PAPR Reduction Schemes for Lifting Scheme based Orthogonal Wavelet Packet Multiplexing System

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Abstract: Lifting scheme based orthogonal wavelet packet multiplexing (LOWPM) system can increase computation efficiency and save storage space, but as a new multicarrier modulation system, it also has the problem of high peak to average power ratio (PAPR). In order to reduce computational complexity and improve the effect of PAPR suppression for Partial Transmit Sequence (PTS) technology, PAPR reduction schemes based on the technology of pruning wavelet packet modulation and the improved threshold search for PTS are put forward in the paper, through proper pruning of the full-tree structure of wavelet packet modulation in the PTS to reduce the number of nodes in the system, and finally improve the effect of PAPR reduction, and on the other hand, through improving threshold search to increase the processing efficiency. Simulation results show that the proposed algorithm compared with the original algorithm has corresponding performance improvement, and on the condition of not affecting the original bit error rate of system, can better reduce PAPR, while decreasing the computational complexity.

Keywords: Lifting scheme, Multicarrier modulation, Orthogonal wavelet packet multiplexing, Partial transmit sequence, Peak to average power ratio, Pruning wavelet packet modulation.

1. INTRODUCTION

With the growing demand for wireless communication broadband network, new communication technology continues to emerge as per the requirement. Compared to orthogonal frequency division multiplexing (OFDM) system, orthogonal wavelet packet multiplexing (OWPM) system has more advantages in these aspects such as utilization rate of spectrum, anti-interference and allocation of sub-channel. Lifting scheme is a more efficient method to implement wavelet transform than Mallat algorithm, so lifting scheme based OWPM system has practical significance for improving transmission rate and operational efficiency.

The theory of lifting wavelet was proposed by Swedens and Daubechies in the mid 90’s, which is a kind of simple and general wavelet construction scheme that does not depend on the Fourier transform [1]. It has the advantages of simple structure, low computational complexity, in situ operation and more storage space. Compared with the Mallat algorithm, the lifting scheme is an efficient method to implement the wavelet transform [2]. Therefore, lifting scheme based orthogonal wavelet packet multiplexing system has practical significance for improving the transmission rate and the operational efficiency.

However, as a new technology of multicarrier communication, LOWPM system also has the problem of high PAPR which is generated by phase accumulation of multiple subcarriers at some moment, requiring very large linear range of front-end power amplifier. If the requirement is not met, once the peak of signal reaches the non-linear range of power amplifier, signal distortion and intermodulation interference between subcarriers can occur, resulting in the destruction of the orthogonality among subcarriers, ultimately reducing the transmitting performance of the system. Therefore, it is necessary to find a method to reduce PAPR effectively [3].

There is an extensive amount of literature studying the problem of PAPR. Literature [4-7] attempted to solve the problem from the optimization of wavelet packet modulation. Some studies [8] and [9] present the method of probability to reduce PAPR, not only to affect the BER performance of system but to have high complexity, such as PTS technology. Methods of pre-distortion [10] and [11], which implement nonlinear transform to the demodulated signals to decrease PAPR such as the method of µ-law companding transform are also studied.

Aiming at the shortcomings of PTS technology, this paper presented corresponding improved schemes. The proposed methods can better reduce PAPR of LOWPM system and decrease the computational complexity, while not affecting the original BER performance.

2. LOWPM SYSTEM AND PEAK TO AVERAGE POWER RATIO

2.1. Lifting Scheme Based Orthogonal Wavelet Packet Multiplexing System

Using inverse lifting wavelet transform (ILWT) and lifting wavelet transform (LWT) to implement modulation and demodulation of multicarrier communication not only overcomes the weaknesses that the OFDM system is very sensi-
packet functions; tal signal scale function of scale space the following equation:

