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RESEARCH ARTICLE

Decision-making Method for Clay-brick Selection Based on Subtraction Operational Aggregation Operators of Intuitionistic Fuzzy Values

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Abstract: The subtraction operation of intuitionistic fuzzy sets (IFSs) has been scarcely used for practical applications since it was introduced. Therefore, it is necessary to propose an aggregation operator based on the subtraction operation of IFSs for engineering applications. Then, clay-brick selection is an important decision-making problem for better building construction. To handle the decision-making problem based on the subtraction operation of IFSs in an intuitionistic fuzzy environment, this paper firstly introduces an intuitionistic fuzzy subtraction operational weighted arithmetic averaging (IFSOWAA) operator and investigates its properties. Then, we propose the IFSOWAA operator-based decision-making method as a supplement for existing decision-making methods under an intuitionistic fuzzy environment. Finally, an actual example about a clay-brick selection problem is provided to show the applicability and effectiveness of the proposed method.

Keywords: Clay-brick selection, Decision making, Intuitionistic fuzzy set, Intuitionistic fuzzy subtraction operational weighted arithmetic averaging (IFSOWAA) operator, Subtraction operation.

1. INTRODUCTION

Multiple attribute decision-making problems are usually to find the most satisfactory alternative from all the feasible alternatives. Owing to the fuzziness of human thinking and cognition about complex decision-making problems, it is difficult to express the attribute values by crisp numbers. Then, a fuzzy set introduced by Zadeh [1] can express fuzzy information in real world. After that, Atanassov [2] considered the non-membership degree and presented an intuitionistic fuzzy set (IFS) as a generalization of the fuzzy set. IFS is composed of a membership degree and a non-membership degree to describe vague and incomplete information. Therefore, IFS is a very useful tool for dealing with fuzziness and uncertainty in decision-making problems. Many methods have been developed to solve the complex multiple attribute decision-making problems with the IFS information [3 - 26]. As a supplement of basic operational laws over IFSs, Atanassov and Riecan [27] and Chen [28] introduced the subtraction and division operations over IFSs. However, the subtraction and division operations over IFSs are scarcely applied in science and engineering fields since they were presented. Therefore, it is necessary to propose some aggregation operators based on the subtraction and division operations of IFSs for engineering applications.

Clay-brick selection is an important decision-making problem for better building construction. To construct a building, a traditional selection method for clay-bricks provided from various brick fields is to select clay-bricks roughly based on their color, size, and total cost, without considering other quality factors of clay-bricks. In this case, the building construction may produce some dangerous problems regarding low quality clay-bricks. Therefore, it is

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necessary to formulate a scientific selection method. In order to select the most suitable brick to construct a building, we have to consider the solidity, color, size and shape, strength, cost of brick etc. as their evaluation indices (attributes) [29, 30]. Hence, some researchers have proposed decision-making methods for clay-brick selection problems under intuitionistic fuzzy and single-valued neutrosophic environments [29, 30].

Since existing various intuitionistic fuzzy aggregation operators cannot handle the information aggregation of the intuitionistic fuzzy subtraction operation and existing subtraction operation of intuitionistic fuzzy values (IFVs), which are basic elements in IFSs, lacks the practical applications, this paper presents an intuitionistic fuzzy subtraction operational weighted arithmetic averaging (IFSOWAA) operator and its decision-making method as a supplement of existing decision-making methods, and then applies the IFSOWAA operator-based decision-making method to the decision-making problem of clay-brick selection under an intuitionistic fuzzy environment.

The remainder of this paper is structured as follows. Section 2 reviews some basic knowledge of IFSs and operations of IFVs. Section 3 proposes an IFSOWAA operator based on the subtraction operation of IFVs and investigates its properties. In Section 4, a multiple attribute decision-making method is developed based on the IFSOWAA operator. In Section 5, an actual example about a clay-brick selection problem is provided to show the applicability and effectiveness of the proposed method. Some conclusions and future research are discussed in Section 6.

2. SOME BASIC KNOWLEDGE OF IFSS AND OPERATIONS OF IFVS

Atanassov [2] extended fuzzy set to IFS and introduced its definition.

