Designing Couplings for Synchronization

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Abstract: Master-slave synchronization and mutual synchronization of two identical oscillators respectively are presented. We believe that this method could be adopted for the teaching of the topic. Numerical results are given for the synchronization of two Sprott's chaotic electric circuits.

Keywords: Synchronization, Sprott's circuits.

1. INTRODUCTION

In biology, medicine and agriculture many systems can be modelled as oscillators or vibratory systems and those systems show a tendency towards synchronous behaviour. From the control point of view, the controlled synchronization is the most interesting. That means to design a controller or interconnections that guarantee synchronization of the multi-composed systems with respect to certain desired functional. Jackson and Grosu [1] developed a powerful method of control: the open-plus-closed-loop (OPCL) method. This method is very general and is mathematically based. It offers a driving in order to determine a general system to reach a desired dynamics. If the goal dynamics is the dynamics of an identical system (master system) then the driving is simpler. The driving term contains an arbitrary Hurwitz matrix. Choosing with care this matrix the driving term can be even simpler. This method was used for Chua systems [2], for neural systems [3] and for Sprott's collection [4]. A similar strategy can be used for mutual synchronization [5]. More than this it can be extend to 3 systems [6] and very recent to several systems. This method can be used for Parameter Estimation [7]. Sprott 's collection can be rewritten into third-order ODE in a single variable [8]. In addition the author in [9] presents some chaotic electronic circuits which can be described in the same manner, by the equation:

$$\dddot{x} + A\ddot{x} + \dot{x} = G(x) \quad (1)$$

These circuits contain resistors, capacitors, diodes and operational amplifiers. Here we present one method for master-slave synchronization and one for mutual synchronization of identical oscillators. We apply these methods to the synchronization (master-slave and mutual respectively) of two identical oscillators from Sprott's circuits.

2. MASTER-SLAVE SYNCHRONIZATION

A general master system is of the form:

$$\frac{dX}{dt} = F(X), X \in Rn \quad (2)$$

and the slave system coupled to the master is:

$$\frac{dx}{dt} = F(x) + (H - \frac{dF(X)}{dX})_{x=X} \quad (3)$$

where H is a constant Hurwitz matrix (a matrix with negative real part eigenvalues). The matrix H should be chosen in such a manner in order that the coupling to be as simple as possible. If the characteristic equation of the matrix H is:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \quad (4)$$

then the Ruth-Hurwitz conditions are

$$a_2\rangle 0; a_1a_2 - a_3\rangle 0; a_3\rangle 0 \quad (5)$$

One big disadvantages of this general method is that the coupling term could be complicated and hard to be implemented in practical/engineering applications [4].

3. MUTUAL SYNCHRONIZATION

Let's consider two identical general oscillators:

$$\frac{dx}{dt} = F(x) \quad (6)$$

and

$$\frac{dy}{dt} = F(y) \quad (7)$$

In order to obtain synchronization it is necessary to couple the two systems. The coupled systems are:

$$\frac{dx}{dt} = F(x) + u(x.y) \quad (8)$$

$$\frac{dy}{dt} = F(y) + u(x.y) \quad (9)$$

where

$$u(x, y) = (H - \frac{dF(s)}{ds})_{(x-y)/2} \quad (10)$$

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and s=(x+y)/2 and H a Hurwitz matrix.

We use the notations: s=(x+y)/2 and r=(x-y)/2 and the Taylor expansions:

$$F(s+r)=F(s)+\frac{dF(s)}{ds}r+\ldots$$
$$F(s-r)=F(s)+\frac{dF(s)}{ds}(-r)+\ldots \qquad (11)$$

Subtracting (8) from (7) we obtain

$$\frac{dr}{dt}=Hr \qquad (12)$$

4. COMPARISON BETWEEN MASTER-SLAVE SYNCHRONIZATION AND MUTUAL SYNCHRONIZATION

For the simplest choice of G(x) in (1) of the form

$G(x)=0.58(x^2-1)$,

the master system:

$$\dot{X}_1=X_2$$
$$\dot{X}_2=X_3 \qquad (13)$$
$$\dot{X}_3=-\mathrm{A}X_3-X_2+0.58(X_1^{\,2}-1)$$

The Routh-Hurwitz conditions (4) give:

$p\in(-0.6;0)$

for A =0.6

The strange attractor for this system is (Fig. **1**):

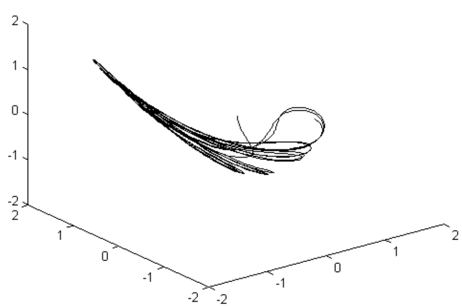

Fig. (1). The strange attractor for the system 13.