The information bits are firstly grouped and mapped into MPSK or M-QAM. Then the serial data stream is transformed to M parallel lines, where M is the number of subcarrier which is dependent on the channel state. The obtained M lines of signals can be modulated through ILWT. So the signal of time domain to be transmitted can be calculated by the following equation:

\[ d(n) = \sum_{(l,m)\in T} \sum_{n} s_{l,m}(n) \phi_{l,m}(t-nT) e^{j2\pi f_c t} \]  

Where, \( \phi_{l,m} \) represents the basis function of the \( m \) branch of the \( l \) layer of wavelet packet decomposition, namely the scale function of scale space \( V_l \); \( x_{l,m}(n) \) represents the digital signal modulating the scale function \( \phi_{l,m} \) at node \((l,m)\); \( T \) represents the collection of sequence \((l,m)\) of wavelet packet functions; \( f_c \) is the carrier frequency.

The orthogonality between sub carriers is expressed as,

\[ \phi_{l,m}(t-j), \phi_{l,m}(t-k) = \delta_{j,k}, \quad j, k \in Z \]  

\[ \phi_{l,2m}(t-j), \phi_{l,2m+1}(t-k) = \delta_{j,k}, \quad j, k \in Z \]  

Fig. (1). Block diagram of wavelet packet multicarrier modulation system based on lift scheme.

Formula 2 indicates that wavelet packet functions are self orthogonal for integer translation, satisfying that this condition can eliminate the inter symbol interference. Formula 3 shows that the odd and even subscript wavelet packets function is orthogonal, satisfying that this condition can eliminate the adjacent channel interference.

2.2. Implementation of Lifting Wavelet Transform

Lifting wavelet transform is divided into three steps that are split, predicted and updated, and the corresponding inverse transform is composed of inverse update, inverse predict and merge. Any discrete wavelet transform of two channel filter groups with finite length can be decomposed into a series of simple lifting steps, so the wavelet lifting algorithm can replace the Mallat algorithm. It is possible to realize multistage filter groups using multiple lifting steps. In LOWPM system, Daubechies (9, 7) lifting wavelet is used to realize the modulation and demodulation of multicarrier signal, which includes two steps prediction and two steps updat-
ing. The structure of (9, 7) lifting wavelet transform is shown in Fig. (2).

Split: input signal is split as even sequence \( s(2n) \) and odd sequence \( s(2n+1) \), this process is also called lazy wavelet transform.

Predict: Using even sequence to predict odd sequence and obtaining the new odd sequence, is also called the prediction error.

\[
d(2n+1) = s(2n+1) + \alpha [s(2n) + s(2n+2)]
\]

(4)

Where, the prediction operator \( \alpha = -1.586134342 \). After the first lifting, new odd sequences are constituted by the error which is also called the wavelet coefficient.

Update: updating even sequence using the prediction error.

\[
c(2n) = s(2n) + \beta [d(2n-1) + d(2n+1)]
\]

(5)

Where, the update operator \( \beta = -0.052980118 \). The updated result is approximation of original signal, namely the scale coefficient; the step which generates new even sequences is also called dual lifting.

The second prediction: using new even sequence to predict odd sequence generated by the first step and gaining new prediction error is also the final odd sequence before scale transformation.

\[
d(2n+1) = s(2n+1) + \gamma [c(2n) + c(2n+2)]
\]

(6)

The second update: using the newest prediction error to update even sequence generated by the second step

\[
c(2n) = c(2n) + \delta [d(2n-1) + d(2n+1)]
\]

(7)

Transform of scale:

\[
c(2n) = c(2n) / k , \quad d(2n+1) = k \times d(2n+1)
\]

(8)

Where, \( \gamma = 0.882911075 \), \( \delta = 0.443506852 \), and the proportion coefficient is \( k = 1.149604398 \).

The reverse execution of the above process and inverse addition and subtraction can implement the inverse wavelet transform, as shown in Fig. (2).