Definition 1 [2]. Let X be a universal of discourse. An IFS N in X is characterized by a membership function $u_N(x)$, a non-membership function $v_N(x)$, where the values of the two functions $u_N(x)$ and $v_N(x)$ are real numbers in the interval [0, 1], such that $u_N(x) \in [0, 1]$ and $v_N(x) \in [0, 1]$ and $0 \leq u_N(x) + v_N(x) \leq 1$. Thus, an IFS N is denoted by the mathematical symbol:

Then,

$$N = \left\{ \langle x, u_N(x), v_N(x) \rangle \mid x \in X \right\}.$$

the intuitionistic index (hesitancy) is represented as $m_N(x) = 1 - u_N(x) - v_N(x)$ and $m_N(x) \in [0,1]$ for $x \in X$. For convenience, a basic element $\langle x, u_N(x), v_N(x) \rangle$ in an IFS N is denoted by $a = \langle u_a, v_a \rangle$ for short, which is called IFV [4].

Let $a = \langle u_a, v_a \rangle$ and $b = \langle u_b, v_b \rangle$ be two IFVs, then there are the following relations [2]:

(1) $a^c = \langle v_a, u_a \rangle$ (complement of a);

(2) $a \leq b$ if and only if $u_a \leq u_b$ and $v_a \geq v_b$;

(3) $a = b$ if and only if $u_a = u_b$ and v_a v_b

After that, the basic operational laws of the two IFVs $a = \langle u_a, v_a \rangle$ and $b = \langle u_b, v_b \rangle$ are introduced as follows [7]:

(1) $a + b = \langle u_a + u_b - u_a u_b, v_a v_b \rangle$;

(2) $a \times b = \langle u_a u_b, v_a + v_b - v_a v_b \rangle$;

(3) $\rho a = \langle 1 - (1 - u_a)^\rho, v_a^\rho \rangle$ for $\rho > 0$;

(4) $a^\rho = \langle u_a^\rho, 1 - (1 - v_a)^\rho \rangle$ for $\rho > 0$.

For any IFV $a = \langle u_a, v_a \rangle$ its score and accuracy functions [31, 32] are introduced, respectively, as follows:

$$s(a) = u_a - v_a, \quad s(a) \in [-1,1], \tag{1}$$

$$h(a) = u_a + v_a, \quad h(a) \in [0,1]. \tag{2}$$

Definition 2 [4, 7]. Let $a = \langle u_a, v_a \rangle$ and $b = \langle u_b, v_b \rangle$ be two IFVs, then according to the score values of $s(a)$ and $s(b)$ and the accuracy degrees of $h(a)$ and $h(b)$, there are the following relations:

(1) If $s(a) < s(b)$, then $a < b$;
(2) If $s(a) = s(b)$ and $h(a) < h(b)$, then $a < b$;
(3) If $s(a) = s(b)$ and $h(a) = h(b)$, then $a = b$.

Let $a_j = \langle u_{a_j}, v_{a_j} \rangle$ (j = 1, 2,..., n) be a collection of IFVs, then the following intuitionistic fuzzy weighted arithmetic averaging (IFWAA) operator [7] are introduced as follows:

$$IFWAA(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j a_j = \left\langle 1 - \prod_{j=1}^{n} (1 - u_{a_j})^{w_j}, \prod_{j=1}^{n} (v_{a_j})^{w_j} \right\rangle, \tag{3}$$

where w_j (j = 1, 2, …, n) is the weight of a_j (j = 1, 2, …, n) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

3. SUBTRACTION OPERATIONAL WEIGHTED AGGREGATION OPERATOR OF IFVS

Based on the subtraction operation over IFVs, this section proposes its aggregation operator.

