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RESEARCH ARTICLE

A Triangular Cloud Model and Cloud Generator Algorithm

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Abstract: Cloud model is a cognitive model which represents the fuzziness and randomness with three digital characteristics, *i.e.* expectation, entropy and hyper-entropy. The normal cloud model is one of the most important cloud models, which is composed of 2nd-order normal distribution. The normal cloud model is usually expressed in qualitative concept with symmetrical features, and for the concept of fasymmetric only uses the complex combination of cloud model. Therefore, it is more complex in computing and processing in practical application. A new simple cloud model, *i.e.* triangular cloud model is proposed, and the forward triangular cloud transformation algorithm is presented. Then, a backward triangular cloud transformation algorithm is designed based on the fuzzy discrete expectation. Finally, two simulation examples are given to state the feasibility of the triangular cloud, and the result shows that the new generator algorithm is simple and effective.

Keywords: Cloud generator, Fuzzy discrete expectation, Normal cloud model, Triangular cloud model.

1. INTRODUCTION

Knowledge representation has been a bottleneck for years in artificial intelligence. The difficulty is uncertainty hidden in qualitative concepts, in particular, randomness and fuzziness [1]. The mathematic methods for handing uncertainty knowledge mostly are probability theory and fuzzy set theory. Probability theory mainly studies the stochastic phenomena, which can deal with stochastic uncertainties well. The normal distribution can be regarded as approximation of a large number of random phenomena [2]. Fuzzy set theory mainly studies the fuzzy phenomena, which mainly use membership degree of the membership function to describe fuzzy events [3]. The Gaussian membership function is considered as one of the most suitable membership function for many fuzzy events. Cloud model was proposed based on probability theory and fuzzy sets theory to describe the uncertainty knowledge in 1995 [4]. The normal cloud model uses three digital characteristics to represent the uncertainty transition between quantitative and qualitative descriptions, which will integrate the randomness and the fuzziness in 2nd-order normal distribution in unified way [5 - 8].

The normal cloud model uses three digital characteristics (Ex , En , He) to depict the intension of a concept, which just accords with human thought. Expected value Ex is used to show the center position of the normal cloud, whose elements are fully compatible with the uncertainty linguistic concept. The standard variance En is a measure of the granularity scale, which can be used to calculate the membership degree. The hyper-entropy He is a measure of the dispersion on the normal cloud drops, which can also be considered as the entropy of the standard variance En . In order to transform the uncertainty concept of quantitative and qualitative description, Li *et al.* [1] presented two basic cloud transformation tools, namely forward normal cloud generator (FNCG) and backward normal cloud generator (BNCG). By inputting the number of its characteristics, FNCG produces normal cloud droplets described by the specific concept. BNCG can obtain the qualitative description of 2nd-order normal distribution from a series of quantitative cloud droplets which have been given. The two cloud generators can also provide a way to simulate the human bidirectional cognition process for uncertainty concepts by computer program.

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At present, the normal cloud model has been successfully applied in many fields, such as intelligent control [9, 10], data mining [11 - 13], qualitative evaluation [14, 15], image segmentation [16, 17], decision making [18 - 21], and so on. However, the normal cloud model is usually expressed in qualitative concept with symmetrical features, and for the concept of asymmetry only uses the complex combination of cloud model. In fact, a lot of linguistic variables or qualitative concepts are asymmetric in quantification, for example, the concept “Young” in Fig. (2). Normal cloud model needs complex calculation and requires linguistic concept with symmetry in the practical application. Therefore, a simple triangular cloud model is proposed in this paper. The characteristic of triangular cloud model is that the function has the advantage of simple structure, and can unify qualitative representation of the linguistic concept of symmetry and asymmetry, which is conducive to the practical application of uncertain description and problem solving.

The rest of the paper is organized as follows: In Section 2, the concept and generator algorithms of the normal cloud model are introduced. In Section 3, the concepts about triangular cloud model are described. In Section 4, the efficiency of triangular cloud generator algorithms is discussed through two case studies. Finally, Section 5 presents our conclusions and suggestions for future research.