2.3. Peak to Average Power Ratio

Since the probability of occurrence of signal peak is relatively small, so the ratio of signal peak power to average power is often used to represent signal distribution of multilayer system. PAPR is defined as follows [12, 13]:

\[
PAPR = 10 \log_{10} \left( \frac{\max|S(t)|}{E[|S(t)|]} \right)(dB)
\]

(9)

Where, \( E[|S(t)|] \) is the average power of multilayer signal.

Under normal circumstances, the complementary cumulative distribution function (CCDF) is used to measure the distribution of PAPR within the LOWPM system, which is defined as:

\[
CCDF\left(PAPR_0\right) = P\left\{ PAPR(dB) > PAPR_0 \right\}
\]

(10)

Where, \( PAPR_0 \) is the threshold of PAPR and \( P\{ \} \) is the probability distribution of signal.

3. PAPR REDUCTION SCHEMES FOR LOWPM SYSTEM

3.1. Pruning Wavelet Packet Modulation

Wavelet packet modulation is a type of modulation scheme using wavelet packet basis function as subcarrier. Since different wavelet packet bases have different time frequency localization abilities, these can reflect the different signal characteristics, therefore, for a given signal, it is crucial to choose a wavelet packet basis with good performance to express the characteristics of signal.

Wavelet packet modulation can use binary tree form to indicate figuratively. Because of the diversity of the binary tree structure of the wavelet packet, the full tree structure is only one of them, as shown in Fig. (3). As the full tree of wavelet packet structure has high complexity and a certain amount of redundancy, so it is needed to prune the full tree structure to find the optimal binary tree structure of the wavelet packet. Its essence is to merge the adjacent two or more child nodes into a new node by pruning the unnecessary sub-tree to construct the optimal binary tree structure. At the same time, this merger of nodes does not lose and influence the information stored in the pruning trees. For pruning wavelet packet modulation, it has been proved [14] that for the merged nodes, the more close to the root node (0, 0) with higher frequency selection, the better the effect to reduce PAPR; the more close to the terminal nodes with lower frequency selection, the worse the effect to reduce PAPR, so through appropriate pruning, the purpose of reducing PAPR can be achieved.

Figure (3). Three-level binary tree structure of full-tree wavelet packet.

Under normal circumstances, a three-level wavelet packet decomposition tree can be taken and cost function can be used as a measure. Through pruning the full tree of the wavelet packet, the optimal binary tree structure of wavelet packet can be obtained, namely the optimal wavelet packet basis.

Here, a bottom-up fast searching method is used. First, by using the wavelet packet rapid decomposition algorithm, the signal’s wavelet packet functions on each subspace is calculated; second, the value of information cost function at
each node in the wavelet packet tree is calculated; then, from the lowest level start of wavelet packet, comparing the value of cost function of parent node with the sum of the value of cost function of its two child nodes, if the sum of the value of cost function of the two child nodes is greater than that of the parent node, the parent node is marked by “*”, otherwise, it is not marked but the sum of the values of cost function of two child nodes is used to replace the value of cost function of the parent node, followed by enclosing the original value of information cost function of parent node in parentheses. After that, the value outside the parentheses is only considered and the former step is repeated until the topmost level is reached. At last, from the topmost level start of the wavelet packet tree, the nodes nearest to the tree root marked by “*” from top to bottom are retained, and at the same time, the sub-trees are cut present in the root of those nodes. To this point, the optimal wavelet packet basis can be sought. For three-level decomposition of wavelet packet example, the final results of the best wavelet packet tree are drawn as shown in Fig. (4). Where, the value in round brackets is the cost function value of original parent node, the value outside the round brackets is the sum of the value of cost function of the two child nodes, * denotes the selected node and the gray box indicates the best wavelet packet basis to be sought.

By comparing Figs. (3) and (4), it can be known that the pruned binary tree structure on the basis of meeting the system requirements greatly reduces the implementation complexity of the system, and due to the combination of nodes, the input sub-carriers are also reduced correspondingly, following a low probability of generating large peaks by system phase accumulation. Ultimately, the PAPR is reduced.