Definition 3. Let $a = \langle u_a, v_a \rangle$ and $b = \langle u_b, v_b \rangle$ be two IFVs, then the subtraction operation of the IFVs a and b is defined as follows [27, 28]:

$$c = a - b = \langle u_c, v_c \rangle = \left\langle \frac{u_a - u_b}{1 - u_b}, \frac{v_a}{v_b} \right\rangle, \text{ if } a \geq b,\ u_b \neq 1,\ v_b \neq 0,\ u_a v_b - u_b v_a \leq v_b - v_a. \tag{4}$$

Based on the basic operational laws of IFVs, we introduce the following theorem:

Theorem 1. Let $a = \langle u_a, v_a \rangle$ and $b = \langle u_b, v_b \rangle$ be two IFVs, ρ > 0. Then, there are the following operational laws of $(a - b)$:

$$\rho(a - b) = \left\langle 1 - \left(1 - \frac{u_a - u_b}{1 - u_b}\right)^{\rho}, \left(\frac{v_a}{v_b}\right)^{\rho} \right\rangle, \text{ if } a \geq b,\ u_b \neq 1,\ v_b \neq 0,\ u_a v_b - u_b v_a \leq v_b - v_a. \tag{5}$$

$$(a - b)^{\rho} = \left\langle \left(\frac{u_a - u_b}{1 - u_b}\right)^{\rho}, 1 - \left(1 - \frac{v_a}{v_b}\right)^{\rho} \right\rangle, \text{ if } a \geq b,\ u_b \neq 1,\ v_b \neq 0,\ u_a v_b - u_b v_a \leq v_b - v_a. \tag{6}$$

Obviously, Eqs. (5) and (6) are true according to the basic operational laws of IFVs.

Let $a_j = \langle u_{a_j}, v_{a_j} \rangle$ and $b_j = \langle u_{b_j}, v_{b_j} \rangle$ (j = 1,2,..n) be two collections of IFVs and $c_j = a_j - b_j = \langle u_{c_j}, v_{c_j} \rangle$ (j = 1, 2, …, n) be a collection of c_j. Based on the intuitionistic fuzzy weighted arithmetic averaging aggregation operator of Eq. (3) and Theorem 1, if these conditions $a_j \leq b_j$, $u_{b_j} \neq 1$, $v_{b_j} \neq 0$, $u_{a_j} v_{b_j} - u_{b_j} v_{a_j} \leq v_{b_j} - v_{a_j}$ are satisfied, we can introduce the intuitionistic fuzzy subtraction operational weighted arithmetic averaging (IFSOWAA) operator:

$$IFSOWAA(c_1, c_2, ..., c_n) = \sum_{j=1}^{n} w_j c_j = \sum_{j=1}^{n} w_j (a_j - b_j)$$
$$= \left\langle 1 - \prod_{j=1}^{n} \left(1 - \frac{u_{a_j} - u_{b_j}}{1 - u_{b_j}} \right)^{w_j}, \prod_{j=1}^{n} \left(\frac{v_{a_j}}{v_{b_j}} \right)^{w_j} \right\rangle \tag{7}$$

where w_j (j = 1, 2, …, n) is the weight of $c_j = a_j - b_j$ (j = 1, 2, …, n) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Especially when $w_j = 1/n$ ($j = 1,2,...,n$) the IFSOWAA operator is degenerated to the intuitionistic fuzzy subtraction operational arithmetic averaging operator.

Based on the properties of the IFWAA operator [7], it is obvious that the IFSOWAA operator also satisfy the properties of idempotency, boundedness and monotonicity:

(1) Idempotency:

If $c_j = c$ for j = 1, 2, . . ., n, then there is $IFSOWAA(c_1, c_2, \cdots, c_n) = \sum_{j=1}^{n} w_j c_j = c$.

(2) Boundedness:

If $C_{min} = \min(C_1, C_2,...,C_n)$ and $C_{max} = \max(C_1, C_2,...,C_n)$ for j = 1, 2,.., n, then there is $c_{\min} \le IFSOWAA(c_1, c_2, \cdots, c_n) \le c_{\max}$..

(3) Monotonicity:

If $c_j \le c_j$* for j = 1, 2, . . ., n, then there is $IFSOWAA(c_1, c_2, \cdots, c_n) \le IFSOWAA(c_1^*, c_2^*, \cdots, c_n^*)$.

4. DECISION-MAKING METHOD BASED ON THE IFSOWAA OPERATOR

In this section, we present a handling method for multiple attribute decision-making problems based on the IFSOWAA operator.