2. THE NORMAL CLOUD MODEL AND CLOUD GENERATOR

Normal cloud model is a cognitive method of transformation between quantitative data and qualitative concepts, which can formalize a concept into three numbers. Considering the uncertainty and objectivity of the membership degree, normal cloud model automatically produces the membership degrees based on probability distribution to interpret the fuzziness of concepts, thereby disclosing the relationship between randomness and fuzziness [1, 5].

2.1. The Concept of Normal Cloud Model

Definition 1 [1]: Let U be the universe of discourse, and C be a qualitative concept in U . If $x \in U$ is a random instantiation of concept C , which satisfies $x \sim N(Ex, En^2)$, $En' \sim N(En, He^2)$, so the degree of certainty of x belonging to concept C satisfies

$$\mu(x) = e^{-\frac{(x-Ex)^2}{2En^2}} \quad (1)$$

then the distribution of x in the universe U is called a normal cloud or a 2nd-order normal cloud.

The key point in Definition 1 is the 2nd-order relationship, *i.e.* within the two normal random numbers. Specially, if digital characteristic parameter $He=0$, then the cloud distribution of x on U will become a normal distribution. If digital characteristic parameters $He=En=0$, then x will be a constant Ex and $\mu(x) \equiv 1$. If digital characteristic parameter He turns larger, the distribution of random variable X will show a heavier tail, which can be used in economic and social researches.

As shown in Fig. (1), different people have different understanding about the concept of “Young”, so it is very different to give a crisp membership degree. However, the normal cloud model can describe this uncertainty concept. Meanwhile, it can also demonstrate the basic certainty of uncertainty.

In normal cloud model, cloud generator, which is a basic tool to realize the transformation between qualitative concepts and quantitative data, is composed of forward cloud generator and backward cloud generator.

2.2. The Forward Normal Cloud Generator

The FNCG is the most commonly used in cloud generator, which is a mechanism for mapping from its qualitative representation to its quantitative representation. By inputting the number of its digital characteristics (Ex, En, He), FNCG produces normal cloud droplets described by the specific concept. The generating algorithm of FNCG (Ex, En, He, n) can be described as follows [1, 5]:

Algorithm 1: Forward normal cloud generator- FNCG (Ex, En, He, n)

Input: Three parameters, Ex, En , and He , and the number of cloud drops n

Output: n cloud drops and their certainty degrees

Step 1: A normal random number En'_i with expectation of En and a standard deviation of He is generated.

$$En'_i = \text{Normrnd} (En, He) \tag{2}$$

Step 2: A normal random number x_i with expectation of Ex and a standard deviation of En'_i is generated.

$$x_i = \text{Normrnd} (Ex, En'_i) \tag{3}$$

Step 3: The certainty degree of x_i is calculated as follows:

$$\mu_i = e^{-\frac{(x_i - Ex)^2}{2En_i'^2}} \tag{4}$$

Step 4: Then x_i is a cloud drop with the confirmation degree μ_i expressed as drop (x_i, μ_i) .

Step 5: Repeat the step1 to step4 until the cloud drops generated are sufficient for n .

In Fig. (1), this is the result of FNCG (25, 3, 0.5, 2000). It can represent the qualitative concept “Young” or a ratio between 15 and 35 with cloud drops.

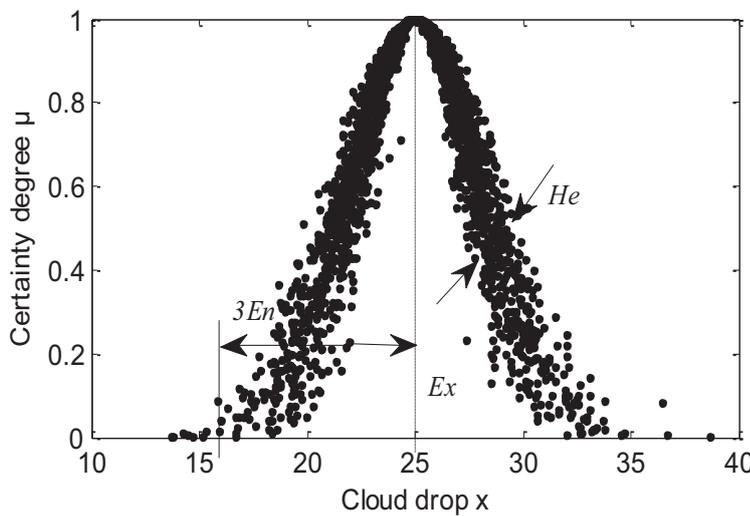


Fig. (1). Describe “Young” by the normal cloud model.