![Fig. (4). The optimal wavelet packet tree structure (pruning wavelet packet).](image)

3.2. Basic Principle of PTS Technology

PTS is the method which reduces the probability of occurrence of transmission signal peak by using linear transformation. Fig. (5) shows the Principle diagram of PTS technology.

In Fig. (5), input signal $X$ is divided into several non-overlapping sub vectors, namely $X = \sum_{i=1}^{V} X_i$, where $V$ is the number of packets, $X_i = \{X_i^1, X_i^2, ..., X_i^N\}$ is the sub vector, $N$ is the length of each sub vector. In order to keep each sub vector at the same length, zero needs to be added for some sub vector.

Then performing inverse discrete wavelet packet transform to each sub vector, the signals can be obtained

$$x = \sum_{i=1}^{V} b_i \cdot x_i \tag{11}$$

Where, $\{b_i = (b_i^0, b_i^1, ..., b_i^{V-1}) \mid l = 1, 2, ..., V\}$ refers to the rotation factor of sub vectors, $b_i^l = \exp(j\phi_i^l)$, $\phi_i \in [0, 2\pi]$, which are mutually statistically independent. The selection for $b_i$ is needed to make the value of PAPR minimum, so the optimal weighting coefficients must meet the condition:

$$\{b_2, b_3, ..., b_{V-1}\} = \arg\min_{[b_2, b_3, ..., b_{V-1}]} \left\{ \max_{l=2,3, ..., V} \left( \sum_{i=1}^{V} b_i \cdot X_i \right) \right\}.$$ 

The technology of PTS can effectively reduce PAPR and does not produce distortion as it is a method of linear processing, having high computational complexity. Therefore, the research for PTS technology with low complexity is very necessary. The paper is mainly focused on how to search the optimal rotation factor to reduce the computational complexity.

3.3. Improved Threshold Search Method

Conventional searching method is an optimal method, but the high computational complexity makes it difficult in

![Fig. (5). Block diagram of PTS technology at sending end.](image)
application. Cimini proposed a suboptimal algorithm of Cimini searching method, which is a good solution to the high computational complexity, but whose performance of PAPR suppression decreases. In order to solve the problem, a threshold searching method was proposed [15] which can avoid the searching of phase factor being trapped in the local optima when the threshold gets smaller, thus increasing the probability of searching for the optimal phase factor sequence, eventually greatly enhancing the performance of PAPR suppression.

Pointing at the deficiency in the study [15], the paper put forward a method of improved threshold searching which increases the module block of acceptable probability to reduce the cycle operation of system in the premise of guaranteeing the performance of PAPR suppression, so as to achieve fast convergence of the optimal value, finally reducing the complexity of the system.

Supposing the peak to average power ratio of the current phase factor sequence is $P_{APR}$, the acceptable probability is [16],

$$P = \begin{cases} 
1, & \lambda \leq 0 \\
\lambda^{-T}, & \lambda > 0 
\end{cases} \tag{12}$$

Where $\lambda = \frac{P_{APR}}{\text{threshold}}$, $T$ is a variable decreasing with the increase of the number of loop iterations. In the paper, processing for $T$ by geometry cooling of the simulated annealing [17], that is $T_{i+1} = nT_i$, where $i$ is the number of loop iterations, $n = (T_j / T_r)^{1/3}$ is the declining rate of $T$, $j$ is the total number of loop iterations, $T_j$ and $T_r$ are the initial and final values of $T$, respectively.

The principle of the algorithm is: if $\lambda \leq 0$, then $P = 1$, which indicates accepting the current phase factor sequence and stopping the search; if $\lambda > 0$, calculating the value of acceptable probability $P = \lambda^{-T}$, at the same time randomly generating a number $r$ which is evenly distributed in $(0,1)$. It can be seen from formula 9 that the greater the value of $\lambda$, the smaller the value of $P$. So, as long as the value of $\lambda$ is in an acceptable range, namely as long as $P > 0.5$, the probability of $r < P$ will be greater than that of $r > P$, accepting the phase factor sequence and stopping the search, otherwise continuing the search. Processing flow of the method of improved threshold searching is shown in Fig. (6).