The slave system:

$$\dot{x}_1=x_2$$
$$\dot{x}_2=x_3 \qquad (14)$$
$$\dot{x}_3=-0.6x_3-x_2+0.58(x_1^{\,2}-1)+(-0.5-1.16X_1)(x_1-X_1)$$

In Fig. (**2**) the numerical results are shown for master slave synchronization for initial conditions $X_1(0)=X_2(0)=X_3(0)=0.1$ and $x_1(0)=x_2(0)=x_3(0)=-0.1$

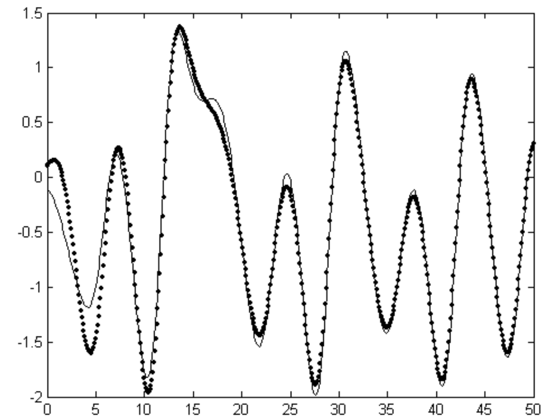

Fig. (2). $X_1(t)$, $x_1(t)$ from (13) and (14) and p=-0.5.

The systems (5), (6) for the system (11) are:

$$\dot{x}_1=x_2$$
$$\dot{x}_2=x_3$$
$$\dot{x}_3=-0.6x_3-x_2+0.58(x_1^{\,2}-1)+$$
$$(-0.5-0.58(x_1+y_1))(x_1-y_1)/2 \qquad (15)$$
$$\dot{y}_1=y_2$$
$$\dot{y}_2=y_3$$
$$\dot{y}_3=-0.6y_3-y_2+0.58(y_1^{\,2}-1)+$$
$$(-0.5-0.58(x_1+y_1))(-x_1+y_1)/2$$

Numerical results are shown in Fig. (**3**) with p=-0.5, $x_1(0)=1$; $x_2(0)=0.1$; $x_3(0)=0.01$ and $y_1(0)=1$; $y_2(0)=0.1$; $y_3(0)=-0.01$

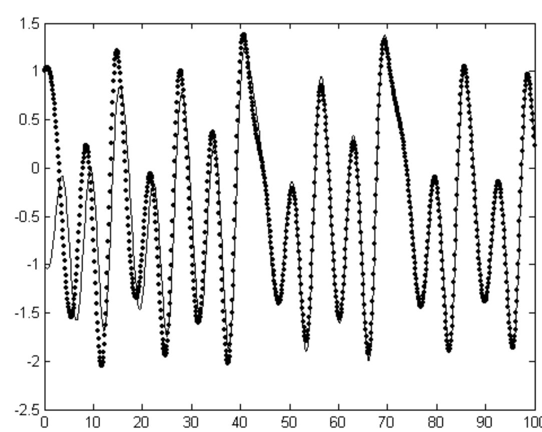

Fig. (3). $x_1(t)$, $y_1(t)$ from (15) and p=-0.5.

5. CONCLUSIONS

Figs. (**2**, **3**) showed that the synchronization is faster for master-slave synchronization than the mutual synchronization ($t\approx 45$ time unities in the first case and $t\approx 80$ time unities for the second case, this means is obtained two times rapidly). Formulation of the Sprott's oscillator chaotic presented in [8], using a jerk function, is simple as form and the synchronization is simpler and easily to learn.

REFERENCES

[1] E.A. Jackson, and I. Grosu, "An Open-Plus-Closed-Loop (OPCL) control of conplex dynamic systems", *Phys. D,* vol. 85, pp. 1-9, 1995.

[2] E.A. Jackson, and I. Grosu, "Toward experimental implementation of migration control", *Int. J. Bifurcat. Chaos,* vol. 5, pp. 1767-1772, 1995.

[3] V. Dragoi, and I. Grosu, "Synchronization of Locally Coupled neural Oascillator", *Neural Process. Lett.,* vol. 7, pp. 199-210, 1998.

[4] A.I. Lerescu, N. Constandache, S. Oancea, and I. Grosu, "Collection of master-slave synchronized chaotic systems", *Chaos, Solitons Fractals,* vol. 22(3), pp. 599-604, 2004.

[5] A.I. Lerescu, S. Oancea, and I. Grosu, "Collection of mutually synchronized chaotic systems", *Phys. Lett. A*, vol. 352, pp. 222-228, 2006.

[6] I.Grosu, "General coupling for synchronization of 3 idendical oscillators", *Int. J. Bifurcat. Chaos,* vol. 17(10), pp. 3519-3522, 2007.

[7] I.Grosu, "Parameter Estimation from time series using OPCL autosynchronization", *Int. J. Bifurcat. Chaos,* vol. 14, no. 6, pp. 2133-2141, 2004.

[8] J.C. Sprott, "Some simple chaotic jerk functions", *Am. J. Phys.,* vol. 65, no. 6, pp. 537-543, 1997.

[9] J.C. Sprott, "A new class of chaotic circuits", *Phys. Lett. A*, vol. 266, pp. 19-23, 2000.

Received: November 30, 2008 Accepted: April 9, 2009