2.3. The Backward Normal Cloud Generator

The BNCG is an algorithm based on probability statistics theory. It is used to obtain the quantitative description of three digital characteristic parameters. It generates the cloud parameters (Ex, En, He) based on the cloud drops (x_i, μ_i) . Wang *et al.* [22] constructed a new algorithm of backward cloud based on the theory of probability statistics, which is more precise than the old. The generating algorithm of the one-dimension normal cloud $\text{BNCG}(x_i, \mu_i)$ can be described as follows [1, 5]:

Algorithm 2: Backward normal cloud generator- BNCG (Ex, En, He, n)

Input: n cloud drops and their certainty degrees (x_i, μ_i)

Output: Three parameters, $Ex, En,$ and He

Step 1: Calculate the mean Ex of cloud drops x_i

$$Ex = \text{Mean}(x_i) = \frac{1}{n} \sum_{i=1}^n x_i \tag{5}$$

Step 2: Calculate the standard deviation En of cloud drops x_i

$$En = Stdev(x_i) = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{X})^2} \tag{6}$$

Step 3: For each cloud drop (x_i, μ_i) , calculate the normal random number En'_i :

$$En'_i = \sqrt{-\frac{(x_i - Ex)^2}{2 \ln(\mu_i)}} \tag{7}$$

Step 4: Then the hyper entropy He can be calculated by the standard deviation of En'_i :

$$He = Stdev(En'_i) \tag{8}$$

3. A TRIANGULAR CLOUD MODEL AND CLOUD GENERATOR

Normal cloud model is an effective tool in uncertain transformation between qualitative concepts and their quantitative expressions. However, compared to the normal cloud model, triangular cloud model has extensive existence in everyday life and its own merits to represent numerical concept. For example, the distribution of the concept “Young” between 15 and 35 is not symmetrical. Obviously, the certainty degree is not the same between interval (15, 25) and interval (25, 35). For example, 20 years younger than 30 years, so the certainty degree of 20 is larger than 30. Triangular cloud model can be expressed with symmetry or asymmetry uncertainty linguistic concept.

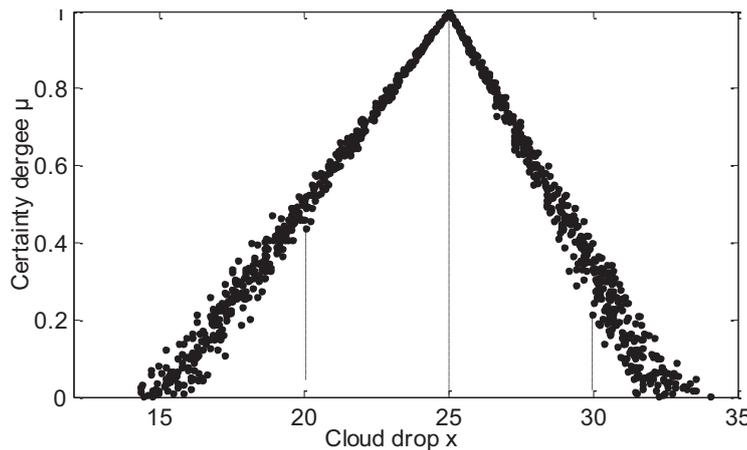


Fig. (2). Describe “Young” by the triangular cloud model.

3.1. The Concept of Triangular Cloud Model

Definition 2: Let U be the universe of discourse, and C be a qualitative concept in U . If $x \in U$ is a random instantiation of concept C , then the degree of certainty of x belonging to concept C satisfies

$$\mu(x) = \begin{cases} \frac{x - Ea'}{Ex - Ea'}, & Ea' \leq x < Ex \\ \frac{x - Ec'}{Ex - Ec'}, & Ex \leq x \leq Ec' \end{cases} \tag{9}$$

where $Ea' \sim N(Ea, He^2)$ and $Ec' \sim N(Ec, He^2)$, then the distribution of x in the universe U is called a triangular cloud model.