Specific steps of the algorithm are as follows:

1. At the start, all phase factor $b_i = 1 \{ l = 1, 2, ..., V \}$, calculating the peak to average ratio $P_{APR}$.

2. Supposing $\text{threshold} = 4dB$. If $P_{APR} \leq \text{threshold}$, the search is stopped; otherwise cycle search and the process is: let $i = 1$, $l = l + 1$, where $i$ is the cycle number, turning $b_i = -b_i$, and the peak to average ratio $P_{APR}$ is recalculated. If $P_{APR} \leq \text{threshold}$, the search is stopped; otherwise randomly a number $r$ is generated which is evenly distributed in $(0,1)$, and the acceptable probability $P = \lambda^{-T}$ is computed; if $r < P$, the search is stopped; otherwise $b_i = -b_i$ is recovered, at the same time turning $b_{i+1} = -b_{i+1}$ and computing the current peak to average ration $P_{APR}$, until the optimization of all phase factors $b_i$, $l = 2, 3, ..., V$ is finished. The remaining steps are the same as the original method of threshold search, finally getting the optimized phase factor sequence.

$$\{ b_1, b_2, \cdots, b_V \} = \min (P_{APR_1}, P_{APR_2}, \cdots, P_{APR_V})$$

The improved method of threshold search adds a module block of acceptable probability, which decides whether to accept the current phase factor sequence by comparing the value of a random number $r$ and acceptable probability $P$, so as to prevent cycle operating when the selection of threshold is too low, finally to achieve fast convergence to the optimal value.

3.4. Improved PTS Technology Based on Pruning WPM and Improved Threshold Search Method

Conventional PTS technology uses full-tree wavelet packet modulation (WPM), requiring not only high cost of hardware, but also a great amount of computation. Based on the literature [14], a new technology was proposed to reduce the PAPR. It can improve the wavelet packet modulation links of PTS system by properly pruning full-tree wavelet packet structure which reduces the number of internal nodes and subcarrier, and thus reduces the probability of generating large peaks by phase accumulation, ultimately improving the effect of PAPR reduction. WPM in Fig. (5) is replaced by pruning wavelet packet modulation which uses the binary tree structure of Fig. (4) and converts it into a two-channel filter bank model.

Improved PTS technology based on pruning wavelet packet modulation and improved threshold search method is shown in Fig. (7).

4. SYSTEM SIMULATIONS AND ANALYSIS

In order to prove the effectiveness and feasibility of the proposed algorithms, simulations for the performance of LOWPM system are carried out. Simulation parameters are shown in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The length of original signal sequence</td>
<td>N=2048</td>
</tr>
<tr>
<td>The number of total symbols</td>
<td>10000</td>
</tr>
<tr>
<td>Lifting Wavelet packet function</td>
<td>(9, 7) lifting wavelet</td>
</tr>
<tr>
<td>Mapping mode</td>
<td>4QAM</td>
</tr>
<tr>
<td>Number of sub channel</td>
<td>8</td>
</tr>
<tr>
<td>Comprising factor</td>
<td>$\mu = 5$</td>
</tr>
</tbody>
</table>
Fig. (6). PTS algorithm based on improved threshold search method.
Fig. (7). Block diagram of PTS technology based on pruning WPM and improved threshold search method at sending end.

![Block Diagram](image)

Fig. (8). PAPR simulation for LOWPM system based on pruning wavelet packet and full-tree wavelet packet.

4.1. Simulation of PAPR for LOWPM System Based on Pruning Wavelet Packet and Full-tree Wavelet Packet

By pruning the full tree of wavelet packet reasonably, the best wavelet packet basis can be found. The multiplexing system with pruning wavelet packet modulation is shown in Fig. (8) and the simulation results of PAPR of the two kinds of wavelet packet tree structure are shown in Fig. (8).

