ is residual of survey values, and in survey teaching materials, $[vv]$ generally represents $\sum_{i=1}^{n} v_i^2$.(Behind of this paper, the meaning of $[vv]$ still is $\sum_{i=1}^{n} v_i^2$.).

But, when we deduce Bessel mean square error formula in survey teaching materials of China, we often select survey values with same mathematics expectation [1-18], and generally survey values with different mathematics expectation are not selected. The literatures [4,6,8,12-15,17] are all national programming teaching materials of Chinese common higher education, and literatures [12,13,15,17] are all called as Chinese excellent surveying teaching materials. But these methods may give students viewpoint that Bessel mean square error formula based on survey values with different mathematics expectation cannot be deduced. Though the viewpoint is wrong, because nobody has finished generally deducing Bessel mean square error formula, but the viewpoint is still used for Bessel mean square error formula, and influenced survey teaching materials and students of China. So, in this paper, it is meaningful to perfectly deduce Bessel mean square error formula.

2. PERFECTIVELY DEDUCING BESSEL MEAN SQUARE ERROR FORMULA

If there are equal precision independent surveying values such as $l_1,l_2,\cdots,l_n$, their residuals are $v_1,v_2,\cdots,v_n$,their mathematics expectation are $\mu_1,\mu_2,\cdots,\mu_n$, their variance is σ^2, and their density is [19]

$$f(l_i)=\frac{1}{\sqrt{2\pi}\sigma}\exp\frac{-(l_i-\mu_i)^2}{2\sigma^2} \quad (i=1,2,\cdots,n) \tag{2}$$

2.1. $\mu_1\neq\cdots\neq\mu_i\neq\cdots\neq\mu_n$ ($i=1,2,\cdots,n$) isn't true

When $\mu_1\neq\cdots\neq\mu_i\neq\cdots\neq\mu_n$ ($i=1,2,\cdots,n$) isn't true, based on Eq.2, we can receive maximum likelihood function[19].

$$f\left(l/\mu_1,\mu_2,\cdots,\mu_n,\sigma^2\right)=\prod_{i=1}^{n}f(l_i)=\frac{1}{\left(\sqrt{2\pi}\right)^n\sigma^n}\exp\left[-\sum_{i=1}^{n}\frac{(l_i-\mu_i)^2}{2\sigma^2}\right] \tag{3}$$

Then,

$$\ln f\left(l/\mu_1,\mu_2,\cdots,\mu_n,\sigma^2\right)=-n\ln\sqrt{2\pi}-\frac{n}{2}\ln\sigma^2-\sum_{i=1}^{n}\frac{(l_i-\mu_i)^2}{2\sigma^2} \tag{4}$$

2.1.1. μ_i ($i=1,2,\cdots,n$) is already known

From Eq.4, we can get

$$\left.\frac{\partial\ln f\left(l/\mu_1,\mu_2,\cdots,\mu_n,\sigma^2\right)}{\partial\sigma^2}\right|_{\sigma^2=\hat{\sigma}^2}=-\frac{n}{2\hat{\sigma}^2}+\sum_{i=1}^{n}\frac{(l_i-\mu_i)^2}{2\hat{\sigma}^4}=0 \tag{5}$$

Therefore;

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$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}\left(l_i - \mu_i\right)^2}{n} \tag{6}$$

Then;

$$m = \hat{\sigma} = \pm\sqrt{\frac{\sum_{i=1}^{n}\left(l_i - \mu_i\right)^2}{n}} = \pm\sqrt{\frac{\sum_{i=1}^{n} v_i^2}{n}} = \pm\sqrt{\frac{[vv]}{n}} \tag{7}$$

If we want Eq.7 to have an unprejudiced character, we should change Eq.7 into Eq.8

$$m = \pm\sqrt{\frac{[vv]}{n\text{-}1}} \tag{8}$$

2.1.2. μ_i ($i = 1,2,\cdots,n$) *isn't already known*

From Eq.4, we can get

$$\left.\frac{\partial \ln f\left(l \middle/ \mu_1,\mu_2,\cdots,\mu_n,\sigma^2\right)}{\partial \sigma^2}\right|_{\sigma^2=\hat{\sigma}^2,\mu_i=\hat{\mu}_i} = -\frac{n}{2\hat{\sigma}^2} + \sum_{i=1}^{n}\frac{\left(l_i - \hat{\mu}_i\right)^2}{2\hat{\sigma}^4} = 0 \tag{9}$$

$$\left.\frac{\partial \ln f\left(l \middle/ \mu_1,\mu_2,\cdots,\mu_n,\sigma^2\right)}{\partial \mu_i}\right|_{\sigma^2=\hat{\sigma}^2,\mu_i=\hat{\mu}_i} = \frac{l_i - \hat{\mu}_i}{\hat{\sigma}^2} = 0 \tag{10}$$