From the Definition 3.1, triangular cloud model represents the qualitative concept by four digital characteristics (Ea, Ex, Ec, He). The expected value Ex is the point which is the most suitable to represent the domain of the concept. The left and right spreads Ea and Ec are used to figure the granularity scale of the concept. The hyper-entropy He is used to

depict the uncertainty of the concept granularity. Moreover, the simple operation rules of triangular cloud model described below make them feasible in practical applications. Specially, if digital characteristic parameter $He=0$, then the cloud distribution of x on U will become a triangular distribution. The explicit expressions of fuzziness and uncertainty by parameters(Ea, Ex, Ec, He)make the triangular cloud model easy to understand and compute directly.

Definition 3: Given two triangular clouds C_1 and C_2 , their digital characters are $C_1(Ea_1, Ex_1, Ec_1, He_1)$ and $C_2(Ea_2, Ex_2, Ec_2, He_2)$, respectively. The arithmetic operation rules of C_1 and C_2 can be defined as follows:

$$\begin{aligned}
 C_1 + C_2 &= (Ea_1 + Ea_2, Ex_1 + Ex_2, Ec_1 + Ec_2, \sqrt{He_1^2 + He_2^2}) \\
 C_1 - C_2 &= (Ea_1 - Ec_2, Ex_1 - Ex_2, Ec_1 - Ea_2, \sqrt{He_1^2 + He_2^2}) \\
 C_1 \times C_2 &= (Ea_1 \times Ea_2, Ex_1 \times Ex_2, Ec_1 \times Ec_2, |Ex_1 Ex_2| \times \sqrt{\left(\frac{He_1}{Ex_1}\right)^2 + \left(\frac{He_2}{Ex_2}\right)^2}) \\
 C_1 \div C_2 &= (Ea_1 \div Ec_2, Ex_1 \div Ex_2, Ec_1 \div Ea_2, |Ex_1 Ex_2| \times \sqrt{\left(\frac{He_1}{Ex_1}\right)^2 + \left(\frac{He_2}{Ex_2}\right)^2})
 \end{aligned}
 \tag{10}$$

Similar to the normal cloud, the uncertain concept of C_1 and C_2 must be in the same universe of discourse such that the triangular cloud operation involved in the algorithm above is meaningful.

3.2. The Forward Triangular Cloud Generator (FTCG)

In order to be able to use computer to calculate the triangular cloud model, it is necessary to research the triangular membership cloud generator algorithm. It generates cloud drops based on the cloud parameters(Ea, Ex, Ec, He). As shown in Fig. (3), the generating algorithm of the one-dimension triangular cloud $FTCG(Ea, Ex, Ec, He, N)$ can be described as follows:

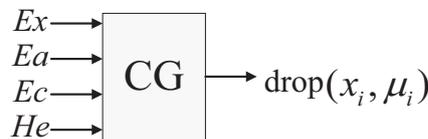


Fig. (3). A forward triangular cloud generator.

Algorithm 3: Forward triangular cloud generator- $FTCG(Ea, Ex, Ec, He, n)$

Input: Three parameters, Ea, Ex, Ec , and He , and the number of cloud drops n

Output: n cloud drops and their certainty degrees

Step 1: Two normal random numbers Ea', Ec' with expectation of Ea, Ec and a standard deviation of He is generated.

$$\begin{aligned}
 Ea'_i &= Normrnd(Ea, He) \\
 Ec'_i &= Normrnd(Ec, He)
 \end{aligned}
 \tag{11}$$

Step 2: Two uniformly distributed random numbers x_{1i}, x_{2i} are generated.

$$\begin{aligned}
 x_{1i} &= Ea'_i + (Ex - Ea'_i) \times unit \\
 x_{2i} &= Ec'_i + (Ex - Ec'_i) \times unit
 \end{aligned}
 \tag{12}$$

where $unit$ is the random number in interval $[0, 1]$.

Step 3: The certainty degree of x_i is calculated as follows:

$$\mu_i = \begin{cases} \frac{x_{1i} - Ea'}{Ex - Ea'}, Ea' \leq x_i < Ex \\ \frac{x_{2i} - Ec'}{Ex - Ec'}, Ex \leq x_i \leq Ec' \end{cases} \tag{13}$$

Step 4: Repeat the step1 to step3 until the cloud drops generated are sufficient for n .