The results show that, after pruning the full-tree wavelet packet tree structure, the effect is better and the value of PAPR can be more efficiently reduced. With the original full tree structure, not only the system complexity can be reduced, also the performance is not damaged.

4.2. Simulation of PAPR Suppression for PTS Technology Based on the Improved Threshold Search

Simulations for the performance of PAPR suppression of PTS technology and for the BER performance of LOWPM system based on PTS technology are as follows. In the experiment, signal packet uses random segmentation, and the segmentation number \( V = 4 \), while phase rotation factor \( b_i \in \{+1,-1\} \). PTS technology based on the improved threshold search algorithm selects signal variance as the initial value \( T_s \), making \( T_f / T_s = 0.2 \), \( I = 9 \).

Fig. (9) shows that the performance of PAPR suppression has a certain degree of improvement after the signals are processed by PTS technology, and Cimini search for the performance improvement of PAPR reduction is the worst, while the improving performance of the other three kinds of search algorithm is almost the same.

Fig. (10) presents the statistical results of 1000 times simulation in Gauss white noise channel. As can be seen, the BER curve of LOWPM system based on PTS technology is almost the same as that of the LOWPM system without PTS.
Fig. (9). Comparison of the distribution curves of PAPR for PTS technology based on different search methods.

Fig. (10). Comparison of BER for PTS based different search methods.
Table 2. The number of loop iterations of various search algorithms.

<table>
<thead>
<tr>
<th>No.</th>
<th>The Minimum Number of Iterations</th>
<th>The Maximum Number of Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of segmentation module</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Original search</td>
<td>$2^4$</td>
<td>$2^8$</td>
</tr>
<tr>
<td>Cimini search</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>Threshold search(threshold=4dB)</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>Improved threshold search(threshold=4dB)</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

Fig. (11). Comparison of the distribution curves of PAPR for different schemes.

algorithm when SNR is less than 12dB; but when SNR is more than 12dB, the curve shows better BER performance of the improved threshold search algorithm. Simulation results prove that the PTS algorithm can gain accurate demodulation signals on the condition of not affecting the BER performance of the system.

The measure of complexity only considers the number of calculation times of PAPR in the paper. Table 2 shows that the complexity of the three kinds of methods of Cimini search, threshold search and the improved threshold search can be greatly reduced than that of the original search method.

4.3. Simulation for Improved PTS Technology Based on Pruning WPM and Improved Threshold Search Method

By the previous simulation results, it can be known that the pruning wavelet packet modulation and PTS technique based on improved threshold search, both have a very good effect in reducing PAPR. So, if combining this two technologies, applied in the wavelet packet multicarrier modulation system, the effect then?

Fig. (11) shows that the LOWPM system based on the combined algorithm has better effect of PAPR suppression than that of the system only based on the pruning wavelet packet modulation or only based on the improved threshold search method.

Fig. (12) shows the statistical results of simulating 1000 times in Gauss white noise channel. As can be seen, the BER curve of LOWPM system based on the combined algorithm is almost the same as that of the LOWPM system without PTS algorithm when SNR is less than 12dB; but when SNR is more than 12dB, the curve shows better BER performance of the proposed PAPR reduction schemes. Simulation results prove that the PTS algorithm can gain accurate demodulation signals on the condition of not affecting the BER performance of the system.
CONCLUSION

Lifting wavelet packet multicarrier system will become a key technology for future mobile communication because of its higher spectrum efficiency and good resistance to ISI and ICI. However, multi-carrier system has high PAPR. On the basis of studying pruning wavelet packet modulation and the improved PTS technology, this paper presented an improved method of PAPR reduction for LOWPM system. Theoretical analysis and simulation results show that the proposed algorithm can more effectively reduce PAPR and decrease the complexity of process without affecting the system’s BER performance.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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Declared none.

REFERENCES