In a multiple attribute decision-making problem, we suppose that $T = \{T_1, T_2, …, T_m\}$ be a set of alternatives and $M = \{M_1, M_2, …, M_n\}$ be a set of attributes. The weight of each attribute M_j (j = 1, 2, …, n) is considered as w_j, satisfying $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Then, the characteristic of each alternative T_i (i = 1, 2, …, m) with respect to each attribute M_j (j = 1, 2, …, n) is evaluated by the decision-maker and the evaluation values are expressed by the IFV $a_{ij} = \langle u_{ij}, v_{ij} \rangle$, where $0 \le u_{ij} + v_{ij} \le 1$, $u_{ij} \ge 0$, $v_{ij} \ge 0$ (j = 1, 2,…, n; i = 1, 2,…, m), and then $u_{ij} \in [0,1]$ indicates the degree that the alternative T_i is satisfactory to the attribute M_j and $v_{ij} \in [0,1]$ indicates the degree that the alternative T_i is unsatisfactory to the attribute M_j. Therefore, we can establish an IFV decision matrix $D = (a_{ij})_{m\times n}$.

As for the multiple attribute decision-making problem, we propose a decision-making method, which is described by the following steps:

Step 1. Based on the IFV decision matrix $D = (a_{ij})_{m\times n}$, the j-th IFV positive ideal solution can be determined $a_j^+ = \langle u_j^+, v_j^+ \rangle = \langle \max_i(u_{ij}), \min_i(v_{ij}) \rangle$ (j = 1, 2, …, n) and the j-th IFV negative ideal solution can be determined by $a_j^- = \langle u_j^-, v_j^- \rangle = \langle \min_i(u_{ij}), \max_i(v_{ij}) \rangle$ (j = 1, 2, …, n). Thus they are constructed as both the ideal alternative $M^+ = \{a_1^+, a_2^+,...,a_n^+\}$ and the non-ideal alternative $M^- = \{a_1^-, a_2^-,...,a_n^+\}$.

Step 2. According to Eq. (7), two collective values C_i^+ and C_i^- (i = 1, 2, …, m) for each alternative T_i (i = 1, 2, …, m) can be calculated by the following IFSOWAA operators:

$$c_i^+ = \left\langle u_{c_j^+}, v_{c_j^+} \right\rangle = IFSOWAA(c_{i1}^+, c_{i2}^+, ..., c_{in}^+) = \sum_{j=1}^{n} w_j c_{ij}^+ = \sum_{j=1}^{n} w_j (a_j^+ - a_{ij}) \tag{8}$$
$$= \left\langle 1 - \prod_{j=1}^{n} \left(1 - \frac{u_j^+ - u_{ij}}{1 - u_{ij}}\right)^{w_j}, \prod_{j=1}^{n} \left(\frac{v_j^+}{v_{ij}}\right)^{w_j} \right\rangle$$

$$c_i^- = \left\langle u_{c_j^-}, v_{c_j^-} \right\rangle = IFSOWAA(c_{i1}^-, c_{i2}^-, ..., c_{in}^-) = \sum_{j=1}^{n} w_j c_{ij}^- = \sum_{j=1}^{n} w_j (a_{ij} - a_j^-) \tag{9}$$
$$= \left\langle 1 - \prod_{j=1}^{n} \left(1 - \frac{u_{ij} - u_j^-}{1 - u_j^-}\right)^{w_j}, \prod_{j=1}^{n} \left(\frac{v_{ij}}{v_j^-}\right)^{w_j} \right\rangle$$

Step 3. We calculate the values of $H(C_i^+)$ and $H(C_i^-)$ (i = 1, 2, …, m) by the hybrid functions of the score and accuracy functions with a real parameter $0 \le \rho \le 1$:

$$H(c_i^+) = \rho(1 + u_{c_i^+} - v_{c_i^+})/2 + (1-\rho)(u_{c_i^+} + v_{c_i^+}), \quad H(c_i^+) \in [0,1], \tag{10}$$

$$H(c_i^-) = \rho(1 + u_{c_i^-} - v_{c_i^-})/2 + (1-\rho)(u_{c_i^-} + v_{c_i^-}), \quad H(c_i^-) \in [0,1]. \tag{11}$$

Step 4. The relative closeness degree of each alternative with respect to the ideal alternative (i = 1, 2, …, m) is calculated by:

$$R_i = \frac{H(c_i^-)}{H(c_i^-) + H(c_i^+)} \quad \text{for } R_i \in [0, 1], \tag{12}$$

Obviously, the larger value of R_i reveals that the alternative is closer to the ideal alternative and farther from the non-ideal alternative simultaneously. Therefore, all the alternatives can be ranked by the values of R_i (i = 1, 2, …, m) in a descending order. The alternative with the largest value is the best choice.