The algorithm mainly uses a normal random number and a uniform random number. A random number is the foundation of another random number. It is the compound relationship which is the key of the forward triangular cloud generator algorithm. If $He=0$, the algorithm step1 always generates a certain value of Ea and Ec , then x has become a triangular membership function. In Fig. (2), this is the simulation result of FTCG (15, 25, 32.5, 0.5, 500). It can represent the qualitative language “Young” or a ratio between 15 and 35 with triangular cloud drops.

3.3. The Backward Triangular Cloud Generator (BTCG)

The BTCG is an algorithm based on fuzzy set theory. It is used to obtain the quantitative description of four digital characteristic parameters. As shown in Fig. (4), it generates the triangular cloud parameters (Ea, Ex, Ec, He) based on the cloud drops (x_i, μ_i).

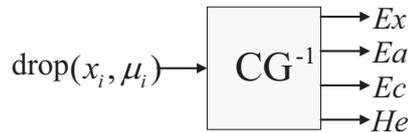


Fig. (4). A backward triangular cloud generator.

Definiton 4 [23]: Let ξ be a normalized fuzzy variable, the expected value of ξ is defined as:

$$E(\xi) = \int_0^\infty Cr\{\xi \geq r\}dr - \int_{-\infty}^0 Cr\{\xi \leq r\}dr \tag{14}$$

where $Cr\{A\} = 0.5(Pos\{A\} + Nec\{A\})$ is the credibility measure.

Let ξ be a normalized discrete fuzzy variable whose possibility distribution function is defined by:

$$\mu(x) = \begin{cases} \mu_1, \text{ if } x = x_1 \\ \mu_2, \text{ if } x = x_2 \\ \dots \\ \mu_n, \text{ if } x = x_n \end{cases} \tag{15}$$

Assume that. It follows form (15) that the expected value of ξ is:

$$E(\xi) = \sum_{i=1}^n x_i p_i \tag{16}$$

Then the weights are given by:

$$p_i = \frac{1}{2} (\bigvee_{j=1}^i \mu_j - \bigvee_{j=0}^{i-1} \mu_j) + \frac{1}{2} (\bigvee_{j=i}^n \mu_j - \bigvee_{j=i+1}^{n+1} \mu_j) \tag{17}$$

which $\mu = \mu_{n+1}$ for $i=1, 2, \dots, n$ and satisfy the following constraints:

$$\sum_{i=1}^n p_i = \bigvee_{i=1}^n \mu_i = 1 \tag{18}$$

If ζ is triangular fuzzy variable(Ea, Ex, Ec), then the expected value of ζ is:

$$E(\zeta) = \frac{Ea + 2Ex + Ec}{4} \tag{19}$$

The backward triangular cloud generator algorithm is based on the fuzzy mathematical expectation. The generating algorithm of the one-dimension triangular cloud BTCG (x_i, μ_i) can be described as follows:

Algorithm 4: Backward triangular cloud generator- BTCG (Ea, Ex, Ec, He, n)

Input: n cloud drops and their certainty degrees (x_i, μ_i)

Output: Four parameters, Ea, Ex, Ec and He

Step 1: Sort n cloud drops from small to large, $i.e. x_1 \leq x_2 \leq \dots \leq x_n$.

Step 2: Let:

$$Ea = \frac{x_1 + x_2}{2}, Ec = \frac{x_{n-1} + x_n}{2} \tag{20}$$

and,

$$\frac{Ea + 2Ex + Ec}{4} = \sum_{i=1}^n x_i p_i \tag{21}$$

Then, calculate the three numerical characters(Ea, Ex, Ec). If $\sum_{i=1}^n p_i \neq 1$, then let $P_i = p_i / \sum_{i=1}^n p_i$.