Therefore;

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}\left(l_i - \hat{\mu}_i\right)^2}{n} \tag{11}$$

$$\hat{\mu}_i = l_i (i = 1,2,\cdots,n) \tag{12}$$

From Eq.11, we can get

$$m = \hat{\sigma} = \pm\sqrt{\frac{\sum_{i=1}^{n}\left(l_i - \hat{\mu}_i\right)^2}{n}} = \pm\sqrt{\frac{\sum_{i=1}^{n} v_i^2}{n}} = \pm\sqrt{\frac{[vv]}{n}} \tag{13}$$

If we want Eq.13 to have an unprejudiced character, we should change Eq.13 into Eq.14

$$m = \pm\sqrt{\frac{[vv]}{n\text{-}1}} \tag{14}$$

So, we can say that when $\mu_1 \neq \cdots = \mu_i \neq \cdots \neq \mu_n$ ($i = 1,2,\cdots,n$) isn't true, and they aren't already known, Bessel mean square error formula has been testified in this paper. But, no matter how bad or good survey values are, when we take Eq.12 into Eq.14 or Eq.13, we can only obtain in a certain case:

$$m = 0 \tag{15}$$

This is a kind of special circumstance.

2.2. $\mu_1 = \cdots = \mu_i = \cdots = \mu_n$ ($i = 1,2,\cdots,n$) is true

Because independent surveying values ($l_1, l_2, \cdots, l_n$) have equal precision, and $\mu_1 = \cdots = \mu_i = \cdots = \mu_n$ ($i = 1,2,\cdots,n$) is true, we can assume survey values come from same mother body and assume $\mu_i = \mu (i = 1,2,\cdots,n)$. So, we can receive maximum likelihood function based on Eq.2 [19].

$$f\left(l \middle/ \mu_1,\mu_2,\cdots,\mu_n,\sigma^2\right) = \prod_{i=1}^{n} f(l_i) = \frac{1}{\left(\sqrt{2\pi}\right)^n \sigma^n}\exp\left[-\sum_{i=1}^{n}\frac{\left(l_i-\mu\right)^2}{2\sigma^2}\right] \tag{16}$$

Then;

$$\ln f\left(l \middle/ \mu_1,\mu_2,\cdots,\mu_n,\sigma^2\right) = -n\ln\sqrt{2\pi} - \frac{n}{2}\ln\sigma^2 - \sum_{i=1}^{n}\frac{\left(l_i-\mu\right)^2}{2\sigma^2} \tag{17}$$

2.2.1. μ_i ($i = 1,2,\cdots,n$) *is already known*

When μ_i ($i = 1,2,\cdots,n$) is already known, we can get Eq.18 from Eq.17

$$\left.\frac{\partial \ln f\left(l \middle/ \mu_1,\mu_2,\cdots,\mu_n,\sigma^2\right)}{\partial \sigma^2}\right|_{\sigma^2=\hat{\sigma}^2} = -\frac{n}{2\hat{\sigma}^2} + \sum_{i=1}^{n}\frac{\left(l_i - \mu\right)^2}{2\hat{\sigma}^4} = 0 \tag{18}$$

Therefore;

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}\left(l_i - \mu\right)^2}{n} \tag{19}$$

Then;

$$m = \hat{\sigma} = \pm\sqrt{\frac{\sum_{i=1}^{n}\left(l_i - \mu\right)^2}{n}} = \pm\sqrt{\frac{\sum_{i=1}^{n} v_i^2}{n}} = \pm\sqrt{\frac{[vv]}{n}} \tag{20}$$

If we want Eq.20 to have an unprejudiced character, we should change Eq.20 into Eq.21

$$m = \pm\sqrt{\frac{[vv]}{n\text{-}1}} \tag{21}$$

2.2.2. μ_i ($i = 1,2,\cdots,n$) *isn't already known*

When $\mu_1 = \cdots = \mu_i = \cdots = \mu_n$ ($i = 1,2,\cdots,n$) is true, and they aren't already known, the paper [20] has deduced Bessel mean square error formula.

5. CONCLUSIONS

In survey teaching materials of China, deducing Bessel mean square error formula is based on survey values with same mathematics expectations. Those teaching materials haven't solved survey values with different mathematics expectation so it isn't deemed to be perfect.