Step 5. End.

5. ACTUAL EXAMPLE OF CLAY-BRICK SELECTION

In this section, an actual example about a clay-brick selection problem (adapted from [30]) in a construction company is provided under an intuitionistic fuzzy environment to demonstrate the applicability and effectiveness of the IFSOWAA operator-based multiple attribute decision-making method in realistic scenarios.

For constructing a building, a construction company needs to select the four types of clay-bricks, which are provided from various brick fields, as a set of alternatives $T = \{T_1, T_2, T_3, T_4\}$. To select the most suitable brick for constructing a building, it is necessary to evaluate the four types of clay-bricks by the six attributes of clay-bricks obtained from experts' opinions [30]: (1) M_1 is solidity, (2) M_2 is color, (3) M_3 is size and shape, (4) M_4 is strength, (5) M_5 is cost, (6) M_6 is carrying cost. The weight vector of the six attributes is given by $\boldsymbol{w}$ = (0.275, 0.175, 0.2, 0.1, 0.05, 0.2). Experts or decision makers are required to evaluate the four possible alternatives under the above six attributes by suitability judgments.

To indicate the evaluation of an alternative T_i (i =1, 2, 3, 4) with respect to an attribute M_j (j =1, 2, …, 6), it can be obtained from the questionnaire or score law of domain experts. For example, when we ask the opinion of an expert about an alternative T_1 with respect to an attribute M_1, he/she may say that the possibility in which the statement is suitable is 0.7 and the statement is unsuitable is 0.2. By the intuitionistic fuzzy notation, it can be expressed as $a_{11} = \langle 0.7, 0.2 \rangle$. Similarly, when the four possible alternatives with respect to the above six attributes are evaluated by the expert, based on [30] we can construct the following IFV decision matrix:

$$D=(a_{ij})_{4\times 6}=\begin{array}{c} \\ T_1 \\ T_2 \\ T_3 \\ T_4 \end{array}\begin{array}{c} \begin{array}{cccccc} M_1 & M_2 & M_3 & M_4 & M_5 & M_6 \end{array} \\ \begin{bmatrix} <0.7,0.2> & <0.8,0.1> & <0.7,0.2> & <0.6,0.2> & <0.7,0.1> & <0.6,0.3> \\ <0.7,0.1> & <0.7,0.2> & <0.8,0.1> & <0.7,0.3> & <0.8,0.1> & <0.5,0.3> \\ <0.8,0.1> & <0.8,0.1> & <0.7,0.2> & <0.6,0.3> & <0.6,0.3> & <0.7,0.2> \\ <0.8,0.1> & <0.8,0.1> & <0.8,0.1> & <0.8,0.1> & <0.6,0.2> & <0.7,0.1> \end{bmatrix} \end{array}.$$

In the decision-making problem of the clay-brick selection, the proposed decision-making method can be applied and the decision steps are described as follows:

Step 1. By $a_j^+=\langle u_j^+,v_j^+\rangle=\langle \max_i(u_{ij}),\min_i(v_{ij})\rangle$ and $a_j^-=\langle u_j^-,v_j^-\rangle=\langle \min_i(u_{ij}),\max_i(v_{ij})\rangle$ (i = 1, 2, 3, 4;j = 1,2,...,6) we can determine both the IFV positive ideal solutions in the ideal alternative and the IFV negative ideal solutions in the non-ideal alternative, respectively, as follows:

$$\begin{aligned} M^+ &= \{a_1^+,a_2^+,a_3^+,a_4^+,a_5^+,a_6^+\} \\ &= \{<0.8,0.1>,<0.8,0.1>,<0.8,0.1>,<0.8,0.1>,<0.8,0.1>,<0.7,0.1>\}, \\ M^- &= \{a_1^-,a_2^-,a_3^-,a_4^-,a_5^-,a_6^-\} \\ &= \{<0.7,0.2>,<0.7,0.2>,<0.7,0.2>,<0.6,0.3>,<0.6,0.3>,<0.5,0.3>\}. \end{aligned}$$