Step 3: For each cloud drop (x_i, μ_i), calculate the normal random number Ea'_i, Ec'_i :

$$Ea'_i, Ec'_i = \frac{\mu_i Ex - x_i}{\mu_i - 1} \tag{22}$$

Step 4: Then the hyper entropy He can be calculated by the standard deviation of Ea'_i, Ec'_i :

$$He = Stdev(Ea'_i) = Stdev(Ec'_i) \tag{23}$$

4. EXPERIMENT ANALYSIS

Example 1: Let $n=500$, using Matlab programming algorithm can realize the forward triangular cloud generator. For example, let $(Ea, Ex, Ec, He)=(0.3, 0.5, 0.8, 0.01)$, the forward triangular cloud generator FTCTG (Ea, Ex, Ec, He, n) is shown in Fig. (5).

The source code is as follows:

Input: (Ea, Ex, Ec, He)=(0.3,0.5,0.8,0.01)

$r_1 = \text{normrnd}(Ea, He, [1 \ 500]);$

$r_2 = \text{normrnd}(Ec, He, [1 \ 500]);$

$\text{unit} = \text{unifrnd}(0, 1, [1, 500]);$

$x_1 = r_1 + (Ex - r_1) .* \text{unit};$

$x_2 = r_2 + (Ex - r_2) .* \text{unit};$

output: $\text{plot}(x_1, 1 + (x_1 - Ex) ./ (Ex - r_1), 'k', x_2, 1 - (x_2 - Ex) ./ (r_2 - Ex), 'k')$

Example 2: There is a group of cloud drops (x_i, μ_i), and assume that the cloud drops x_i have been sorted, as shown in Table 1.

Table 1. A group of cloud drops.

x_i	0.3276	0.3373	0.3641	0.3735	0.3742	0.3821	0.3950	0.3999
μ_i	0.1359	0.1493	0.3351	0.2853	0.3584	0.3987	0.4374	0.6264
x_i	0.4103	0.4222	0.4490	0.4747	0.5226	0.6148	0.6501	0.6762
μ_i	0.5325	0.6264	0.7258	0.8686	0.9275	0.6226	0.5113	0.3911
x_i	0.6790	0.7086	0.7157	0.7331	0.7514	0.7562	0.7584	0.7666
μ_i	0.3965	0.3100	0.2411	0.2233	0.1326	0.1351	0.1348	0.1102