Based on survey values which don't have the same mathematics expectation, this paper has put forward a sort of method to prove Bessel mean square error formula. The method has solved problems which haven't been solved by survey teaching materials of China, include national programming teaching materials of Chinese common higher education and Chinese excellent surveying teaching materials.

In addition, the paper has studied all sorts of circumstances to deduce Bessel mean square error formula. So, the paper is complete.

Obviously,the paper is good for Bessel mean square error formula to be perfective, for students to understand Bessel mean square error formula, for survey teaching materials of China, include national programming teaching materials of Chinese common higher education and Chinese excellent surveying teaching materials, to be renewed

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CONFLICT OF INTEREST

The author confirms that this article content has no conflict of interest.

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REFERENCES

[1] Z.K. Feng,L. Zhou, J.L. Zhao and C.-J. Zhao, *Principle of Surveying*, Chinese Forestry Press: Beijing, 2002, pp. 44-45.

[2] X.Q. Fu, *Surveying*, Beijing University of Technology Press: Beijing, 2006, pp. 78-80.

[3] N.W. Deng, *Surveying*, 2nd ed., China Electric Power Press: Beijing, 2010, pp. 95-96.

[4] X.W. Cheng, X.Y. Chen, G. Chen, Z.T. Ming and J.T. Xu, *Surveying*, Geological Publishing House: Beijing, 2008, pp. 62-64.

[5] Surveying edition group of Hehai University, *Surveying*, 2nd ed., National Defence Industry Press: Beijing, 2013, pp. 83-84.

[6] T.W. Li, *et al*., *Model Surveying*, 2nd ed., Science Press: Beijing, 2014, pp. 132-134.

[7] Z.Q. Luo, *Surveying*, 2nd ed., Yunnan University Press: Kunming, 2010, pp. 94-97.

[8] L. Chen and H.F. Lou, *Model Surveying Technique*, China Water & Power Press: Beijing, 2011, pp. 92-94.

[9] S.L. Yang, T.F. Yang and H.Y. Shi, *Surveying*, 2nd ed., Press of China Railway: Beijing, 2013, pp. 118-120.

[10] X.M. Zhang, X.L. Li and Z.X. Liao, *Surveying,* Press of Heifei University of Technology: Heifei, 2007, pp. 96-98.

[11] L.H. Chen, G.B. Du, H. Zhang and L.R. Zhao, *Surveying*, Zhe jiang University Press: Hangzhou, 2009, pp. 96-99.

[12] N. Wang and J.J. Guo, *Modern General Surveying*, 2nd ed., Tsinghua University Press: Beijing, 2009, pp. 114-116.

[13] X.L. Gu, F. Bao and X.J. Cheng, *Surveying*, 4th ed., Tongji University Press: Tongji, 2011, pp. 126-127.

[14] Z.F. Pan, Z.Y. Yang, X.J. Cheng, S. Cheng and T.J. Wang, *Principles and Method of Digital Mapping*, Wuhan University Press: Wuhan, 2004, pp. 48-50.

[15] Z.F. Pan, X.J. Cheng, S. Cheng, T.J. Wang, W.D. Song and J.G. Zou, *Principles and Method of Digital Mapping*, 2nd ed., Wuhan University Press: Wuhan, 2011, pp. 30-32.

[16] H. Qin, X. Wu, P.Y. Tang and D.J. Yu, *Civil Engineering Surveying*, 4th ed., Tongji University Press: Tongji, 2013, pp. 141-143.

[17] J.X. Gao, B.L. Xiao, P.Y. Fu, X.X. Yu, M. Liang, H.Y. Liang, F.L. Yang and G.L. Chen, *Principles and Method of Digital Mapping*, China University Of Mining And Technology: Xuzhou, 2010, pp. 100-102.

[18] T.S. Wang and T.Q. Yuan, *Basic Surveying*, Press of Yellow River Water Conservancy: Zhengzhou, 2008, pp. 115-117.

[19] X.Z. Cui, Z.C. Yu, B.Z. Tao, D.J. Liu, Z.L. Yu, H.Y. Sun and X.Z. Wang, *Generalized Surveying Adjustment*, new ed., Wuhan Technical University of Surveying and Mapping Press: Wuhan, 2001, pp. 2-10.

[20] Y. H. Deng, "Deduction of Bessel mean square error formula," *Journal of Geodesy and Geodynamics*, vol. 29, pp. 128-130, 2009.

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