Step 2. By using Eqs. (8) and (9), we can obtain the two collective values c_i^+ and c_i^- (i = 1, 2, 3, 4) for each alternative T_i (i = 1, 2, 3, 4):

c_1^+ = <0.2880, 0.5389>, c_2^+ = <0.2776, 0.6371>, c_3^+ = <0.1689, 0.6427>, and c_4^+ = <0.0341, ,0.9659>;

c_1^- = <0.1219, 0.8051>, c_2^- = <0.1346, 0.6810>, c_3^- = <0.2477, 0.6750>, and c_4^- = <0.3528, 0.4491>.

Step 3. By applying Eqs. (10) and (11) and taking ρ= 0.5, we calculate the values of $H(C_i^+)$ and $H(C_i^-)$ (i = 1, 2, 3, 4):

$H(c_1^+)$ = 0.6007, $H(c_2^+)$ = 0.6175, $H(c_3^+)$ = 0.5374, and $H(c_4^+)$ = 0.5170;

$H(c_1^-)$ = 0.5427, $H(c_2^-)$ = 0.5212, $H(c_3^-)$ = 0.6045, and $H(c_4^-)$ = 0.6269.

Step 4. By using Eq. (12), we calculate the relative closeness degrees of each alternative with respect to the ideal alternative for R_i (i = 1, 2, 3, 4):

R_1 = 0.4746, R_2 = 0.4577, R_3 = 0.5294, and R_4 = 0.5480.

Since the ranking order of the relative closeness degrees is $R_4 > R_3 > R_1 > R_2$, the ranking order of the four alternatives is $T_4 \succ T_3 \succ T_1 \succ T_2$. Hence, the best alternative is T_4.

By a comparison with the decision-making method in [30], although the ranking orders are different, the best alternative is the same result as in [30]. Hence, the decision result of the decision-making method proposed in this paper is suitable. It is obvious that the main advantage of the proposed approach is simpler and more convenient than existing related method in [30].

To further demonstrate the effectiveness and rationality of the proposed method in this paper, we compare the proposed method with the conventional method based on the IFWAA operator introduced in [7] and the score and accuracy functions. By directly using the IFWAA operator of Eq. (3), we can obtain all the collective values of a_i = *IFWAA* (a_{i1}, a_{i2}, a_{i3}, a_{i4}, a_{i5}, a_{i6}) (i = 1, 2, 3, 4) for each alternative T_i (i = 1, 2, 3, 4):

a_1 = <0.6954, 0.1856>, a_2 = <0.6998, 0.1570>, a_3 = <0.7390, 0.1556>, and a_4 = <0.7755, 0.1035>.

By applying Eq. (1), we calculate the score values of $s(a_i)$ for each alternative T_i (i = 1, 2, 3, 4):

$s(a_1) = 0.5098$, $s(a_2) = 0.5428$, $s(a_3) = 0.5834$, and $s(a_4) = 0.6719$.

Since the ranking order of the score values is $s(a_4) > s(a_3) > s(a_2) > s(a_1)$, the ranking order of the four alternatives is $T_4 \succ T_3 \succ T_2 \succ T_1$. Hence, the best alternative is T_4.

For the above two decision results with respect to the two decision-making methods based on the IFSOWAA and IFWAA operators, we can see that the two ranking orders of the alternatives only reveal difference between T_1 and T_2, while the ranking order $T_4 \succ T_3$ and the best alternative T_4 are identical. Therefore, the decision-making method proposed in this paper is effective and provides a useful supplement for existing decision-making methods under an IFV environment.

CONCLUSION

To use the subtraction operation of IFVs for practical applications, this paper presented the IFSOWAA operator for IFVs. Next, we developed a multiple attribute decision-making method based on the IFSOWAA operator. Finally, an actual example about a clay-brick selection problem was provided to demonstrate the applicability and effectiveness of the developed method. However, the proposed decision-making method provides both a useful supplement and another new way for existing decision-making methods under an IFV environment. In the future work, the developed method will be further extended to other fields, such as pattern recognition, image processing and clustering analysis.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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