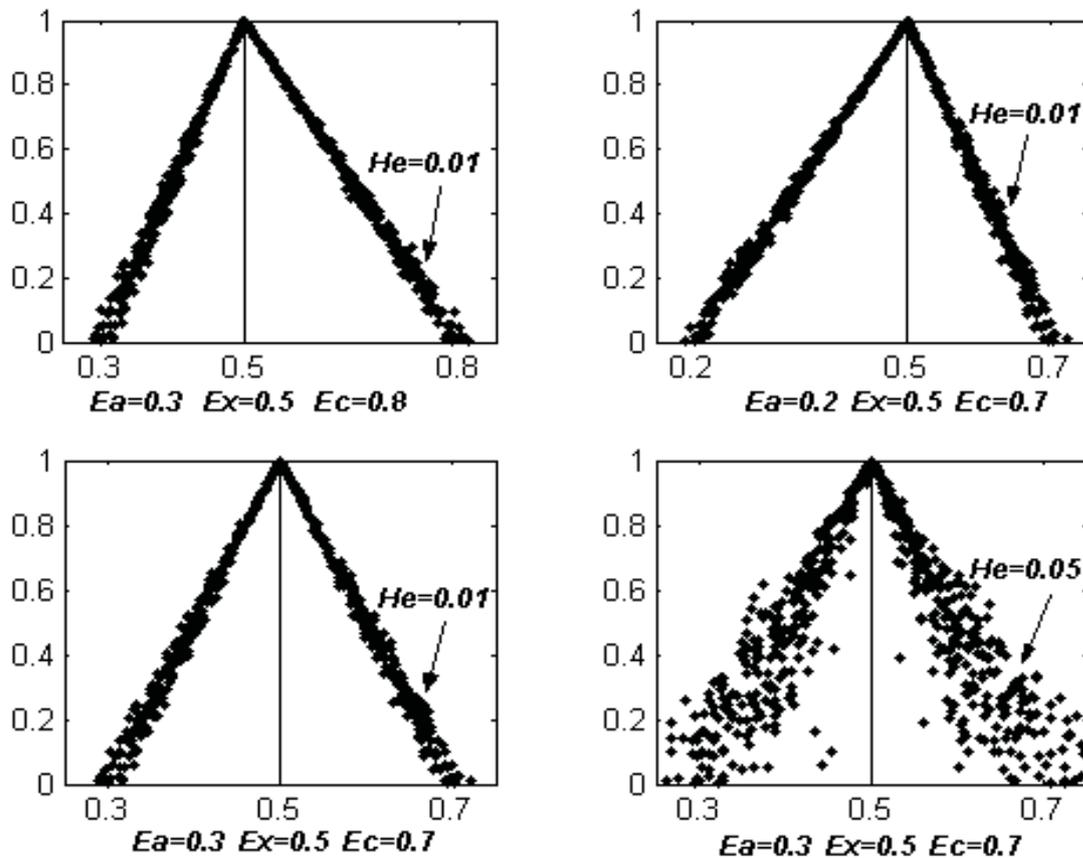


Fig. (5). Different shapes of triangular membership cloud model.

We use the backward triangular cloud generator to calculate the four numerical characters (Ea, Ex, Ec, He).

With Step 2, we can get $Ea=0.3325, Ec=0.7625$, and:

$$p_1 = \frac{1}{2} \mu_1, p_2 = \frac{1}{2} (\mu_2 - \mu_1), p_3 = \frac{1}{2} (\mu_3 - \mu_2), p_4 = 0, \dots$$

If $\sum_{i=1}^n p_i \neq 1$, let $P_i = p_i / \sum_{i=1}^n p_i$, we can get:

$$\sum_{i=1}^n x_i P_i = 0.5205,$$

Then,

$$\frac{Ex}{Ex} = 0.4925$$

From Step 3 and step 4, we can get,

$$He = Var(Ea'_i) = 0.0629$$

Thus, according to this group of cloud drops, the triangle membership cloud model BTCG $(Ea, Ex, Ec, He) = \text{BTCG}(0.3325, 0.4925, 0.7625, 0.0629)$, as shown in Fig. (6).

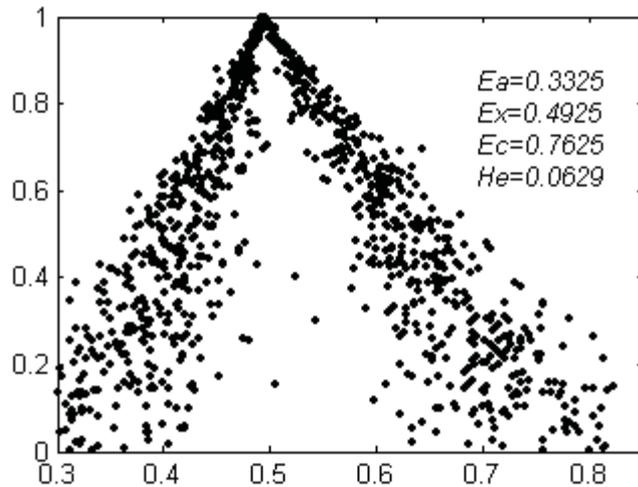


Fig. (6). The backward triangular cloud model.

CONCLUSION

The cloud model is a good tool for the bidirectional cognitive transformation between the qualitative and quantitative description of a concept by using the cloud generator algorithms. In this paper, based on fuzzy triangular membership function, a simpler and practical cloud model - triangular membership cloud model and the cloud generator algorithms are studied. It has very important significance for the promotion of the application of cloud theory and improves and expands of the existing cloud model theory. Compared with the normal cloud model, the triangular cloud model has two advantages. One is that the triangular cloud model can deal with the symmetric and asymmetric qualitative concepts while the normal cloud model can only deal with the symmetric qualitative concept. Another advantage is that the triangular cloud model is simpler than the normal cloud model in computational process, so it is more convenient for practical application.

In this paper, the backward triangular cloud generator algorithm must be given the value of certainty degree in advance. However, in the practical application, we are often only given a group of cloud drops, whose certainty degree was not given. Error analysis of the backward triangular cloud generator algorithm has not been studied theoretically. Therefore, without the value of certainty degree, the implementation of calculating the four parameters of the triangular cloud and error analysis of the backward generator algorithm will be the focus of our future research work.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

ACKNOWLEDGEMENTS

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